OSNOVE ELEKTROTEHNIKE II

zapiski z avditornih vaj

Šolsko leto	2007 / 2008
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Avtor dokumenta	Aljoša Praznik
Sodelavci	Blaž Potočnik, Aljoša Praznik

UREJANJE DOKUMENTA

VERZIJA	01	**REVIZIJA**	01
DATUM	1. 3. 2009		

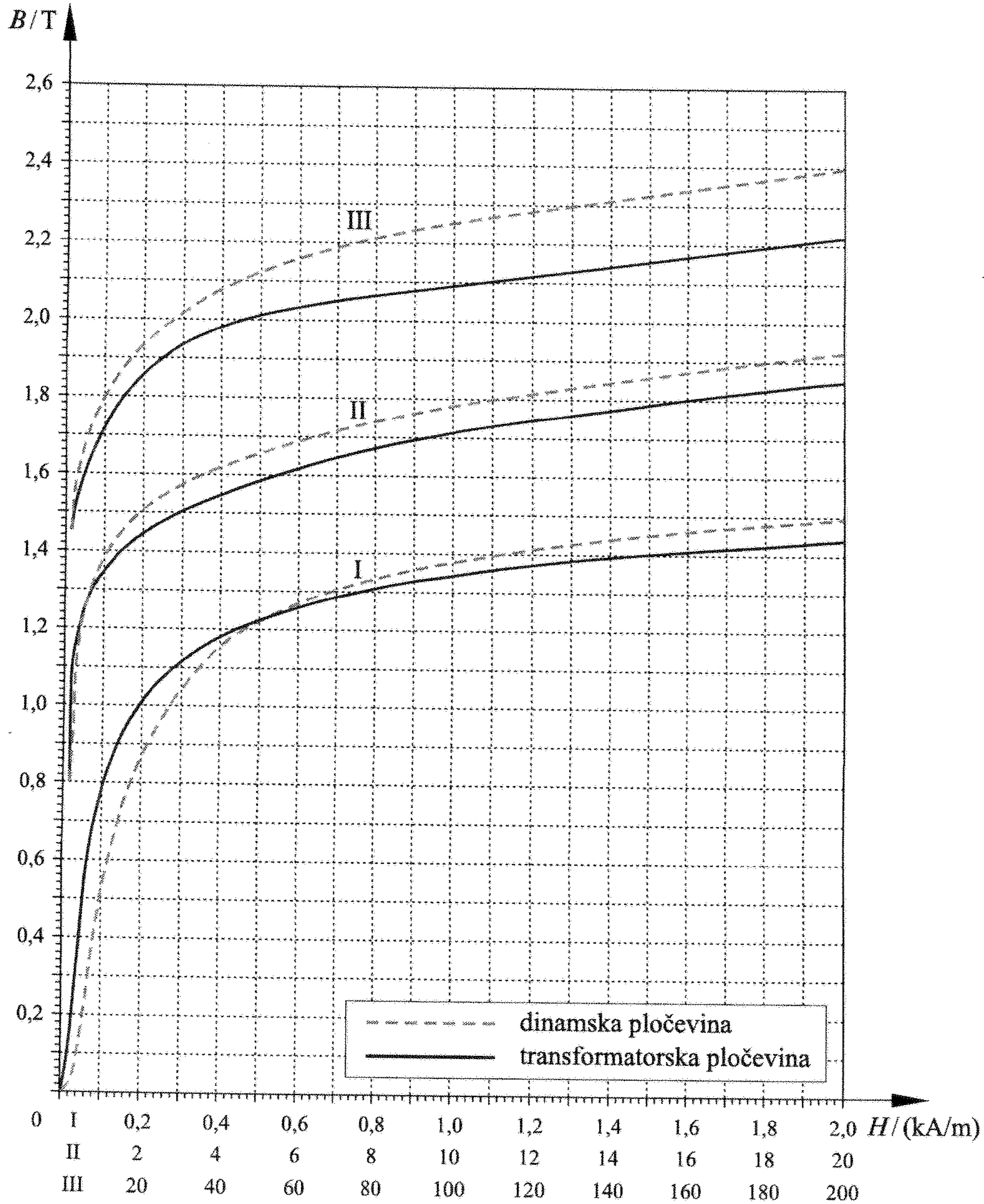
ZADNJI POPRAVLJAL /

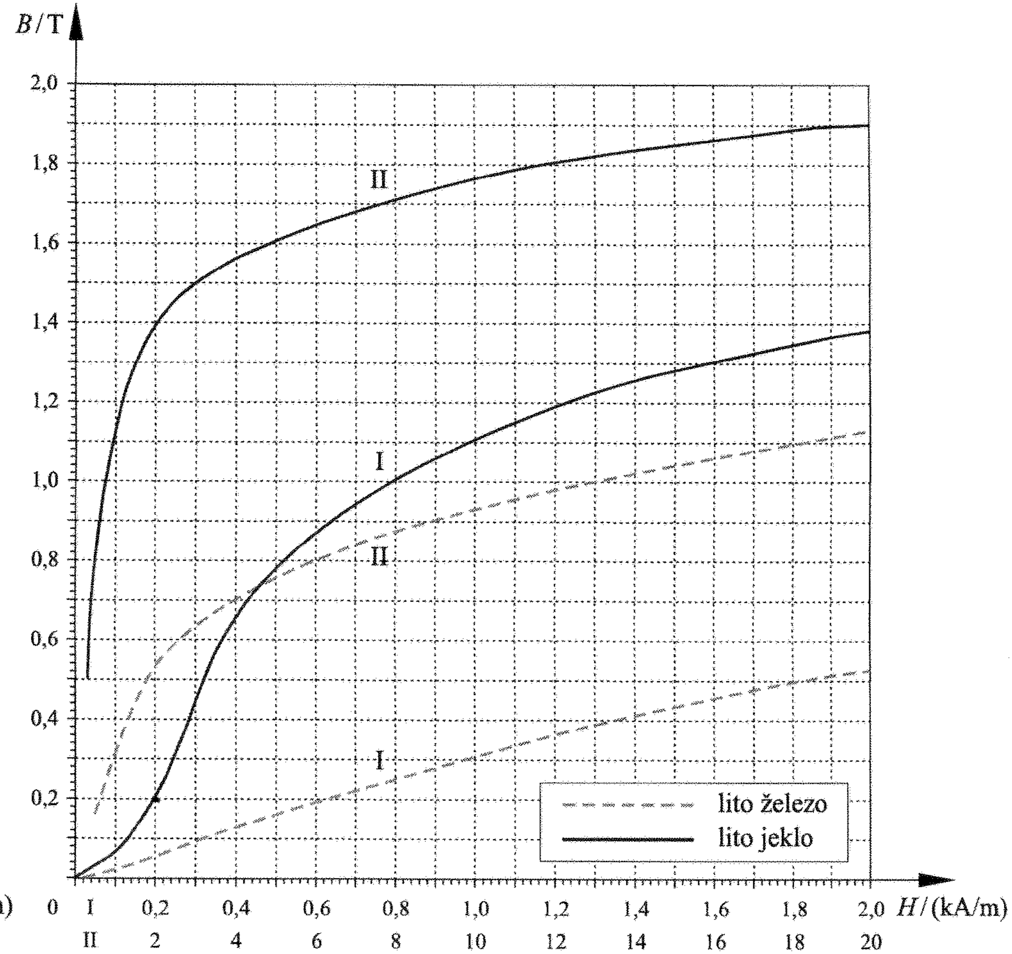
PREGLEDAL Blaž Potočnik, Aljoša Praznik

OPOMBE

POPRAVKI

Srednje krivulje magnetiziranja za mehke magnetne materiale





10.02.2008

Lorentzova sila (na točkasti naboj)

$$\vec{F}_i = Q_i\vec{E} + \underbrace{Q_i\,\vec{v}_i}_{\circ} \times \vec{B}$$

Sila na tokovni element

$$d\vec{F}_i = \underbrace{I_i\,d\vec{l}_i}_{\circ} \times \vec{B}$$ – povzročajo ga vsi ostali tokovi v prostoru

Napotek za izračun $\vec{B}$-ja:

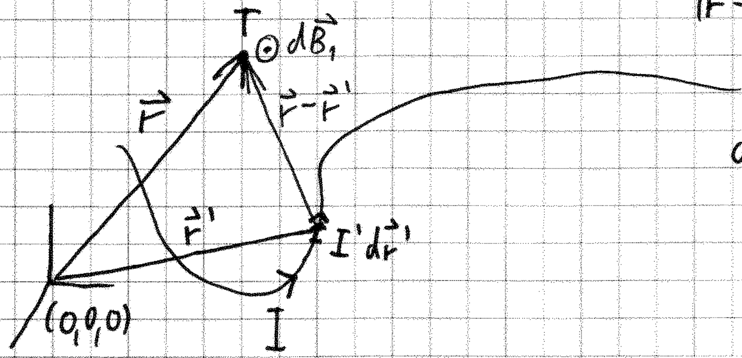
- Biot-Savartov zakon

$$\boxed{B(\vec{r}) = \frac{\mu_0}{4\pi}\int_{\mathcal{L}'} \frac{I'\,d\vec{r}\,' \times (\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}}$$

PRAVILO DESNE ROKE

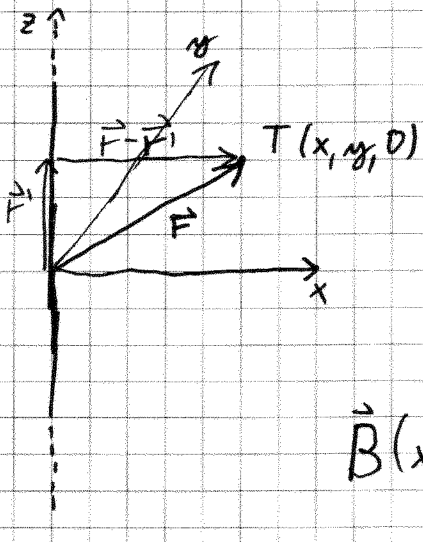
$$;\ dB(\vec{r}) = \frac{\mu_0}{4\pi}\,\frac{\overset{\text{palec}}{I'd\vec{r}\,'} \times \overset{\text{kazalec}}{(\vec{r}-\vec{r}\,')}}{|\vec{r}-\vec{r}\,'|^3}$$

$$\frac{\vec{r}-\vec{r}\,'}{|\vec{r}-\vec{r}\,'|} = \vec{e}_{\vec{r}-\vec{r}\,'}$$



$$dB(\vec{r}) = \frac{\mu_0}{4\pi}\,\frac{I'd\vec{r}\,' \times \vec{e}_{\vec{r}-\vec{r}\,'}}{|\vec{r}-\vec{r}\,'|^2}$$

Izpelji izraz za gostoto mag. pretoka $\vec{B}$ v okolici premega vodnika (v osi z) s tokom I (v smeri $\vec{e}_z$).



$$\vec{r} = (x, y, 0)$$
$$\vec{r}\,' = (0, 0, z')$$
$$d\vec{r}\,' = (0, 0, dz')$$
$$\vec{r}-\vec{r}\,' = (x, y, -z')$$
$$|\vec{r}-\vec{r}\,'|^2 = x^2 + y^2 + z'^2$$

$$d\vec{r}\,' \times (\vec{r}-\vec{r}\,') = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 0 & dz' \\ x & y & -z' \end{vmatrix} = dz'(-y, x, 0)$$

$$\vec{B}(x, y, 0) = \frac{\mu_0}{4\pi}\int_{-\infty}^{\infty} \frac{I\cdot(\overset{B_x}{-y}, \overset{B_y}{x}, \overset{B_z}{0})\,dz'}{(x^2+y^2+z'^2)^{3/2}}$$

$$B_x(x,y,0) = \frac{\mu_0}{4\pi} \int_{-\infty}^{\infty} \frac{-y \cdot I' dz'}{(x^2+y^2+z'^2)^{3/2}} = -\frac{\mu_0 I y}{4\pi} 2 \cdot \int_0^{\infty} \frac{dz'}{(x^2+y^2+z'^2)^{3/2}} =$$

$$= -\frac{\mu_0 I y}{2\pi} \frac{z'}{(x^2+y^2) \cdot \sqrt{x^2+y^2+z'^2}} \Bigg|_0^{\infty} = -\frac{\mu_0 I y}{2\pi (x^2+y^2)} \frac{z'}{\sqrt{x^2+y^2+z'^2}} \Bigg|_0^{\infty} =$$

$$= -\frac{\mu_0 I y}{2\pi (x^2+y^2)} [1-0] = \underline{\underline{-\frac{\mu_0 I y}{2\pi (x^2+y^2)}}}$$

$$B_y(x,y,0) = \underline{\underline{\frac{\mu_0 I x}{2\pi (x^2+y^2)}}} \qquad B_z(x,y,0) = \underline{\underline{0}}$$

$$B(x,y,0) = \frac{\mu_0 I}{2\pi (x^2+y^2)} (-y, x, 0) = \frac{\mu_0 I}{2\pi \rho^2} (-y, x, 0) = \frac{\mu_0 I}{2\pi \rho} \underbrace{\left(-\frac{y}{\rho}, \frac{x}{\rho}, 0\right)}_{\substack{(\sin\varphi, \cos\varphi, 0) \\ \vec{e}_\varphi}}$$

$$x^2 + y^2 = \rho^2$$

$$\boxed{\vec{B}(\rho) = \vec{e}_\varphi \frac{\mu_0 I}{2\pi \rho}}$$

$\vec{B}$ v okolici premega vodnika
(tok v $\vec{e}_z$ smeri)

2

V osi z leži premi vodnik s tokom $I = 10\,A$, ki teče v smeri $\vec{e}_z$. Določi $\vec{B}$ v točkah $T_1(2,5,3)\,m$ in $T_2(2,5,-1)\,m$.

$\rho_1 = \sqrt{x^2+y^2}$ • $\vec{B}_1 = \vec{e}_\rho \frac{\mu_0 I}{2\pi\rho}$

$$\vec{B}_1 = \vec{e}_\rho \frac{4\pi \cdot 10 A\, Vs}{10^7\, Am\, \sqrt{4+25}} = 0{,}36 \cdot 10^{-6} \frac{Vs}{m^2} = \underline{\underline{0{,}36\, \mu T}}$$

$$\vec{B}_1 = \vec{B}_2$$

vir (statični B)	tipične vrednosti
motnje v ionosferi	$1\,\mu T$
zemeljsko mag. polje	$20\,\mu T - 150\,\mu T$
sončne pege	$\sim 1\,T$
NMR	$\sim 20\,T$

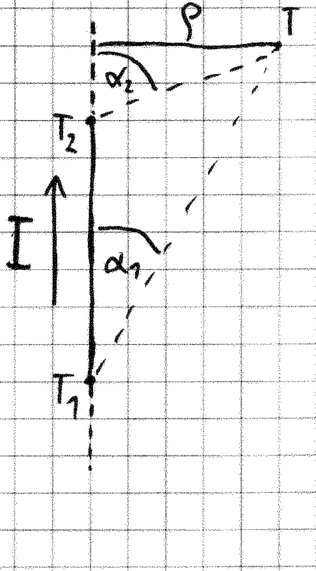
kri v telesu

$B = 20\,T$
$Q = 3{,}2 \cdot 10^{-19}\,As$
$v = 20\,m/s$

$$F = I \cdot l \cdot B$$

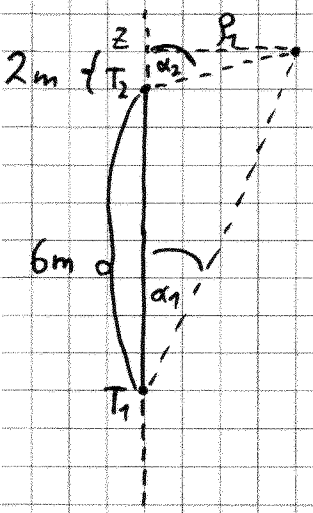
$$F_{KRI} = Q v B = 1{,}2 \cdot 10^{-16}\,N$$

$\vec{B}$ v okolici daljice (tok I v smeri $\vec{e}_z$)



$$\vec{B}(\rho) = \vec{e}_\rho \frac{\mu_0 I}{4\pi\rho} \left(\cos\alpha_1 - \cos\alpha_2\right)$$

V osi z leži med točko $T_1(0,0,-5)\,m$ in $T_2(0,0,1)\,m$ daljica s tokom $I=10\,A$, ki teče v smeri $\vec{e}_z$.
Določi $\vec{B}$ v točkah $P_1(2,5,3)\,m$ in $P_2(2,5,-1)\,m$!



$$\rho_1 = \sqrt{x^2+y^2} = \sqrt{29}\ m$$

$$\cos\alpha_1 = \frac{8}{\sqrt{8^2+29}}$$

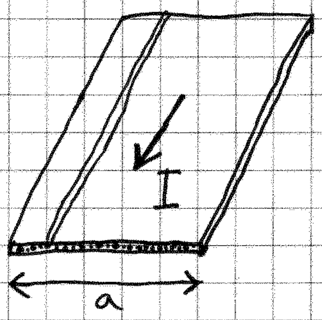
$$\cos\alpha_2 = \frac{2}{\sqrt{2^2+29}}$$

$$\bullet\ \vec{B}(\rho) = \vec{e}_\varphi \frac{\mu_0 I}{4\pi\rho}(\cos\alpha_1 - \cos\alpha_2)$$

$$\vec{B}_1 = \vec{e}_\varphi \frac{4\pi\cdot 10^{-7}\cdot 10}{4\pi\cdot\sqrt{29}}(0{,}83-0{,}35)$$

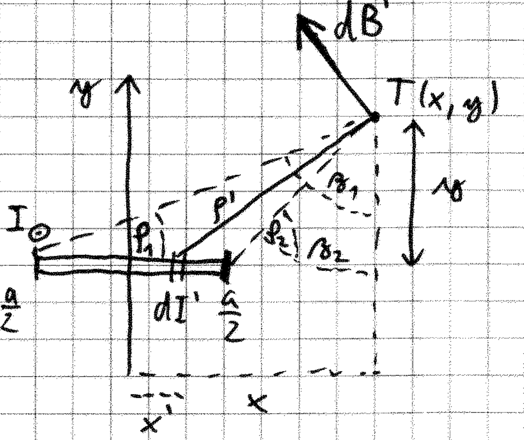
$$\vec{B}_1 = \vec{e}_\varphi \frac{10^{-6}}{\sqrt{29}} = 89\,nT\,\vec{e}_\varphi$$

Izpelji izraz za $\vec{B}$ v okolici tračnega vodnika s tokom I
(enakomerne gostote v prerezu, v smeri $\vec{e}_z$)



$$K = \frac{I}{a} \Rightarrow I = K\cdot a$$

$$dI = K\cdot dx'$$



smer toka

od toka proti točki

$$d\vec{B}'(T) = \vec{e}_{\varphi'} \frac{\mu_0\, dI'}{2\pi\rho'}\ ;\ \vec{e}_{\varphi'} = \vec{e}_z \times \frac{\vec{\rho}'}{|\vec{\rho}'|}$$

$$\vec{\rho}' = (x-x',\ y)$$

$$\rho'^2 = (x-x')^2 + y^2$$

$$\vec{e}_{\varphi'} = (0,0,1)\times(x-x',\ y,\ 0)\,\frac{1}{\rho'} =$$

$$\vec{e}_{\varphi'} = \frac{1}{\rho'}\begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 0 & 1 \\ x-x' & y & 0 \end{vmatrix} = \frac{1}{\rho'}(-y,\ x-x')$$

$$\vec{B}(T) = \int_{-\frac{a}{2}}^{\frac{a}{2}} \frac{1}{\rho'}\left(-y,\ x-x'\right) \cdot \frac{\mu_0 \cdot K \cdot dx'}{2\pi\rho'} = \frac{\mu_0 K}{2\pi} \int_{-\frac{a}{2}}^{\frac{a}{2}} \frac{(-y,\ x-x')\,dx'}{(x-x')^2+y^2} = \dots\dots$$

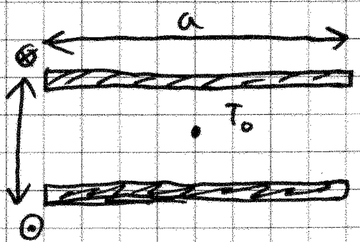
$$B_x(T) = \frac{\mu_0 K}{2\pi}\Big(\underbrace{\beta_2}_{(90-\varphi_2)} - \underbrace{\beta_1}_{(90-\varphi_1)}\Big)$$

$$\underline{\underline{B_x(T) = \frac{\mu_0 K}{2\pi}\left(\varphi_1 - \varphi_2\right)}}$$

$$\vec{B}(x,y) = \frac{\mu_0 K}{2\pi}\left(\varphi_1 - \varphi_2,\ \ln\frac{\rho_1}{\rho_2}\right)$$

$$\underline{\underline{B_y(T) = \frac{\mu_0 K}{2\pi} \ln\frac{\rho_1}{\rho_2}}}$$

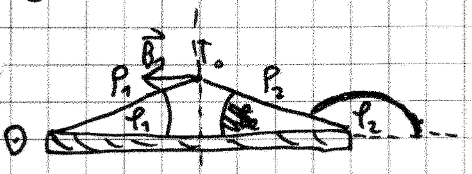
Tanka tračna vodnika ($a = 16\,mm$, $b = 4\,mm$) vodita tok $I = 2A$.
Določite vektor $\vec{B}$ v točki T_0.



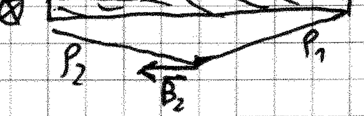
$$K_1 = K_2 = \frac{2\,A}{16\,mm} = 125\,\frac{A}{m}$$

$$\vec{B} = 2\vec{B}_1 = 2 \cdot \vec{e}_x \frac{\mu_0 K}{2\pi}\left(\varphi_1 - \varphi_2\right)$$

$$= \vec{e}_x \frac{\mu_0 K}{\pi}\left(\mathrm{arctg}\frac{b}{a} - \left(\pi - \mathrm{arctg}\frac{b}{a}\right)\right) = -133\,\mu T$$



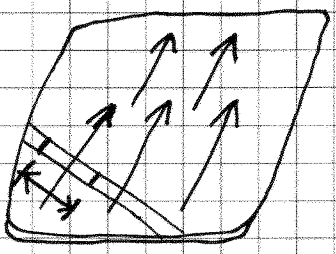
$$\ln\frac{\rho_1}{\rho_2} = 0$$



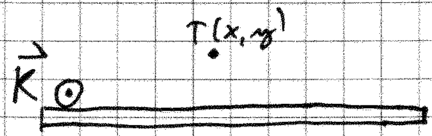
$$\vec{B}_1 = \vec{B}_2$$

27.02.2008

Tokovna ravnina (obloga, plast, ...)



$\vec{K} = \vec{e}_i \cdot \frac{\Delta I}{\Delta l}$ – koliko A na prečno ravnino



$\varphi_1 \to 0$
$\varphi_2 \to \pi$

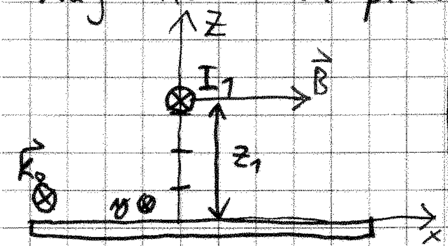
$\frac{\rho_1}{\rho_2} \to 1$

$$\vec{B}(x,y) = \frac{\mu_0 K_z}{2\pi}\left(0 - \pi, \overset{0}{\ln 1}\right)$$

$$\boxed{\vec{B} = -\vec{e}_x \frac{\mu_0 K_z}{2}}$$ (za točke $y>0$)

$$\vec{B} = +\vec{e}_x \frac{\mu_0 K_z}{2}$$ (za točke $y<0$)

Tokovna ravnina (obloga) $z=0$ vodi tok $\vec{K}_0 = \vec{e}_y 2 \frac{A}{m}$. Vzporedno z njo poteka na višini $z_1 = 3m$ vodnik s tokom $I_1 = 6A$ v $\vec{e}_y$ smeri. Določite mag silo/m na premi vodnik



Amperova sila

$$\vec{F}' = I' \vec{l}' \times \vec{B} \quad ; \quad \vec{l}' = l' \cdot \vec{e}_l$$

$$\frac{\vec{F}}{l'} = I' \vec{e}_l \times \vec{B}$$

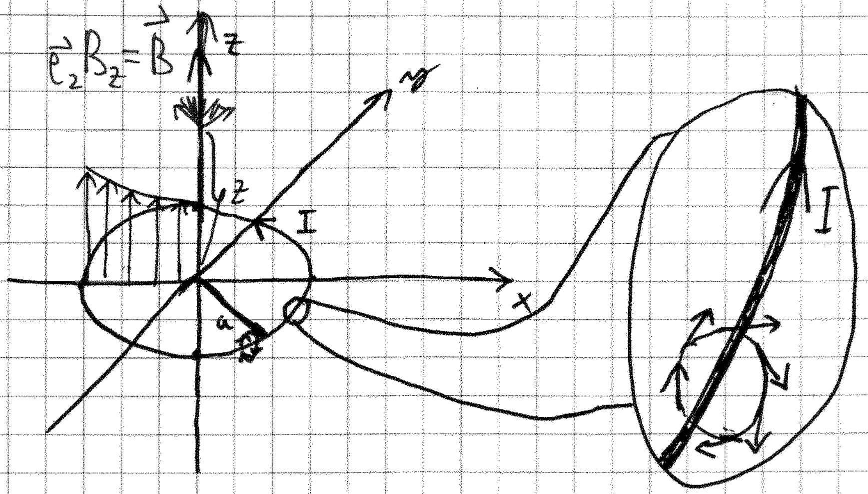
$$\vec{f}' = I' \vec{e}_l \times \vec{B}$$

$\vec{B}$ na mestu vodnika (od ravnine):

$$\vec{B} = \vec{e}_x \cdot \frac{\mu_0 K_0}{2}$$

$$\vec{f}_1 = I_1 \cdot \vec{e}_y \times \vec{B} = I_1 \cdot \vec{e}_y \times \vec{e}_x \frac{\mu_0 K_0}{2} = -\vec{e}_z \frac{1}{2} \mu_0 I_1 K_0 = -\vec{e}_z \cdot 7{,}5 \, \mu N/m$$

Krožna zanka (v xy ravnini, polmera a, središče (0,0,0), tok I teče v $\vec{e}_\varphi$ samo v osi!)

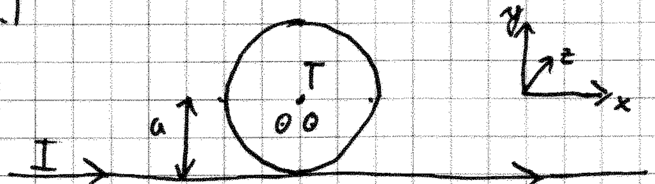


~~Samo za~~

$\vec{B}(z) = \vec{e}_z \frac{\mu_0 I}{2} \frac{a^2}{(a^2+z^2)^{3/2}}$	$\vec{B}(z=0) = \vec{e}_z \frac{\mu_0 I}{2a}$

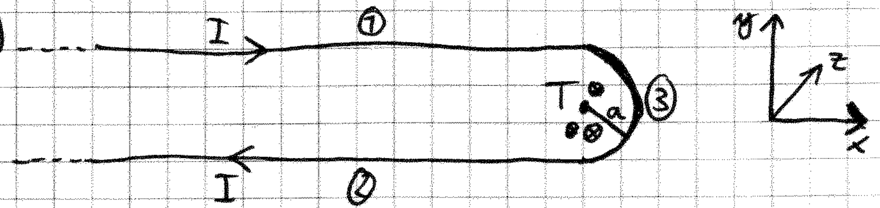
Za posamezne primere tokovodnikov iz ravnih in krožnih delov polmera a določite gostoto B v danih točkah.

a.)



$$\vec{B}(T) = \vec{B}_{premica}(T) + \vec{B}_{krož.\ ovoj}(T) = \vec{e}_z \frac{\mu_0 I}{2\pi a} + \vec{e}_z \frac{\mu_0 I}{2a} = \vec{e}_z \frac{\mu_0 I}{2a}\left[1 + \frac{1}{\pi}\right]$$

b.)

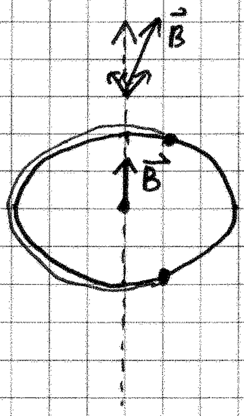


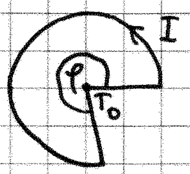
$$\vec{B}(T) = \vec{B}_1(T) + \vec{B}_2(T) + \vec{B}_3(T)$$

$$\vec{B}_1(T) = -\vec{e}_z \frac{\mu_0 I}{4\pi a}\left(\overset{1}{\cos 0^\circ} - \overset{0}{\cos 90^\circ}\right) \qquad \vec{B}_2(T) = -\vec{e}_z \frac{\mu_0 I}{4\pi a}\left(\overset{0}{\cos 90^\circ} - \overset{-1}{\cos 180^\circ}\right)$$

$$\vec{B}_3(T) = -\vec{e}_z \frac{\mu_0 I}{2a}\left(\frac{180^\circ}{360^\circ}\right)$$

Samo za središče zanke velja

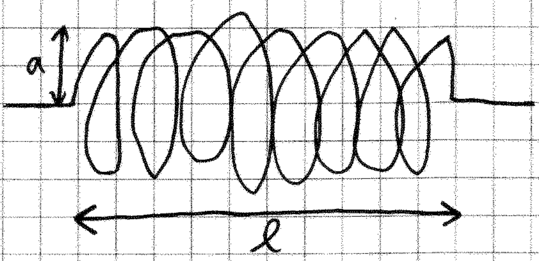




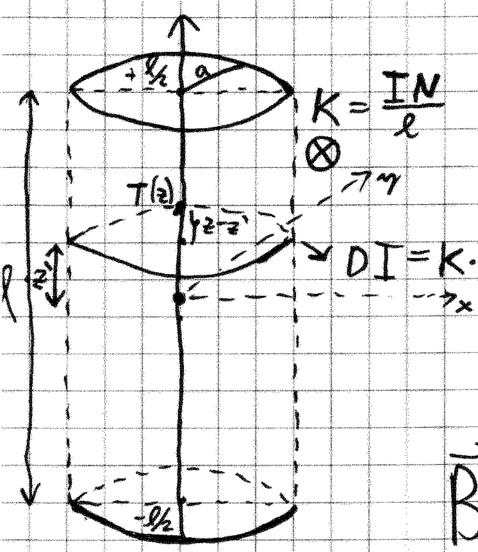
$$\boxed{\vec{B}(z=0) = \vec{e}_z \frac{\mu_0 I}{2a} \cdot \left(\frac{\varphi}{2\pi}\right)}$$

$$\underline{\vec{B}(T) = -\vec{e}_z \frac{\mu_0 I}{4a}\left[1 + \frac{2}{\pi}\right]}$$

Tuljava



Izpelji izraz za splošno tuljavo (model: N krožnih zank)



za ovoj (diskreten):

$$\vec{B}(z) = \vec{e}_z \frac{\mu_0 I}{2} \frac{a^2}{(a^2 + (z-z')^2)^{3/2}}$$

za infinit. dI:

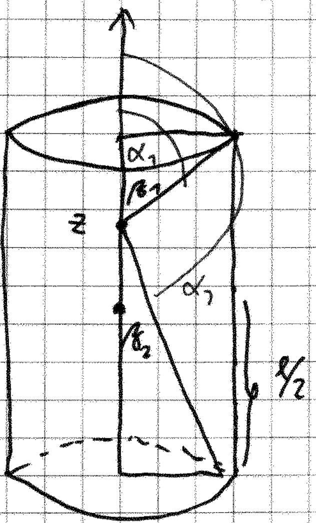
$$d\vec{B}(z) = \vec{e}_z \frac{\mu_0 dI'}{2} \frac{a^2}{(a^2 + (z-z')^2)^{3/2}}$$

$$\vec{B}(z) = \int \vec{e}_z \frac{\mu_0 dI'}{2} \frac{a^2}{(a^2 + (z-z')^2)^{3/2}}$$

$$\vec{B}(z) = \vec{e}_z \frac{\mu_0 a^2}{2} \int_{-\frac{l}{2}}^{+\frac{l}{2}} \frac{K \cdot dz'}{(a^2 + (z-z')^2)^{3/2}}$$

$$\vec{B}(z) = \vec{e}_z \frac{\mu_0 K a^2}{2} \int_{-\frac{l}{2}}^{+\frac{l}{2}} \frac{dz'}{(a^2+(z-z')^2)^{3/2}} \ldots\ldots$$

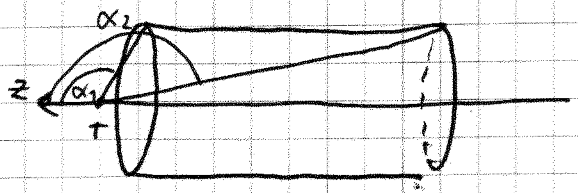
$$\vec{B}(z) = \vec{e}_z \frac{\mu_0 N I}{2l} \left[\cos\beta_1 + \cos\beta_2\right]$$



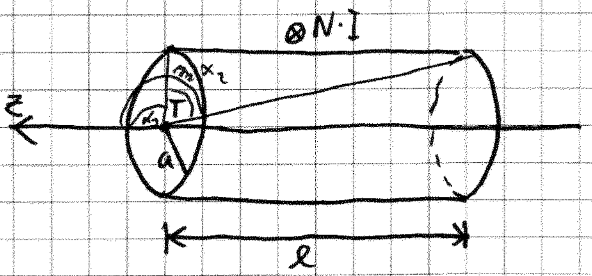
$$\alpha_1 = \beta_1$$

$$\alpha_2 = \pi - \beta_2 \Rightarrow \beta_2 = \pi - \alpha_2$$

$$\boxed{\vec{B}(z) = \vec{e}_z \frac{\mu_0 N I}{2l} \left[\cos\alpha_1 - \cos\alpha_2\right]}$$



Tuljava ($a = 1\,cm$, $l = 4\,cm$, $N = 1000$) ima $I = 2\,A$. Določite B v točki T na skrajnem levem robu daljave.



$$\alpha_1 = 90^\circ$$

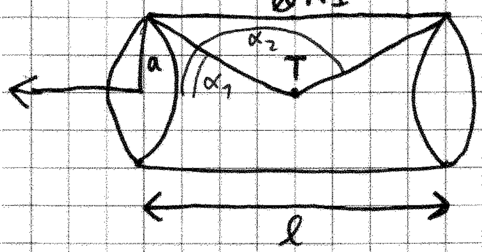
$$\tan\beta_2 = \frac{l}{a} = 76^\circ$$

$$\alpha_2 = 166^\circ$$

$$\vec{B}(z) = \vec{e}_z \frac{\mu_0 N I}{2l}\left[\overset{0}{\cos\alpha_1} - \cos\alpha_2\right] = \vec{e}_z \frac{4\pi\cdot 10^{-7}\cdot 1000\cdot \cancel{2}}{\cancel{2}\cdot 0{,}04\,m}\left[-\cos 166^\circ\right] =$$

$$\vec{B}(z) = \vec{e}_z\; 30{,}5\,mT$$

Z enoslojno superprevodno tuljavo dolžine $l = 2\,m$ in premera $2a = 1\,m$, ki ima $N = 100$ ovojev, želimo v njenem centru doseči $1{,}8\,T$. Kolikšen tok mora teči skozi tuljavo?



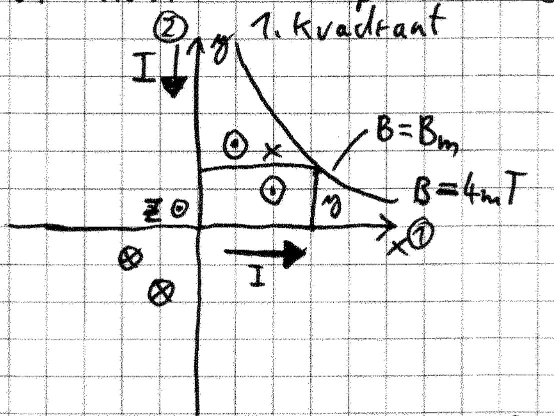
$\cancel{\alpha_1 = 148°}$ $\tan\alpha_1 = \frac{0{,}5}{1} = 26{,}6°$

$\cancel{\alpha_2 = 153{,}4}$ $\tan\alpha_2 = 153{,}4°$

$$\vec{B}(z) = \vec{e}_z \frac{\mu_0 \cdot N \cdot I}{2\,l}\left[\cos\alpha_1 - \cos\alpha_2\right]$$

$$I = 32\ kA$$

V proizvodnji hali tovarne aluminija imajo na višini $h = 1\,m$ od tal napeljavo v obliki križa (glej skico), po kateri teče $I = 2000\,A$. Upravo zanima, kje je meja med varnim in potencialno škodljivim območjem za delavca, ki dela v I. kvadrantu. Določi jim mejo, če je priporočena mejna vrednost v teh primerih $B_m = 4\,mT$.



$$B = B_1 + B_2$$

$$B(x,y) = \frac{\mu_0 I}{2\pi y} + \frac{\mu_0 I}{2\pi x} = B_m$$

$$B_m = \frac{\mu_0 I}{2\pi}\left(\frac{1}{y} + \frac{1}{y}\right)$$

$$\frac{2\pi B_m}{\mu_0 I} = \frac{1}{y} + \frac{1}{x} \qquad \frac{2\pi B_m}{\mu_0 I} = \frac{1}{a}$$

$$\frac{1}{a} = \frac{1}{y} + \frac{1}{x}$$

$$y = \left(\frac{1}{a} - \frac{1}{x}\right)^{-1}$$

$$y = \left(\frac{x-a}{ax}\right)^{-1}$$

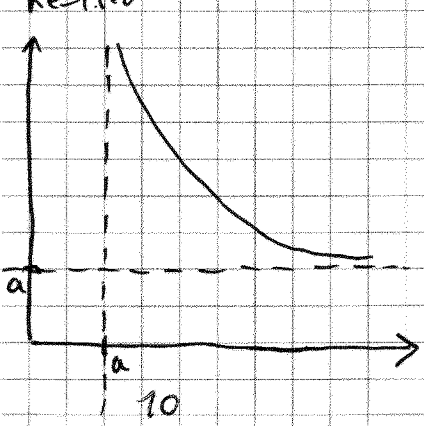
$$y = \frac{ax}{x-a} \qquad \boxed{y = \frac{ax}{x-a}}$$

poli: $x - a = 0$ $x = a$

ničle: $x = 0$

asimptota: $y = a$

Rešitev



$$a = \frac{\mu_0 I}{2\pi B_m} = \frac{\cancel{4\pi}\cdot 10^{-7}\cdot 2000\,A}{\cancel{2\pi}\ \cancel{4}\cdot 10^{-3}} = 0{,}1\,m$$

04.03.2008

Sila na vodnik

$$\vec{F}' = I' \cdot \vec{l}' \times \vec{B}$$

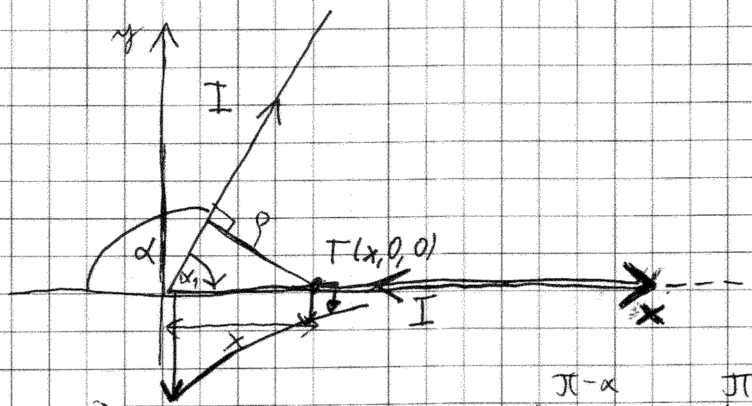
$$d\vec{F}' = I' \cdot d\vec{l}' \times \vec{B}$$

$$\frac{d\vec{F}}{dl} = I \cdot \vec{e}_l \times \vec{B}$$

$$\boxed{\vec{f} = I \cdot \vec{e}_l \times \vec{B}}$$

(točkovna)

Opredelite silo $\vec{f}_m = d\vec{F}_m/dl$ v točki T, ki je za x oddaljena od pregiba vodnika!



$\vec{e}_l = -\vec{e}_x$

K $\vec{B}(T)$ prispeva le en poltrak.

$$\vec{B}(T) = -\vec{e}_z \frac{\mu_0 I}{4\pi\rho}\left(\cos\underset{\pi-\alpha}{\alpha_1} - \cos\underset{\pi}{\alpha_2}\right)$$

$$\vec{B}(T) = -\vec{e}_z \frac{\mu_0 I}{4\pi x \sin\alpha_1}\left(\cos\alpha_1 + 1\right)$$

$$\vec{B}(T) = -\vec{e}_z \frac{\mu_0 I}{4\pi x \sin\alpha}\left(1 - \cos\alpha\right)$$

$$\vec{f}_m(x) = I \cdot (-\vec{e}_x) \times (-\vec{e}_z) \frac{\mu_0 I}{4\pi x} \cdot \frac{1-\cos\alpha}{\sin\alpha}$$

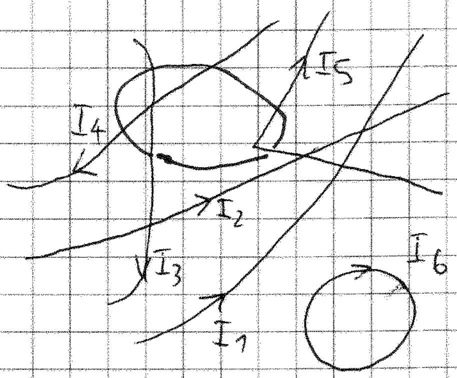
$$\boxed{\vec{f}_m(x) = -\vec{e}_y \frac{\mu_0 I^2}{4\pi x} \frac{1-\cos\alpha}{\sin\alpha}}$$

Kje ga najbolj razteguje:

$$\frac{d\vec{f}_m}{dx} =$$

Amperov zakon (za $\vec{B}$)

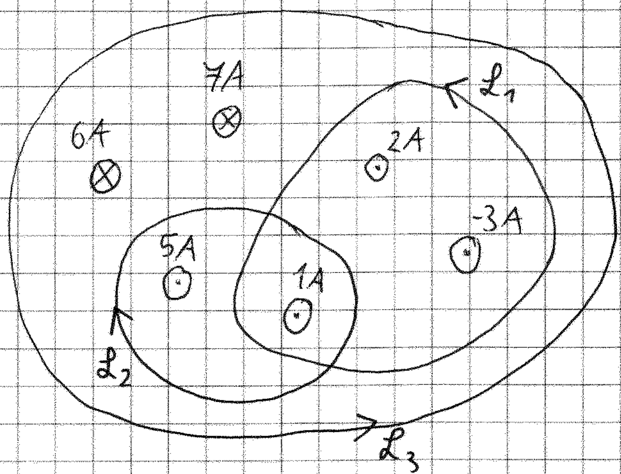
$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{notri oz. skozi}}$$



$$\oint \vec{B} d\vec{l} = \mu_0 (I_3 + I_4 - I_5)$$

Določi $\oint \vec{B} d\vec{l}$ za dane tokove in krivulje:

a)

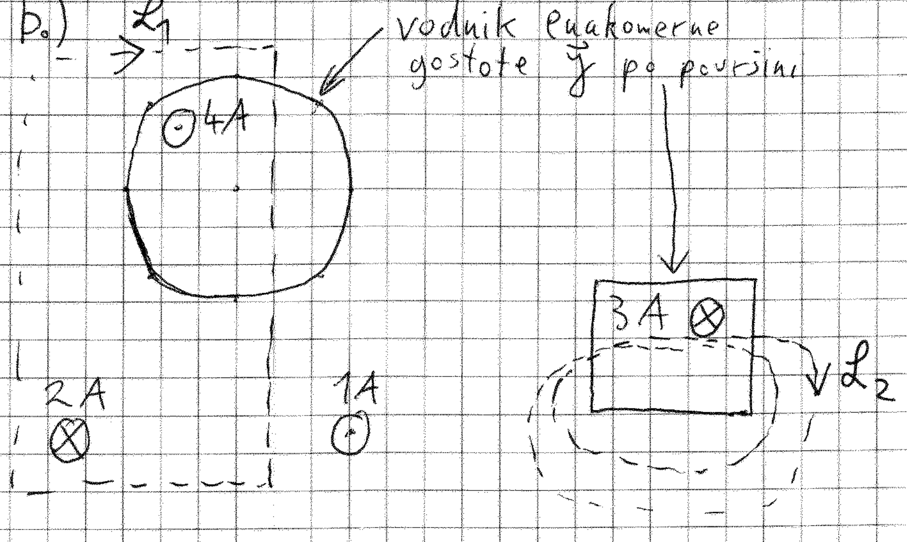


$$\oint_{\mathcal{L}_1} \vec{B}\cdot d\vec{l} = \mu_0 \cdot (2A + 1A - 3A)$$

$$\oint_{\mathcal{L}_2} \vec{B}\cdot d\vec{l} = \mu_0 \cdot (-5A - 1A)$$

$$\oint_{\mathcal{L}_3} \vec{B} d\vec{l} = \mu_0 \cdot (-6A - 7A - 3A + 5A + 2A + 1A)$$
$$= \mu_0 \cdot (-8A)$$

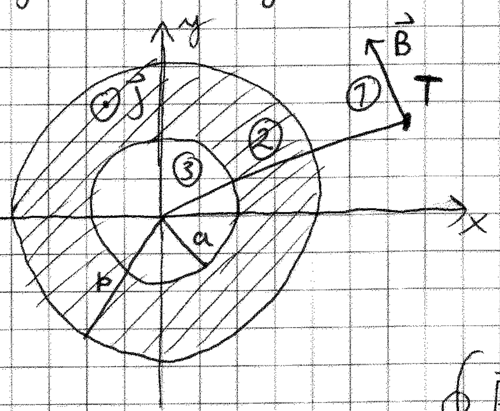
b.)



$$\oint_{\mathcal{L}_2} \vec{B}\, d\vec{l} = \mu_0 (1{,}5A + 1{,}5A)$$

$$\oint_{\mathcal{L}_1} \vec{B}\, d\vec{l} = \mu_0 (-2A + 2A)$$

Izpeljava formule za cevasti vodnik, polmera a in b, enakomerne gostote toka $\vec{J}$ (koncentrični primer).



1.) za $\rho > b$

- predpostavimo
  a.) $\vec{B}$ je v smeri $\vec{e}_\varphi$.
  b.) $|\vec{B}|$ odvisen le od ρ

$\vec{B} = \vec{e}_\varphi \cdot B_\varphi(\rho)$ velja za 1 2 3

za $\mathcal{L}$ izbereno krožnico polmera $\rho > b$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{skozi}$$

krožnica

$$\oint B(\rho)\, dl = \mu_0 I_{skozi}$$

$$B(\rho) \cdot \underbrace{\oint dl}_{2\pi\rho} = \mu_0 I \Rightarrow \boxed{B(\rho) = \frac{\mu_0 I}{2\pi\rho}}$$

2) za $\mathcal{L}$ izberemo $\rho \in (a,b)$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{skozi}$$

$$\oint B \cdot dl = \mu_0 I_{skozi}$$

$$S(\rho) = \pi \rho^2 - \pi a^2$$

$$B_\varphi(\rho) \cdot \oint dl = \mu_0 \cdot J \cdot S(\rho)$$

$$B_\varphi(\rho) \cdot 2\pi\rho = \mu_0 J \cdot (\pi \rho^2 - \pi a^2)$$

$$\boxed{B_\varphi(\rho) = \frac{\mu_0 J}{2} \left(\rho - \frac{a^2}{\rho} \right)}$$

za polni vodnik

$$\boxed{B_\varphi(\rho) = \frac{\mu_0 J}{2} \cdot \rho}$$

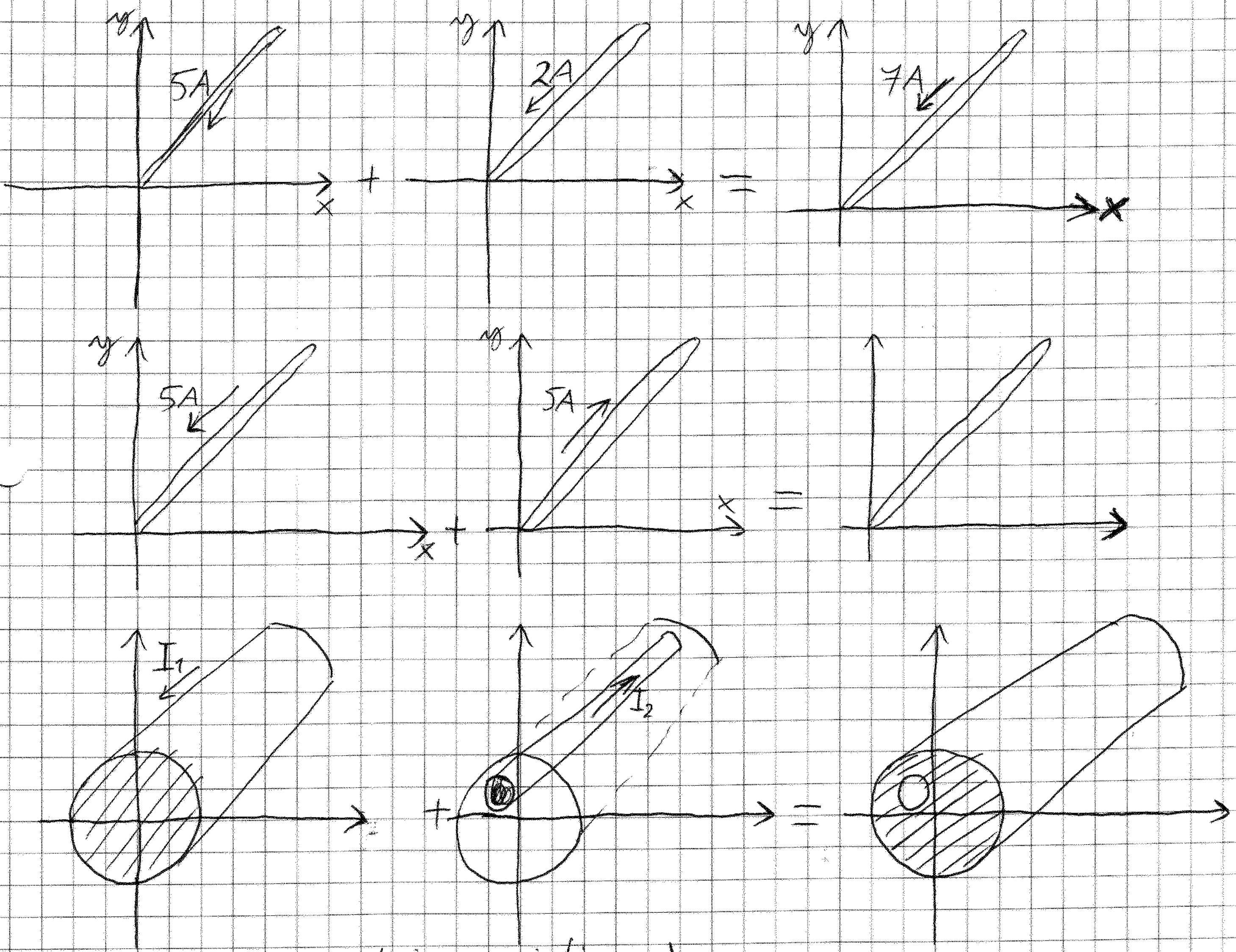
3) za $\mathcal{L}$ izberemo $\rho < a$

$$\oint \vec{B} \, d\vec{l} = \mu_0 I_{skozi}$$

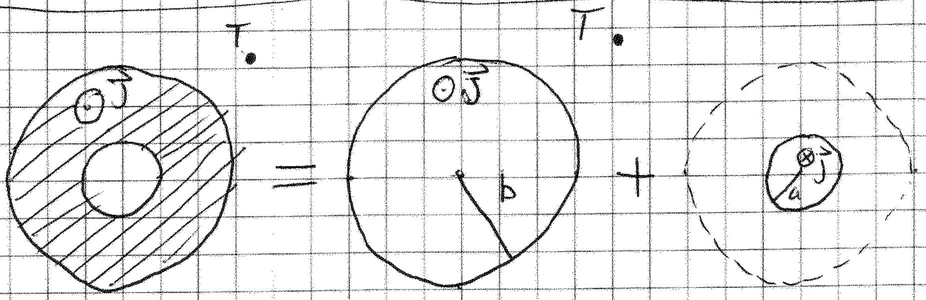
$$B_\varphi \, 2\pi\rho = \emptyset$$

$$\boxed{B_\varphi = 0}$$

Prejšnja naloga s trikom!



Vsi z istim J.



$$\vec{B}(\rho) = \vec{B}_b(\rho) + \vec{B}_a(\rho)$$

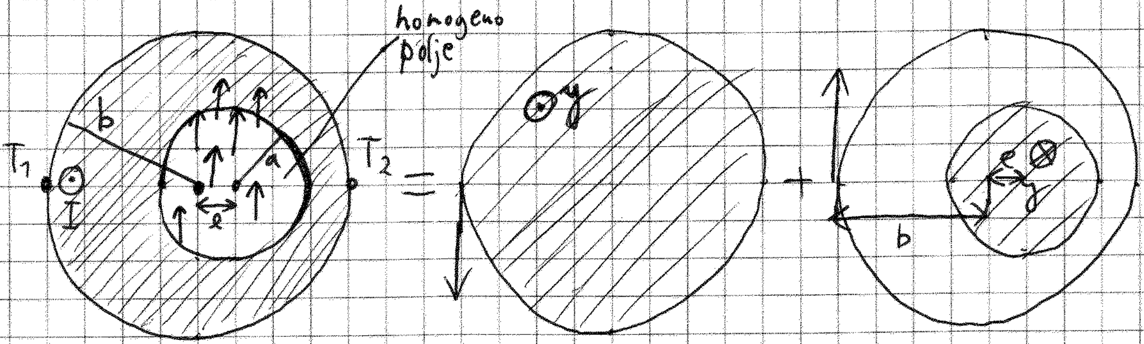
1.) $\vec{B}(\rho) = \vec{e}_\varphi \frac{\mu_0 I_b}{2\pi\rho} + (-\vec{e}_\varphi)\frac{\mu_0 I_a}{2\pi\rho} = \vec{e}_\varphi \frac{\mu_0}{2\pi\rho}(I_b - I_a)$

2.) $\vec{B}(\rho) = \vec{e}_\rho \frac{\mu_0 J}{2}\rho + (-\vec{e}_\rho)\frac{\mu_0 I_a}{2\pi\rho}$ $\qquad I_b - I_a = J(\pi b^2 - \pi a^2) = I$

$$\vec{B}(\rho) = \vec{e}_\rho \left[\frac{\mu_0 J}{2}\rho - \frac{\mu_0 J \pi a^2}{2\pi\rho}\right] =$$

$$\vec{B}(\rho) = \vec{e}_\rho \frac{\mu_0 J}{2}\left[\rho - \frac{a^2}{\rho}\right]$$

Izrazite $\vec{B}$ v luknji, v prerezu in izven ravnega ekscentričnega vodnika enakomerne gostote toka J! (a, b, c)



$$\vec{B}(T_1) = -\vec{e}_y \frac{\mu_0 J}{2}\cdot b + \vec{e}_y \frac{\mu_0 I_a}{2\pi(b+e)}$$

$$I_a = J\cdot\pi a^2$$

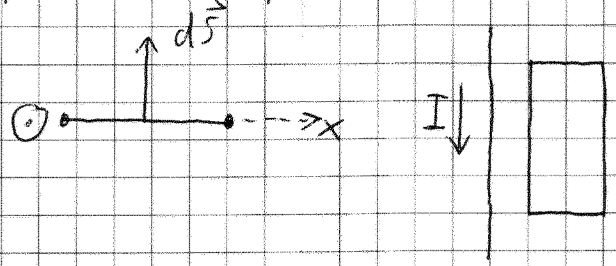
$$\vec{B}(T_1) = \vec{e}_y \frac{\mu_0 J}{2}\left[\frac{a^2}{b+e} - b\right]$$

Magnetni pretok

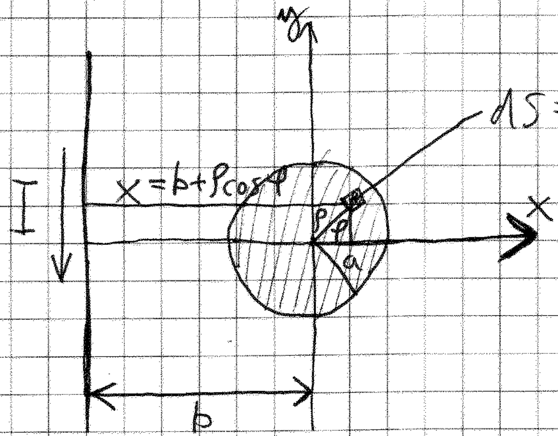
$$\Phi = \iint \vec{B} \cdot d\vec{S}$$

Φ določa ograjo površine S

Primer:
pretok skozi pravokotni okvir ob vodniku



Izpelji pretok skozi krožno zanko v ravnini vodnika s tokom I!



$$\vec{B} = \vec{e}_z \frac{\mu_0 I}{2\pi(x+b)} = \vec{e}_z \frac{\mu_0 I}{2\pi(b+\rho\cos\varphi)}$$

$$d\vec{S} = \vec{e}_z\, \rho\, d\rho\, d\varphi$$

$$\Phi = \iint \vec{B} \cdot d\vec{S} = \int_0^a \int_0^{2\pi} e_\varphi \frac{\mu_0 I}{2\pi(b+\rho\cos\varphi)} \cdot \vec{e}_z\, \rho\, d\rho\, d\varphi =$$

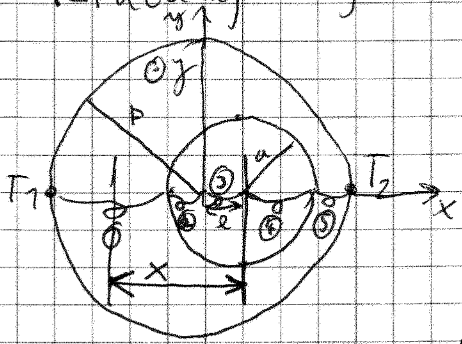
$$\Phi = \frac{\mu_0 I}{2\pi} \int_0^a \int_0^{2\pi} \frac{\rho\, d\rho\, d\varphi}{b+\rho\cos\varphi} = \frac{\mu_0 I}{2\pi} \int_0^a \rho\, d\rho \cdot 2 \int_0^{\pi} \frac{d\varphi}{b+\rho\cos\varphi} =$$

$$\Phi = \frac{\mu_0 I}{\pi} \int_0^a \rho\, d\rho \left[\frac{2}{\sqrt{b^2-\rho^2}} \cdot \operatorname{arctg}\left(\frac{(b-\rho)\cdot \operatorname{tg}\frac{\varphi}{2}}{\sqrt{b^2-\rho^2}} \right) \right]\Bigg|_0^{\pi}$$

$$\boxed{\Phi = \mu_0 I \cdot \left[b - \sqrt{b^2 - a^2} \right]}$$

Nadaljevanje prejšnje naloge

Izračunaj magnetni pretok med T_1 in T_2 (pravokotnik dolžine ℓ).



$\vec{B}$ v poljubni točki na osi $(T(x,0,0))$
po območjih

① $B_y = \frac{\mu_0 J}{2}\cdot x + \frac{\mu_0 I_a}{2\pi(\ell - x)}$

② $B_y = \frac{\mu_0 J}{2} x + \frac{\mu_0 J}{2}\cdot(\ell - x)$

③ $B_y = \frac{\mu_0 J}{2}\cdot x + \frac{\mu_0 J}{2}(\ell - x)$

④ $B_y = \frac{\mu_0 J}{2}\cdot x + \frac{\mu_0 J}{2}\cdot(x - \ell)$

⑤ $B_y = \frac{\mu_0 J}{2} x + \frac{\mu_0 I_a}{2\pi(x-\ell)}$

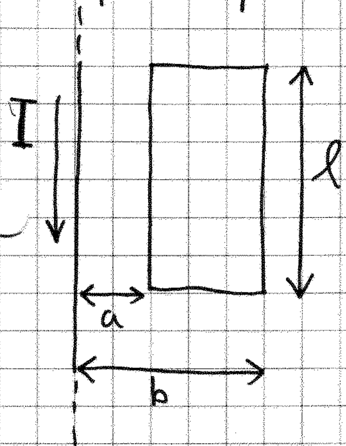
11.03.2008

Magnetni pretok

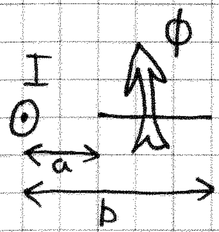
$$\Phi = \iint \vec{B} \cdot d\vec{S}$$

Tipični primeri

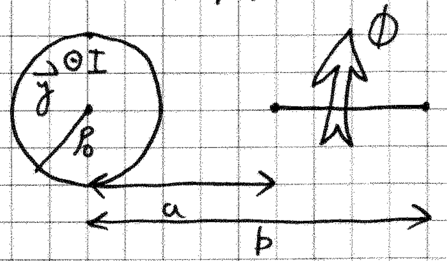
a.) vir: premi vodnik
ploskev: pravokoten okvir v ravnini vodnika in vzporeden



$$\Phi = \frac{\mu_0 I l}{2\pi} \ln \frac{b}{a}$$

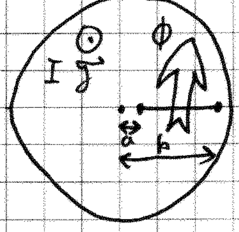


b.) vir: vodnik okroglega prereza, polmera ρ_0
ploskev: pravokotni okvir v ravnini vodnika, 2 stranici vodniku vzporedni + zunaj vodnika



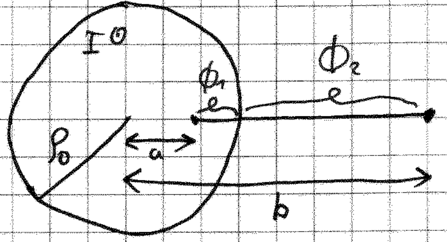
$$\Phi = \frac{\mu_0 I l}{2\pi} \ln \frac{b}{a}$$

c.) vir: vodnik okroglega prereza, polmera ρ_0
ploskev: pravokotni okvir v ravnini vodnika, 2 stranici vodnika vzporedni + znotraj vodnika



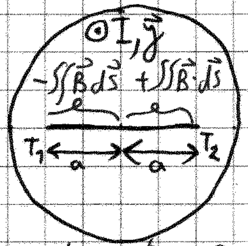
$$\Phi = \frac{\mu_0 J l}{4} (b^2 - a^2)$$

d.) mešan primer

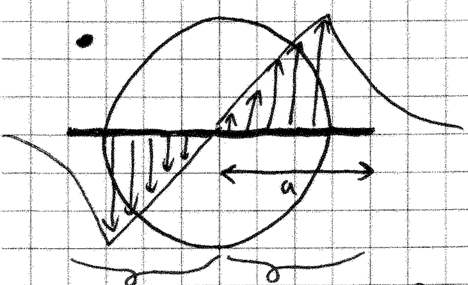


$$\Phi = \frac{\mu_0 J l}{4}\left(\rho_0^2 - a^2\right) + \frac{\mu_0 I l}{2\pi}\ln\frac{b}{\rho_0}$$

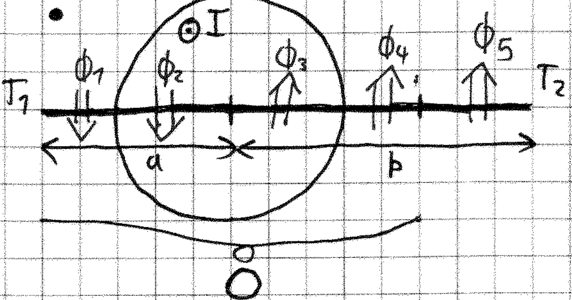
e.) vir: kot prej
ploskev: —||—, simetrični primer

- 

$-\Phi_1 + \Phi_2 = 0$

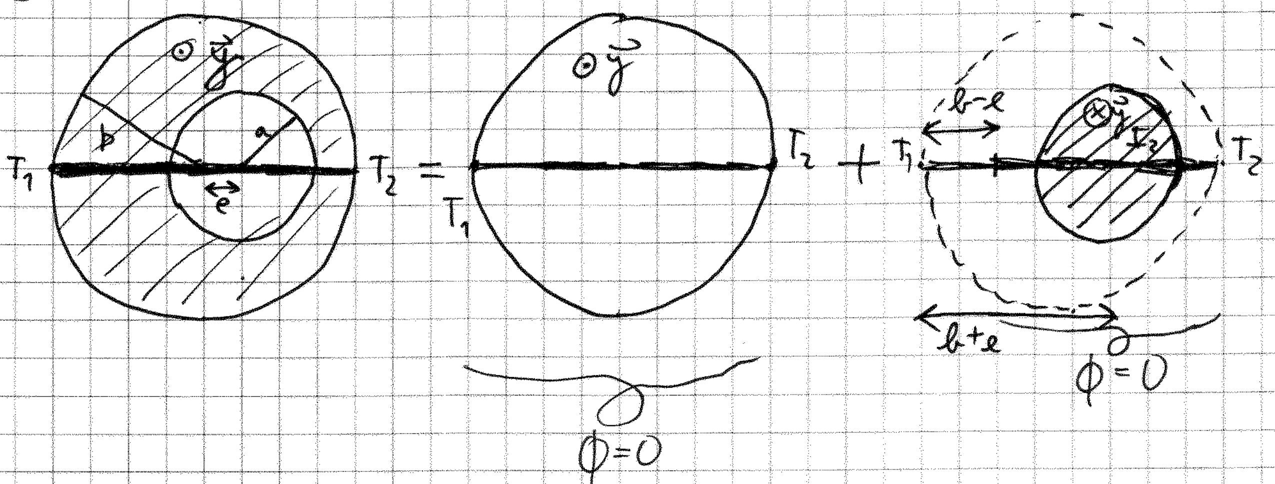
- 

$-\Phi_1 \quad +\Phi_1 = 0$

- 

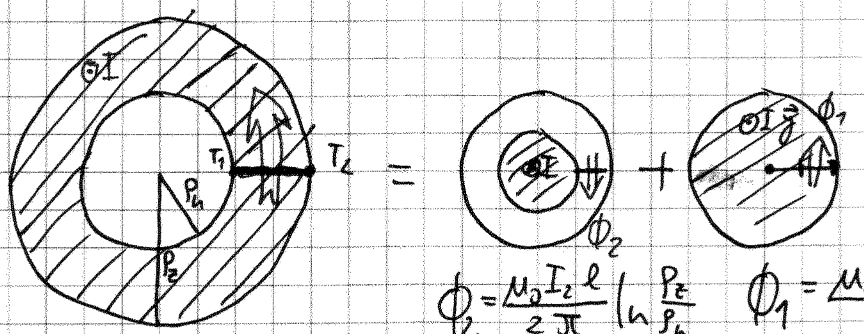
$$\Phi_5 = \frac{\mu_0 I l}{2\pi}\ln\frac{b}{a}$$

Določi ϕ med T_1 in T_2 vzdolž l metrov!



$$\phi = \phi_2 = \frac{\mu_0 I_2 l}{2\pi} \ln \frac{b+e}{b-e} = \frac{\mu_0 (J \cdot \pi a^2) l}{2\pi} \ln \frac{b+e}{b-e}$$

Določi ϕ na enoto dolžine cevi med ρ_n in ρ_z!



$$\phi_2 = \frac{\mu_0 I_2 l}{2\pi} \ln \frac{\rho_z}{\rho_n} \qquad \phi_1 = \frac{\mu_0 J l}{4} (\rho_z^2 - \rho_n^2)$$

$$I_2 = J \pi \rho_n^2 \qquad I_1 = J \pi \rho_z^2$$

$$\phi = \phi_1 - \phi_2$$

$$\frac{\phi}{l} = \frac{\mu_0 J}{4} \left[\rho_z^2 - \rho_n^2 - 2\rho_n^2 \ln \frac{\rho_z}{\rho_n} \right]$$

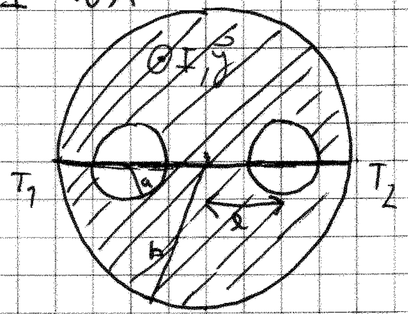
Posebni primer:

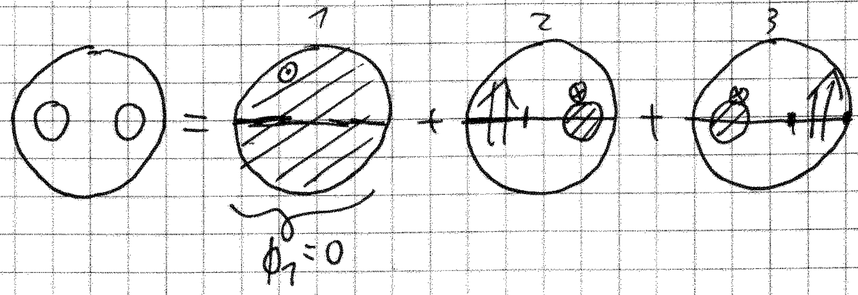
$\rho_n \approx \rho_z \Rightarrow \phi/l \stackrel{?}{=} 0$

$\rho_n \to \emptyset \Rightarrow \phi/l = \frac{\mu_0 I}{4\pi \rho_z^2} \rho_z^2 = \frac{\mu_0 I}{4\pi}$

Določite pretok med T_1 in T_2 na dolžini $\ell = 5\,m$ vzdolž vodnika.

$I = 10\,A$



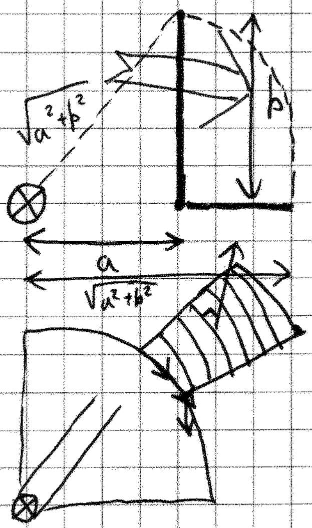


$I_2 = I_3$

$$\phi = \phi_1 + \phi_2 - \phi_3 = 0$$

~~Na okvir~~

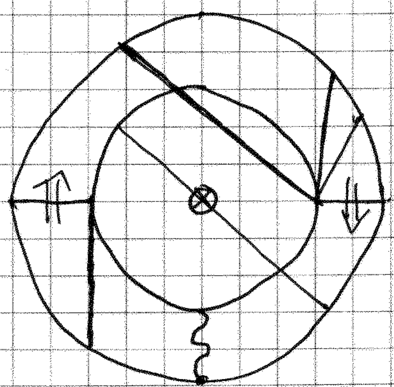
Na oddaljenosti $a = 0{,}05\,m$ od vodnika s tokom $I = 20\,A$ je pravokoten okvir $a = 0{,}2\,m$ $b = 0{,}1\,m$. Določite pretok ϕ skozi okvir!



Uporabimo lastnost mag. polja:
- ϕ je odvisn le od ograje ploskve, skozi katero ga računamo.

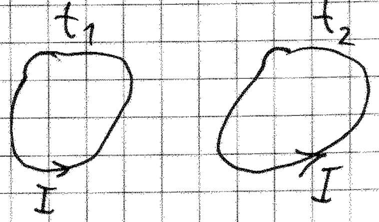
$$\phi = \frac{\mu_0 I \ell}{2\pi} \ln \frac{\sqrt{a^2+b^2}}{a} = \frac{\mu_0 I \ell}{4\pi} \ln\left(1 + \left(\frac{b}{a}\right)^2\right)$$

$$\phi = 6 \cdot 10^{-7}\,Vs$$



Na vseh daljicah enak ϕ!!!!!

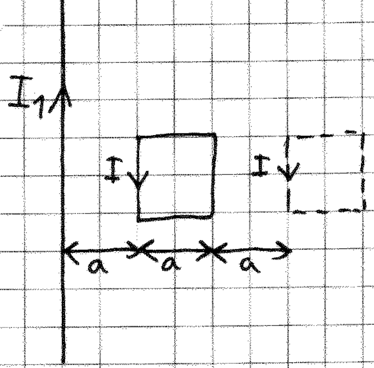
Delo pri premiku zanke v $\vec{B}$



$$A_m = I \cdot (\Phi_2 - \Phi_2)$$

teži k legi, kjer je Φ skoznjo največji (tudi smer!)

Koliko dela bi opravila magnetna sila, da bi kvadratno zanko s tokom $I = 10\,A$ premaknila v črtkano lego? $I_1 = 1A$.

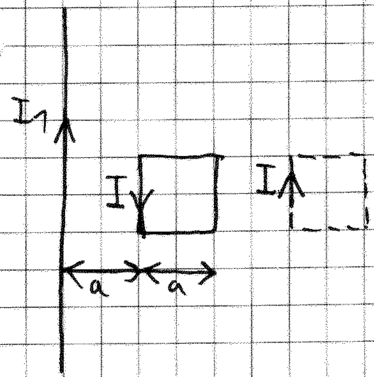


$$\phi(t_1) = -\frac{\mu_0 I_1 \cdot a}{2\pi} \ln \frac{2a}{a}$$

$$\phi(t_2) = -\frac{\mu_0 I_1 a}{2\pi} \ln \frac{4a}{3a}$$

$$A_m = I \cdot (\phi(t_2) - \phi(t_1)) = -I \cdot \frac{\mu_0 I_1 a}{2\pi} \left[-\ln 2 + \ln \frac{4}{3}\right] = \frac{\mu_0 I I_1 a}{2\pi} \ln \frac{3}{2}$$

Podvprašanje: kaj pa v tem primeru?

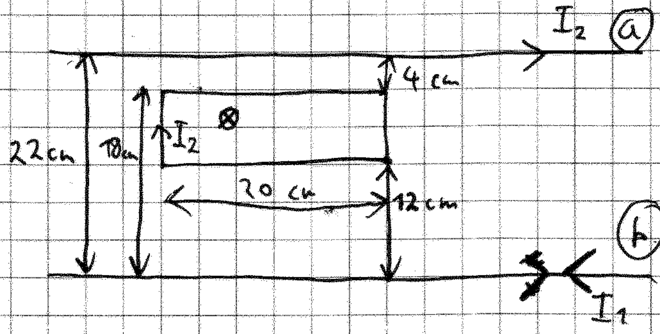


$$\phi(t_1) = -\frac{\mu_0 I_1 a}{2\pi} \ln \frac{2a}{a}$$

$$\phi(t_2) = +\frac{\mu_0 I_1 a}{2\pi} \ln \frac{4a}{3a}$$

$$A_m = I \cdot (\phi(t_2) - \phi(t_1)) = \frac{\mu_0 I I_1 a}{2\pi} \ln \frac{8}{3}$$

Tokovna zanka $I_2 = 100A$ leži med vodnikoma dvovoda $I_1 = 3A$. Koliko dela bi opravila zunanja sila, da bi zanko odvlekli stran od dvovoda.



$$\Phi_a(t_1) = \frac{\mu_0 I_1 l}{2\pi} \ln \frac{10}{4}$$

$$\Phi_b(t_1) = \frac{\mu_0 I_1 l}{2\pi} \ln \frac{18}{12}$$

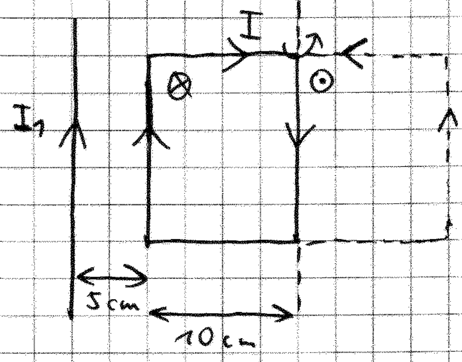
$$\Phi(t_1) = \Phi_a(t_1) + \Phi_b(t_2) = \frac{\mu_0 I_1 l}{2\pi} \ln \frac{10}{4} \cdot \frac{18}{12}$$

$$A = -A_m = -I\,(\overset{0}{\Phi(t_2)} - \Phi(t_1)) = I \cdot \Phi(t_1)$$

$$A = \frac{\mu_0 I I_1 l}{2\pi} \ln \frac{15}{4}$$

$$A = 15.9\,\mu J$$

Kvadratna tuljava $N = 150$, $I = 0{,}3A$ je vrtljiva (glej sliko) ob vodniku s tokom $I_1 = 10A$. Koliko dela opravimo, da zanko zavrtimo za kot $180°$ okrog osi? $l = 10\,cm$



$$\Phi(t_1) = \frac{\mu_0 I_1 l}{2\pi} \ln \frac{15}{5}$$

$$\Phi(t_2) = -\frac{\mu_0 I_1 l}{2\pi} \ln \frac{25}{15}$$

$$A = I \cdot N \cdot |\Phi(t_2) - \Phi(t_1)| = I \cdot N \left| -\frac{\mu_0 I_1 l}{2\pi} \ln \frac{5}{3} - \frac{\mu_0 I_1 l}{2\pi} \ln 3 \right|$$

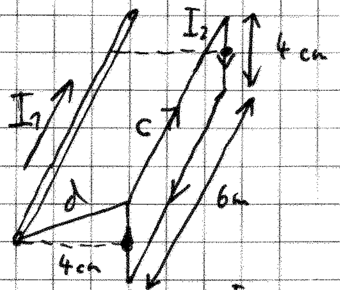
$$A = \frac{\mu_0 I I_1 N l}{2\pi} \ln 5 = 14{,}5\,\mu J$$

18.03.2008

Perendev

Navor na tokovno zanko

Izračunajte navor na zanko!

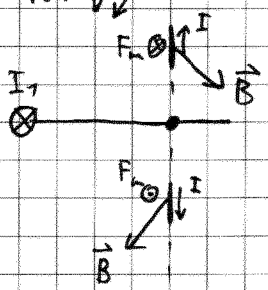


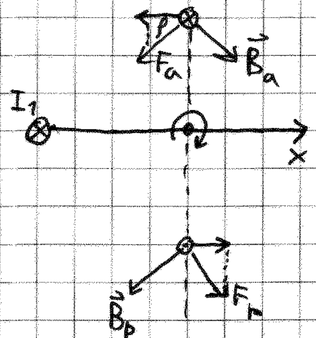
$I_1 = 100\,A$
$I_2 = 20\,A$

$\cos\rho = \frac{4}{\sqrt{4^2+2^2}}$

$|\vec{F_a}| = c\cdot I_2\cdot B_a = c\cdot I_2\cdot\frac{\mu_0 I_1}{2\pi d}$

$d = \sqrt{4^2+2^2}$





$F_x = |\vec{F_a}|\cos\rho$

$|\vec{M}| = 2|\vec{r}\times\vec{F}| = 2\cdot\frac{l}{2}\cdot F_x = l\cdot F_x$

$|\vec{M}| = l\cdot|\vec{F_a}|\cdot\cos\rho =$

$|\vec{M}| = 1{,}92\cdot 10^{-5}\ Nm$

$\vec{M} = \vec{p_m}\times\vec{B}$ $\vec{p_m} = I\cdot\vec{S}$

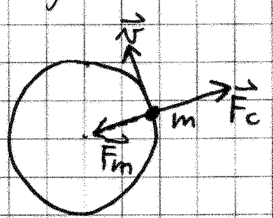
Lorentzova sila

$$\vec{F} = Q\vec{E} + \underbrace{Q\vec{v} \times \vec{B}}_{\text{magnetna sila}}$$

$$\vec{F} = \frac{d}{dt}(m \dot{\vec{r}})$$

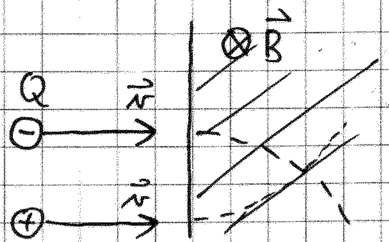
stacionarno stanje (F_m)

Kroženje



$$|\vec{F_c}| = \frac{m \cdot v^2}{R} = m\omega^2 R$$

$$|\vec{F_m}| = Q \cdot v \cdot B$$



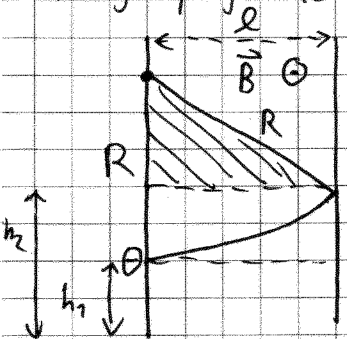
$$|\vec{F_m}| = |\vec{F_c}|$$

$$Q \cancel{v} B = m v^{\cancel{2}} / R$$

$$\boxed{R = \frac{m \cdot v}{Q\,B}}$$ radij kroženja delca

$$\boxed{\omega = \frac{Q\,B}{m}}$$ ciklotronska frekvenca

Elektron s hitrostjo $v \stackrel{!}{=} 3 \cdot 10^{7}\ \frac{m}{s}$ vstopi v območje homogenega mag. polja $\vec{B} = (0,0,B)$ širine $l = 2$ cm na višini $h_1 = 0{,}5$ cm. Določite B_z, da bo delec iz območja mag. polja izstopil na višini $h_2 = 1$ cm.



$$R^2 = (R - \Delta h)^2 + l^2$$

$$R = \frac{m \cdot v}{Q \cdot B}$$

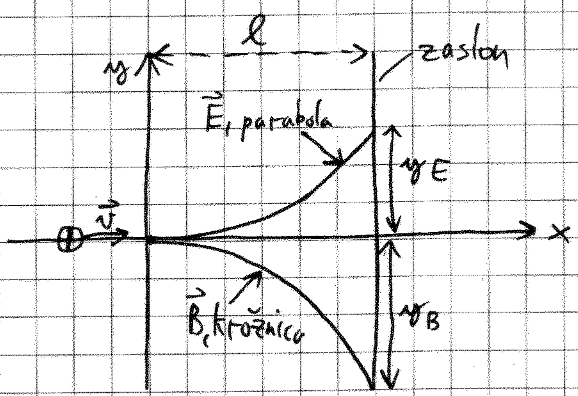
$$\cancel{R^2} = \cancel{R^2} - 2R\Delta h + \Delta h^2 + l^2$$

$$2R\Delta h = \Delta h^2 + l^2$$

$$R = \frac{1}{2\Delta h}(\Delta h^2 + l^2) = 4{,}25\ cm$$

$$B = \frac{m \cdot v}{Q \cdot R} = \frac{9.1 \cdot 10^{-31} \cdot 3 \cdot 10^{7}}{1.6 \cdot 10^{-19} \cdot 4.25 \cdot 10^{-2}} \cong 4\ mT$$

Delce $Q = 1{,}6 \cdot 10^{-18}$ As in $m = 2 \cdot 10^{-21}$ kg upihujemo v prostor $x > 0$ v koordinatnem izhodišču s hitrostjo $\vec{v}_0 = \vec{e}_x \, 4 \, m/s$. Določite razdaljo med pikama na zaslonu $l = 20$ cm, ~~odklo~~ če v prvem primeru vklopimo le $\vec{E} = \vec{e}_y \, 5 \, V/m$, v drugem pa le $\vec{B} = \vec{e}_z \cdot 0{,}25$ T.



$$E: \; t = \frac{l}{v_0}, \quad a = \frac{QE}{m}$$

$$\boxed{y_E = \frac{a t^2}{2} = \frac{1}{2} \frac{QE}{m} \left(\frac{l}{v_0}\right)^2}$$

$$B: \; R^2 = (R - y_B)^2 + l^2$$

$$R^2 - l^2 = (R - y_B)^2$$

$$R - y_B = \pm \sqrt{R^2 + l^2}$$

$$\boxed{y_B = R \mp \sqrt{R^2 - l^2}} \qquad R = \frac{m \cdot v}{QB}$$

$y_E = 5$ m

$y_B = 0{,}2$ m

$\Delta y = y_E + y_B =$

Uporaba lorentzove sile

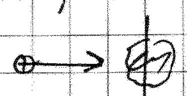
Masni spektrograf

Pospeševalniki

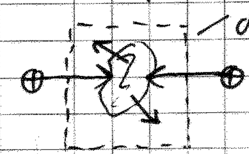
pospeševanje — detekcija

p^+
n^0
e^-

a) mirujoča tarča



b)



25.03.2008

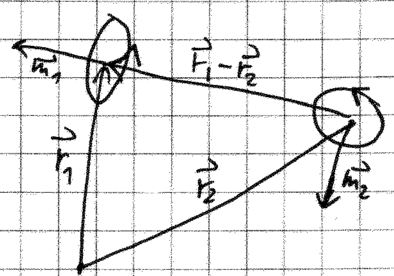
Magnetni dipol

zankica s tokom in površino: vpeljemo magnetni dipolski moment

$\delta\vec{m} = I \cdot \delta\vec{a}$
$(\vec{p}_m)$ površina

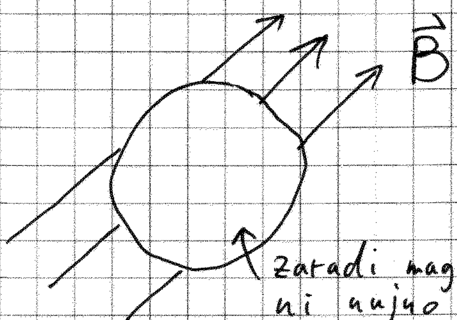
Navor na dipol

$\vec{M}_i = \vec{m}_i \times \vec{B}(r_i)$ glej naloga 4.38



Poišči formulo za $\vec{B}$ v okolici $\vec{m}$!

Snov v mag. polju



zaradi mag. dipolov, ki nastanejo pod vplivom $\vec{B}$, zunanje polje ni nujno enako notranjemu.

Vektor magnetizacije $\vec{M}$

$$\vec{M} = \frac{\vec{B}}{\mu_0} - \vec{H}$$

Vpeljemo (kar ne bo vedno OK): $\mu_0 \Rightarrow \boxed{\vec{B} = \mu_r \mu_0 \vec{H}}$

$$\frac{\vec{B}}{\mu_0} = \vec{H} + \vec{M} \Rightarrow \vec{B} = \mu_0 (\vec{H} + \vec{M})$$

$$\mu_r \vec{H} = \vec{H} + \vec{M}$$

$$\boxed{\vec{M} = \underbrace{(\mu_r - 1)}_{\chi_m} \vec{H}}$$

μ_r v naravi: večina 1 ± 0.001

redki $\mu_r \gg 1$; feromagnetiki $\mu_r \approx 100.000$

V nekem domnevno linearnem materialu so hkrati določili gostoto mag. pretoka $B = 1.3\,T$ in magnetizacijo $M = 650\,kA/m$. Kolikšna je najboljša ocena za μ_r?

$$\vec{B} \propto \vec{H} \text{ — linearno}$$

$$B = \mu_0 \mu_r H \Rightarrow H = \frac{B}{\mu_0 \mu_r}$$

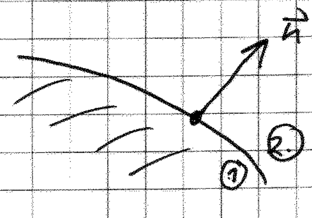
$$M = \frac{B}{\mu_0} - H$$

$$\frac{B}{\mu_0 \mu_r} = \frac{B}{\mu_0} - M \quad / \cdot \mu_0 \mu_r$$

$$B = B\mu_r - M\mu_0\mu_r$$

$$B = \mu_r (B - M\mu_0) \Rightarrow \mu_r = \frac{B}{B - M\mu_0} = 2{,}69$$

Mejna pogoja



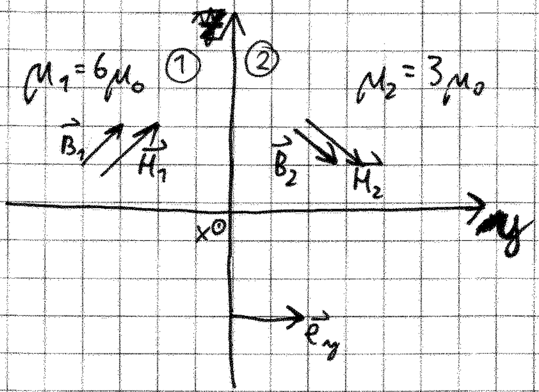
$$\boxed{\begin{array}{l} \vec{n} \cdot (\vec{B}_2 - \vec{B}_1) = 0 \\ \vec{n} \cdot (\vec{H}_2 - \vec{H}_1) = \vec{K} \end{array}}$$

tahitre formule, če je $\vec{K} = 0$:

$$B_{2n} = B_{1n}$$

$$H_{2t} = H_{1t}$$

Ravnina $y=0$ je meja dveh feromagnetikov. V območju $y<0$ je $\vec{H}_1=(50,100,50)\,\frac{A}{m}$. Določite $\vec{B}_2$ v $y>0$, če je meja $y=0$ brez tokovne obloge.



$$\vec{B}_1=\mu_1\vec{H}_1=(\underbrace{300}_{tan},600,\underbrace{300}_{tan})\mu_0$$

$$B_{2n}=B_{1n}=600\mu_0\cong 750\,\mu T$$

$$H_{2t}=H_{1t}\Rightarrow$$

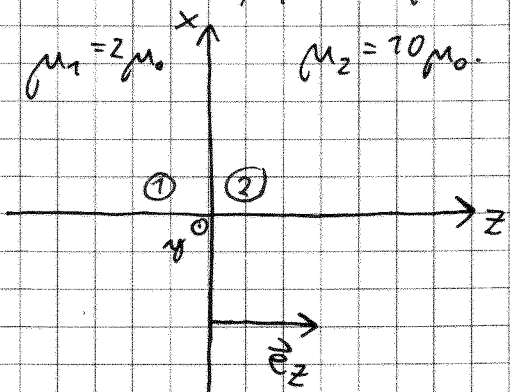
$$H_{2x}=H_{1x}=50\,\frac{A}{m}$$

$$H_{2z}=H_{1z}=50\,\frac{A}{m}$$

$$B_{2x}=\mu_2\cdot H_{2x}=150\mu_0\cong 190\,\mu T$$

$$B_{2z}=\mu_2\cdot H_{2z}=150\mu_0\cong 190\,\mu T$$

Ravnina $z=0$ je meja dveh linearnih feromagnetikov. V območju $z<0$ je $\vec{H}_1=(100,100,125)\,A/m$, v območju $z>0$ sta tangencialni komponenti vektorjev $B_{2x}=8\cdot\pi\cdot10^{-4}\,T$ in $B_{2y}=2\pi\cdot10^{-4}\,T$. Določite B_2 v območju $z>0$ ter vektor $\vec{K}$ na meji.
$\mu_1=2\mu_0$, $\mu_2=10\mu_0$.



$$B_{2z}=B_{1z}=\mu_1\cdot H_{1z}=2\mu_0\cdot125=$$
$$=250\mu_0=\pi\cdot10^{-4}\,T$$

$$\vec{H}_2=\frac{\vec{B}_2}{\mu_2}=\frac{(8,2,1)\cancel{\pi}\cdot\cancel{10^{-4}}}{\cancel{10}\cdot4\cancel{\pi}\cdot\cancel{10^{-7}}}=(200,50,25)\,\frac{A}{m}$$

Vektor $\vec{k}$

$\vec{n}=(0,0,1)=\vec{e}_z$

$\vec{H}_1=(100,100,125)\,\frac{A}{m}$

$\vec{H}_2=(200,50,25)\,\frac{A}{m}$

$$\vec{K}=\vec{n}\times(\vec{H}_2-\vec{H}_1)$$

$$\vec{k}=\vec{n}\times(100,-50,-100)$$

$$\vec{k}=(0,0,1)\times(100,-50,-100)=\vec{e}_x\cdot50+\vec{e}_y\,100+\vec{e}_z\,0$$

$$\vec{k}=(50,100,0)\,\frac{A}{m}$$

$$\begin{vmatrix} e_x & e_y & e_z \\ 0 & 0 & 1 \\ 100 & -50 & -100 \end{vmatrix}$$

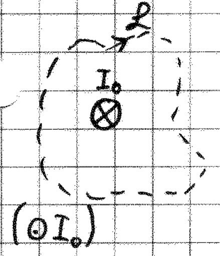
Amperov zakon (edini pravi)

$$\oint \vec{H} \cdot d\vec{\ell} = I_{zaobjeti}$$

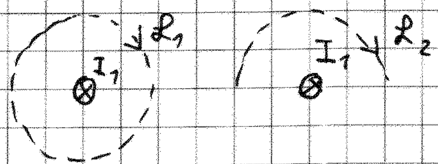
(glej primere za: $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{zaobjeti}$)

Podobnost z $\int_A^B \vec{E} \cdot d\vec{\ell} = U_{AB}$

Vpeljava potenciala



$$\oint \vec{H} \cdot d\vec{\ell} = I_0$$

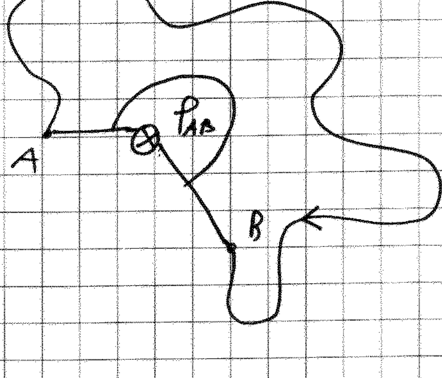


~~Magnetna napetost je vorazmetna~~

$\oint_{\mathcal{L}_1} \vec{H} \cdot d\vec{\ell} = I_1$ $\quad \int_{\mathcal{L}_2} \vec{M} \cdot d\vec{\ell} = \frac{1}{2} I_1$ $\Rightarrow$ pomemben je kot (vsaj pri prenih vodnikih)

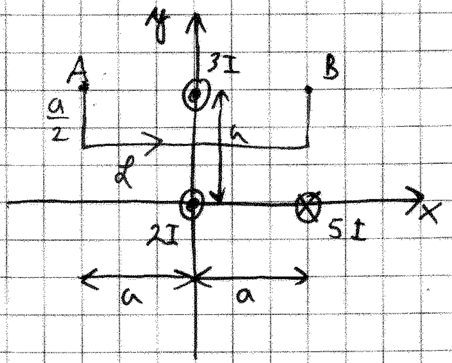
za preni vodnik

$$\int_{\mathcal{L}} \vec{H} \cdot d\vec{\ell} = \frac{\varphi_{AB}}{2\pi} I_0$$



$a^2 + 4a^2$ $\quad 5a^2$ $\quad \sqrt{5}a$

Izračunajte mag. napetost Θ_{AB} vzdolž krivulje $\mathcal{L}$ med točkama A in B v okolici treh premih vodnikov! ($I = 15\,A$, $a = 0{,}2\,m$)



Ločeno za 3I:

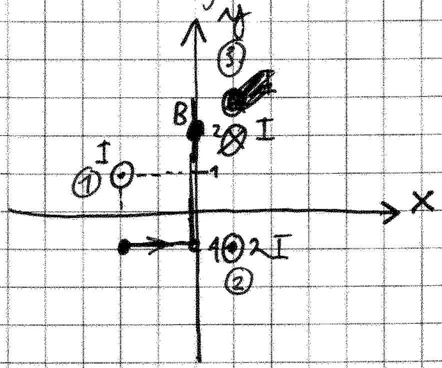
$$\tilde{\Theta}_{AB}^{(3I)} = \frac{\pi}{2\pi} \cdot 3I$$

$$\tilde{\Theta}_{AB}^{(2I)} = \frac{-\frac{\pi}{2}}{2\pi} \cdot 2I$$

$$\tilde{\Theta}_{AB}^{(5I)} = \frac{1{,}1}{2\pi} \cdot 5I$$

$$\tilde{\Theta}_{AB} = \frac{3}{2} I - \frac{1}{2} I + 0{,}88\, I = 28{,}2\, A$$

Kolikšna je Θ_{AB}, če je $I = 1\,A$?



$$\tilde{\Theta}_{AB}^{①} = \frac{117°}{360°} I$$

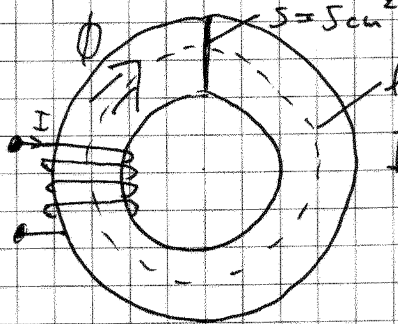
$$\tilde{\Theta}_{AB}^{②} = \frac{-72°}{360°} 2I$$

$$\tilde{\Theta}_{AB}^{③} = \frac{45°}{360°} I$$

$$\tilde{\Theta}_{AB} = \frac{18°}{360°} I = \frac{1}{20} \cdot 1A = 0{,}05\, A$$

01.04.2008

V enozančnem jedru iz transformatorske pločevine preseka $5\,cm^2$ in srednje dolžine poti $50\,cm$ je magnetni pretok $0{,}5\,mVs$. Kolikšen tok teče skozi navitje z 200 ovoji?



$l_s = 50\,cm$

$I = ?$

$\Phi \rightarrow B = \frac{\Phi}{S} \longrightarrow H \longrightarrow I = \frac{1}{N} \cdot H \cdot l_s$

S

N

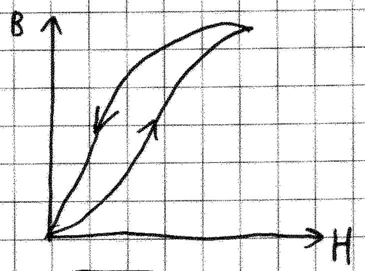
$h = l_s$

$$B = \frac{\Phi}{S} = \frac{0{,}5 \cdot 10^{-3}}{5 \cdot 10^{-4}} = 1T \longrightarrow$$

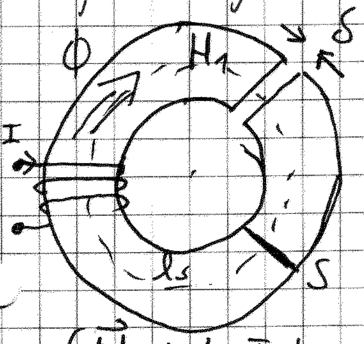
$$\xrightarrow[\text{krivulja}]{\text{mag.}} H = 0{,}2\,\frac{kA}{m} = 200\,\frac{A}{m}$$

$$I = \frac{1}{200} \cdot 200 \cdot 0{,}5 \approx \underline{\underline{0{,}5\,A}}$$

Magnetna krivulja



Kolikšen pa mora biti I (za podatke iz prejšnje naloge), če je v jedru reža širine $\delta = 1\,mm$?



$\Phi \rightarrow B_1$ ($\frac{\Phi}{S_1}$), $\rightarrow B_0$ ($\frac{\Phi}{S_0}$)

$$N \cdot I = H_1 \cdot (l_s - \delta) + H_0 \cdot \delta$$

$$B_1 = \frac{\Phi}{S} = 1T = B_0$$

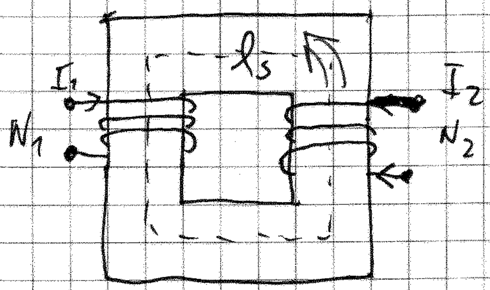
$$H_1 = 200\,\frac{A}{m}$$

$$H_0 = \frac{B_0}{\mu_0} = \frac{1T}{4\pi \cdot 10^{-7}} = 800\,\frac{kA}{m}$$

$$\oint \vec{H} \cdot d\vec{l} = I \cdot N$$

$$I = \frac{1}{N}\left[H_1(l_s - \delta) + H_0 \cdot \delta\right] = 4{,}5\,A$$

Na feromagnetnem jedru iz transformatorske pločevine sta navitji $I_1 = 5A$, $N_1 = 100$, $I_2 = 10A$, $N_2 = 200$. Določite srednjo gostoto magn. pretoka v jedru, če je $l_s = 30cm$ in $S = 20cm^2$.



$$B \leftarrow H \leftarrow IN = H \cdot l_s$$

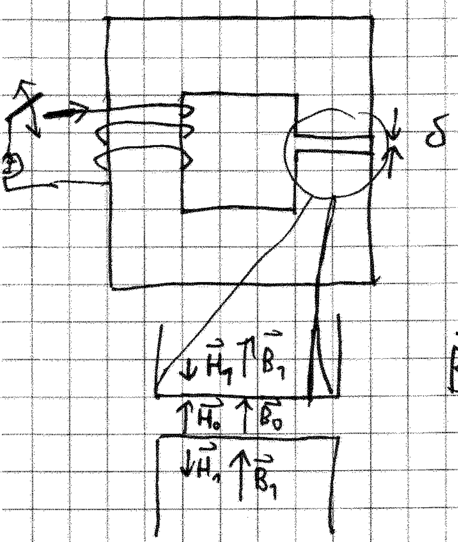
$$H \cdot l_s = I_2 N_2 - I_1 N_1$$

$$H = \frac{I_2 N_2 - I_1 N_1}{l_s}$$

$$H = 5000 \frac{A}{m}$$

$$H = 5000 \frac{A}{m} \xrightarrow{m.kr.} B \cong 1.58T$$

Feromagnetno jedro z režo namagnetimo prehodno z vklopom stikala. Določite povprečno magnetizacijo v jedru, če je v reži $B_0 = 0.6T$! $l_s = 0,3m$ $\delta = 1mm$.



$$\vec{M} = \frac{\vec{B}}{\mu_0} - \vec{H}$$

$$\oint \vec{H} \cdot d\vec{l} = N \cdot I_{zaobjeti}$$

$$H_1 \cdot l_1 + H_0 \cdot \delta = \emptyset$$

$$H_1 = -H_0 \frac{\delta}{l_s}$$

$$\vec{B}_0 = \mu_0 \cdot \vec{H}_0$$

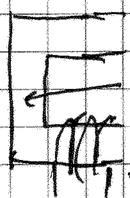
$$M_1 = \frac{\overset{= B_0}{B_1}}{\mu_0} - H_1$$

$$M_1 = \frac{B_0}{\mu_0} - \left(-H_0 \frac{\delta}{l_s}\right) =$$

$$M_1 = \frac{B_0}{\mu_0} + \frac{B_0}{\mu_0} \frac{\delta}{l_s} = \frac{B_0}{\mu_0}\left(1 + \frac{\delta}{l_s}\right) = \frac{0,6 \cdot 10^7}{4\pi}\left(1 + \frac{1}{300}\right)$$

$$M_1 \cong 5 \cdot 10^5 \, A/m$$

Modelna vezja



pretok ujet v jedro

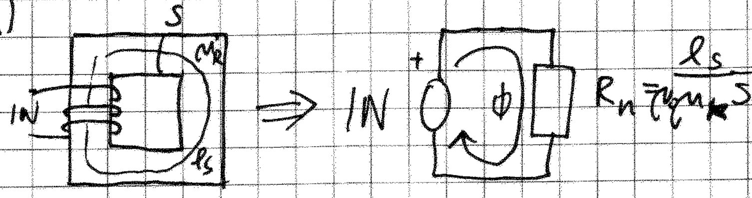
$IN \Rightarrow$ vir $\qquad I \cdot N = H_1 \cdot l_1 + H_2 \cdot l_2 + \ldots H_n \cdot l_n$

analogija z el. vezji

električno v.	magnetno vezje
I	$\Phi = B_K \cdot S_K$
U_g	$I \cdot N$
U	$H_K \cdot l_K$
$R = \frac{U}{I}$	$R_m = \frac{H_K \cdot l_K}{\Phi} = \frac{H_K \cdot l_K}{B_K \cdot S_K} \xrightarrow[B_K = \mu_K \cdot H_K]{\text{če velja}} R_m = \frac{l_K}{\mu_K S_K}$

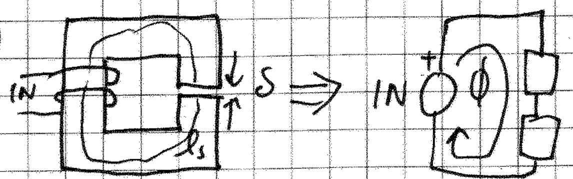
Modeli

a.)



$R_n = \frac{l_s}{\mu_K S}$

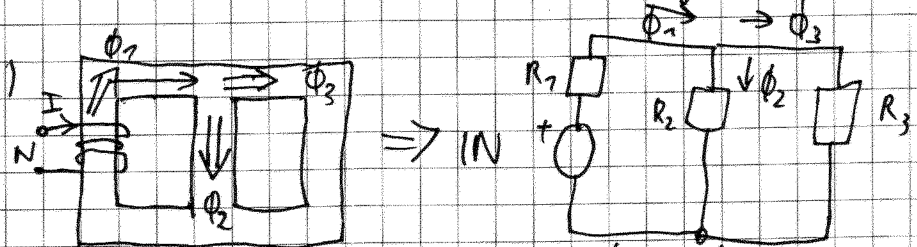
b.)



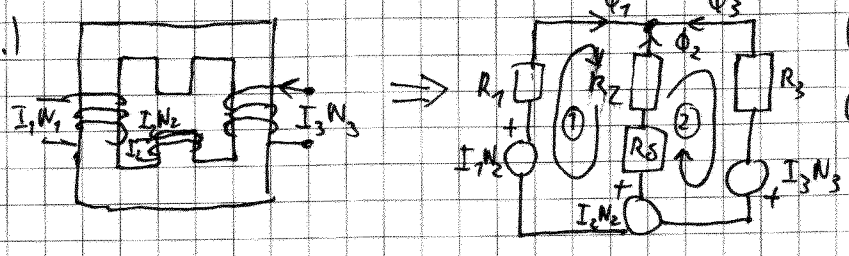
$R_n = \frac{l_s}{\mu_K \cdot S}$

$R_n = \frac{\delta}{\mu_0 \cdot S}$

c.)

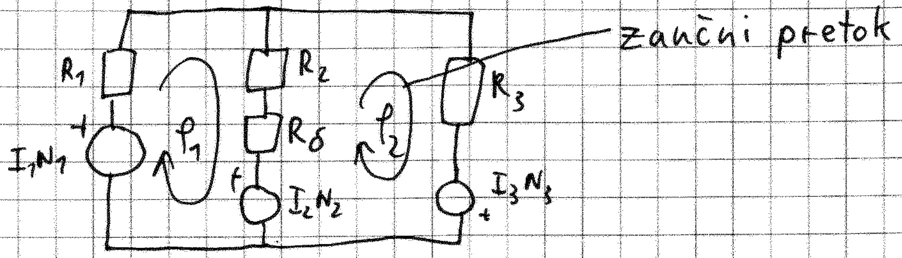


d.)



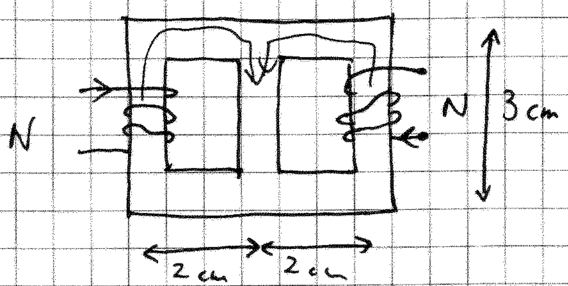
① $I_1 N_1 - I_2 N_2 = R_1 \Phi_1 - (R_2 + R_\delta)\Phi_2$

② $I_2 N_2 + I_3 N_3 = (R_\delta + R_2)\Phi_2 - R_3 \Phi_3$

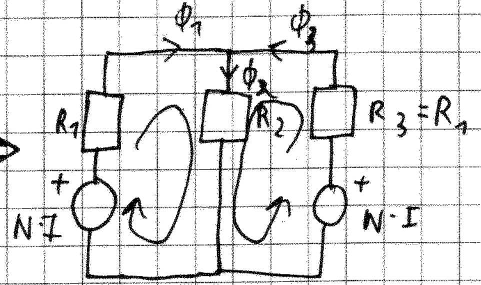


$$\varphi_1 : I_1N_1 - I_2N_2 = (R_1 + R_2 + R_\delta)\varphi_1 - (R_2 + R_\delta)\varphi_2$$

Simetrično jedro $S = 1\,cm^2$ in $\mu_R = 10^4$ ima v srednjem stebru $B_2 = 0{,}8\,T$.
Kolikšen je I skozi dvodelno navitje z $N = 150$ ovoji (2×150)!



$\Longrightarrow$



$a = 7\,cm$
$b = 3\,cm$

$$R_1 = R_3 = \frac{a}{\mu_0 \mu_r \cdot S}$$

$$R_2 = \frac{b}{\mu_0 \mu_R \cdot S}$$

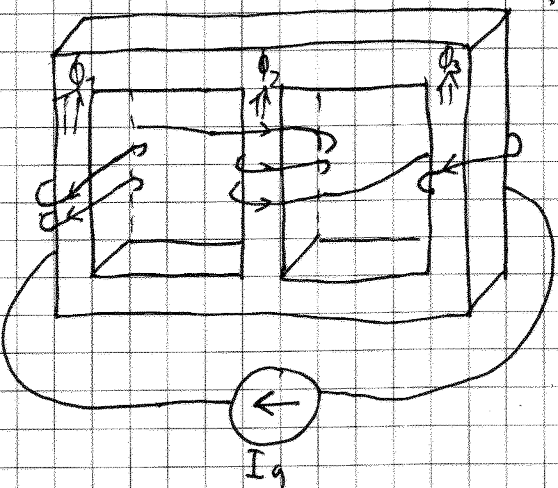
$$\phi_1 + \phi_3 = \phi_2 \Rightarrow = 2\phi_1 \Rightarrow \phi_1 = \frac{\phi_2}{2}$$

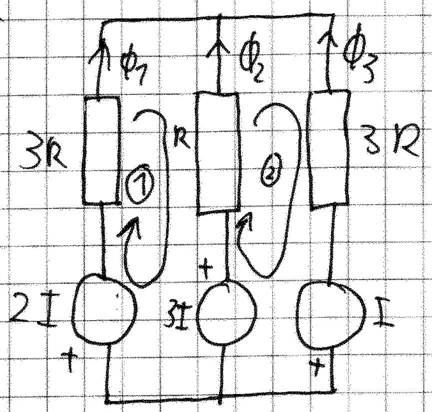
$$\left.\begin{array}{l} (1)\; IN = R_1\phi_1 + R_2\phi_2 \\ (2)\; IN = R_1\phi_3 + R_2\phi_2 \end{array}\right\} \Rightarrow \phi_3 = \phi_1$$

$$I = \frac{\phi_2}{N}\left[\frac{R_1}{2} + R_2\right] = \frac{B_2 \cdot S}{N} \cdot \left[\frac{a}{2\mu_r\mu_0 S} + \frac{b}{\mu_0\mu_R S}\right] = \frac{B_2}{N\mu_r\mu_0}\left[\frac{a}{2} + b\right]$$

$$I = 2\,\cancel{0,}\,\cancel{m}A$$

Tristebrno jedro ovijemo s tokovno pentljo. Določite razmerje $\Phi_1 : \Phi_2 : \Phi_3$, če $R_1 = R_3 = 3R_2$
R





$$\frac{\Phi_1}{\Phi_2} = \alpha \qquad \frac{\Phi_3}{\Phi_2} = \beta \qquad \frac{I}{R} = c$$

1 $\quad -2I - 3I = 3R\Phi_1 - R\Phi_2$

$$-5I = R(3\Phi_1 - \Phi_2)$$

$$\boxed{3\Phi_1 - \Phi_2 = -\frac{5I}{R}}$$

2 $\quad 3I + I = R\Phi_2 - 3R\Phi_3$

$$4I = R(\Phi_2 - 3\Phi_3)$$

$$\boxed{\Phi_2 - 3\Phi_3 = \frac{4I}{R}}$$

$$\boxed{\Phi_1 + \Phi_2 + \Phi_3}$$

$$\boxed{\begin{aligned} 3\Phi_1 - \Phi_2 &= -5c \\ \Phi_2 - 3\Phi_3 &= 4c \\ \Phi_1 + \Phi_2 + \Phi_3 &= 0 \end{aligned}}$$

$$\cancel{\Phi_2 = 3\Phi_1 + 5c}$$
$$\cancel{\Phi_2 = 3\Phi_3 + 4c}$$

$$\Phi_1 = \frac{1}{3}[\Phi_2 - 5c]$$

$$\Phi_3 = \frac{1}{3}[\Phi_2 - 4c]$$

$$\frac{1}{3}[\Phi_2 - 5c] + \Phi_2 + \frac{1}{3}[\Phi_2 - 4c] = 0$$

$$5\Phi_2 = 9c$$

$$\Phi_2 = \frac{9}{5}c$$

$$\Phi_1 = -\frac{16}{15}c$$

$$\Phi_3 = -\frac{11}{15}c$$

$$\frac{\Phi_1}{\Phi_3} = \frac{-\frac{16}{15}c}{-\frac{11}{15}c} = \frac{16}{11}$$

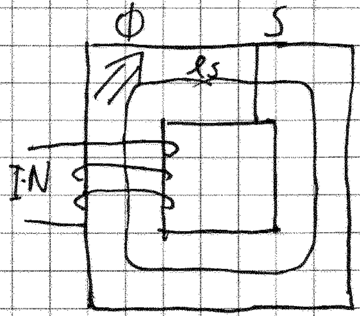
$$\frac{\Phi_2}{\Phi_3} = \frac{\frac{9}{5}c}{-\frac{11}{15}c} = -\frac{27}{11}$$

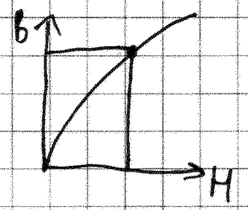
$$\Phi_1 : \Phi_3 = 16 : 11$$

$$\Phi_2 : \Phi_3 = (-27) : 11$$

$$\Phi_1 : \Phi_2 : \Phi_3 = 16 : (-27) : 11$$

Na magnetnem jedru $l_s = 0,5\,m$ in $S = 30\,cm^2$ je navitje $N = 2000$ ovoji. Magnetna krivulja je podana z enačbo $B = K\sqrt{H}$, kjer je $K = 0,05\,T\sqrt{m/A}$. Pri kolikšnem toku je $\Phi = 3\cdot10^{-3}\,Vs$?





$$\Phi \to B \to H \to I =$$

$$I = \frac{H \cdot l_s}{N}$$

$$H = \left(\frac{B}{K}\right)^2$$

$$I = \frac{l_s}{N}\left(\frac{B}{K}\right)^2 = \frac{l_s}{N}\left(\frac{\Phi}{K\cdot S}\right)^2$$

$$I = \frac{0,5}{2000}\left(\frac{3\cdot10^{-3}}{0,05\cdot30\cdot10^{-4}}\right)^2$$

$$I = 0,1\,A$$

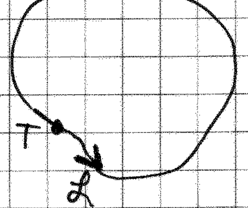
08.04.2008

Indukcija

$\oint \vec{E}\cdot d\vec{l} = 0$ v OET I.

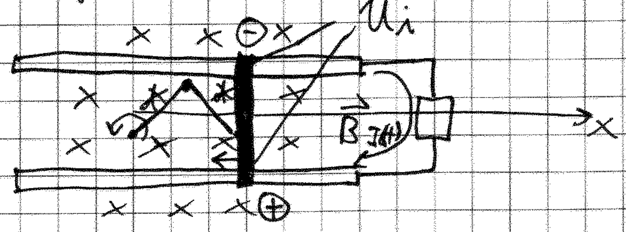
$\oint \vec{E}\cdot d\vec{l} \neq 0$ v OET II.

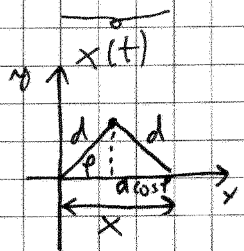
napetost v zanki



$$U_i = -\frac{d\Phi}{dt}$$

Izračunaj povprečno moč na upornika (zanemarite samo indukcijo), ko mehanizem z enako dolgima ročicama pretvarja kroženje v drsenje (kot kaže slika).





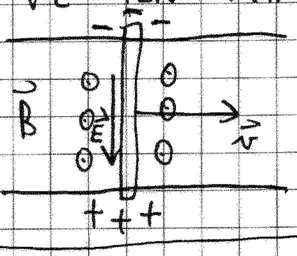
$$\varphi = \omega \cdot t$$

$$x(t) = 2\cdot d\cdot \cos\varphi = 2\cdot d\cdot \cos(\omega\cdot t) \Rightarrow \vec{v} = \dot{x}\,\vec{e}_x \;;\; \vec{B} = -\vec{e}_z \cdot B$$

$$\vec{E}_i = \vec{v}\times\vec{B} = \vec{e}_x \times (-\vec{e}_z)\cdot \dot{x}\, B = \qquad \Rightarrow \dot{x} = -2d\omega \sin(\omega t)$$

$$\vec{E}_i = \vec{e}_y \cdot (-2d\omega \sin(\omega t))\cdot B = -\vec{e}_y\, 2\cdot d\cdot \omega\cdot B\cdot \sin(\omega\cdot t)$$

VEZALNA NAPETOST



$$\boxed{\vec{E}_i = \vec{v}\times\vec{B}}$$

$$U_i = \int_0^l \left(-\vec{e}_y\, 2\cdot d\cdot \omega\cdot B\cdot \sin(\omega t)\right)(-\vec{e}_y)\, dl$$

$$U_i = 2 d\omega B l \cdot \sin(\omega\cdot t)$$

treut. moč: $p(t) = \frac{U_R^2(t)}{R}$

povprečna moč: $\overline{P} = \frac{1}{T}\int_0^T p(t)\, dt$

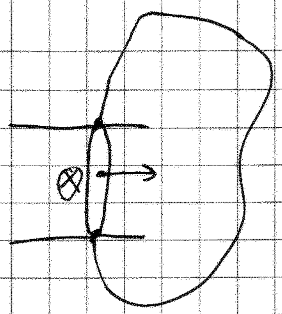
$$\overline{P} = \frac{\overline{(2\cdot\omega\cdot d\cdot B\cdot l\, \overbrace{\sin(\omega\cdot t)}^{\frac{1}{2}})^2}}{R}$$

$$\overline{P} = \frac{2\cdot\omega^2\cdot d^2\cdot l^2\cdot B^2}{R}$$

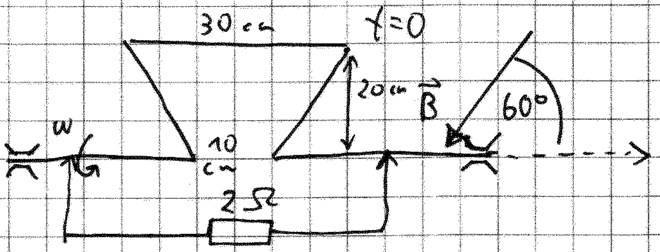
$$\Phi = \vec{B}\cdot\vec{S} = B\cdot S\cdot\cos\varphi$$

$$\frac{d\Phi}{dt} = \frac{d(BS)}{dt} = B\frac{dS}{dt} + \frac{dB}{dt}\cdot S$$

$$\frac{d\Phi}{dt} = \frac{d}{dt}(B\cdot S\cdot\cos\varphi)$$



Trapezna oblikovana pentlja rotira s $\nu = 25\ Hz$ v $B = 100\ mT$. Smer B oklepa z osjo vrtenja kot $\alpha = 60^\circ$. Koliko toplote se sprosti na uporu v $t_1 = 60\ s$?



$$B_y = |\vec{B}|\cdot\sin\alpha$$

$$S = 20\,cm \times 20\,cm$$

$$\Phi = B_y\cdot S\cdot\sin(\omega\cdot t)$$

plan: $\Phi \rightarrow U_i \rightarrow P \rightarrow Q$

$$U_i = -\frac{d\Phi}{dt} = -\frac{d}{dt}(B_y\cdot S\cdot\sin\omega t) = -B_y\cdot S\cdot\omega\cdot\cos\omega t$$

$$P(t) = \frac{U_i^2}{R} = \frac{B_y^2 S^2\omega^2\cos^2(\omega t)}{R}$$

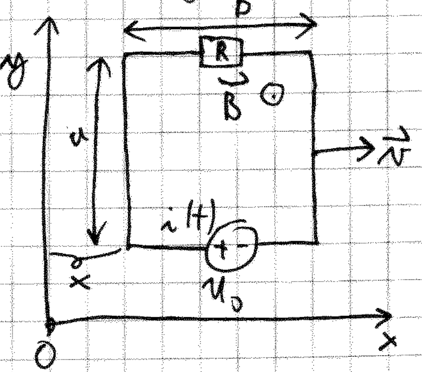
$$Q = \int_0^{t_1} P\cdot dt = \int_0^{t_1}\frac{B_y^2 S^2\omega^2}{R}\cos^2(\omega t)\,dt = \frac{B_y^2 S^2\omega^2}{R}\int_0^{t_1}\frac{1}{2}(1+\cos(2\omega t))\,dt =$$

$$= \frac{B_y^2 S^2\omega^2}{R}\left[\frac{1}{2}\int_0^{t_1}dt + \frac{1}{2}\int_0^{t_1}\cos(2\omega t)\,dt\right] = \frac{B_y^2\cdot S^2\cdot\omega^2 t_1}{R}\left[\frac{1}{2} + \frac{1}{2}\frac{1}{2\omega t_1}\sin(2\omega t_1)\right]$$

$$\underline{\underline{Q = \frac{B_y^2 S^2\omega^2 t_1}{2R}}}$$

$$Q = \frac{B^2\cdot\sin^2\alpha\,\omega^2\cdot t_1}{2R} = \frac{18}{4} \approx 4{,}5\ J$$

Pravokotna zanka $a \times b$, v katero je vgrajen vir napetosti U_g, se giblje s hitrostjo $\vec{v} = \vec{e}_x \cdot v_0$. Določite časovno odvisnost toka $i(t)$ v zanki, če je $\vec{B}(x) = \vec{e}_z \cdot B_0 \cdot e^{-x/b}$, ob $t=0$ pa leva stranica sovpada z osjo y!



$$\phi(x) = \iint \vec{B} \cdot d\vec{S} = \int_x^{x+b} B_0 \cdot e^{-\frac{x'}{b}} a \, dx' =$$

$$= B_0 a \int_x^{x+b} B_0 e^{-\frac{x'}{b}} dx' = B_0 a (-b) e^{-\frac{x'}{b}} \Big|_x^{x+b}$$

$$= -a b B_0 \left[e^{-\frac{x+b}{b}} - e^{-\frac{x}{b}} \right]$$

$$\phi(x) = +a \cdot b \cdot B_0 \cdot e^{-\frac{x}{b}} \left[1 - \frac{1}{e}\right]$$

$$x = v_0 \cdot t$$

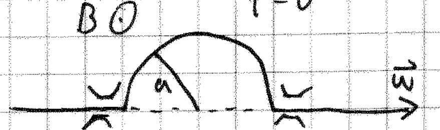
$$\phi(t) = a \cdot b \cdot B_0 \left[1 - \frac{1}{e}\right] \cdot e^{-\frac{v_0}{b} \cdot t}$$

$$\frac{d\phi(t)}{dt} = a \cdot b \cdot B_0 \cdot \left[1 - \frac{1}{e}\right] \cdot \left(-\frac{v_0}{b}\right) \cdot e^{-\frac{v_0}{b} t}$$

$$U_i(t) = a \cdot v_0 \cdot B_0 \left[1 - \frac{1}{e}\right] \cdot e^{-\frac{v_0}{b} \cdot t}$$

$$i(t) = \frac{U_0}{R} - \frac{U_i}{R} = \frac{1}{R}\left[U_0 - a v B_0 \left(1 - \frac{1}{e}\right) \cdot e^{-\frac{v_0}{b} t}\right]$$

Polkrožna zanka $a = 5\,cm$ se enakomerno vrti v $B = 50\,mT$, določite frekvenco vrtenja, da bo maksimalna vrednost inducirane napetosti $U_{max} = 3\,V$!



$$\phi(t) = B \cdot S \cdot \cos\varphi = B \cdot \frac{\pi a^2}{2} \cdot \cos(\omega \cdot t)$$

$$U_i(t) = -\frac{d\phi}{dt} = -\frac{d}{dt}\left(\frac{1}{2}\pi B a^2 \cos(\omega t)\right)$$

$$= \frac{1}{2}\pi B a^2 \omega \sin(\omega t)$$

$$U_{max} = \frac{1}{\cancel{2}} \pi B a^2 \cancel{2}\pi \nu$$

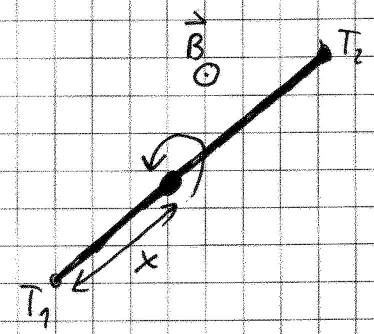
$$U_{ma} = \pi^2 B a^2 \nu$$

$$\nu = \frac{U_{max}}{\pi^2 B a^2}$$

Dimenzijska analiza: $\frac{1}{s}$

$$\nu = 2400\,Hz$$

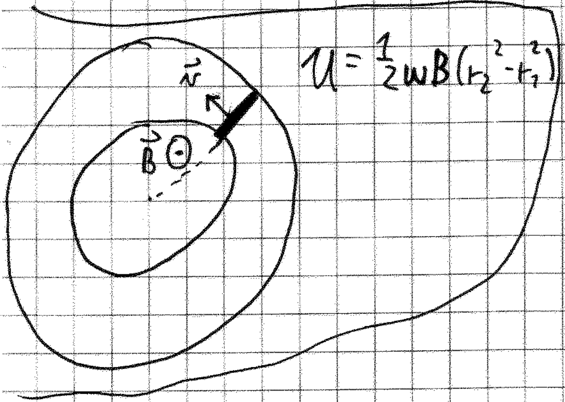
Kovinska palica dolžine $l = 20\,\mathrm{cm}$ bomo vrteli s frekvenco $\nu = 100\,\mathrm{Hz}$ pravokotno na $B = 0{,}1\,\mathrm{T}$. Določite položaj osi, da bo napetost med T_1 in T_2 enaka $U_{12} = 0{,}5\,\mathrm{V}$.



$$U_i = \frac{1}{2}\omega B\left((l-x)^2 - x^2\right)$$
$$= \frac{1}{2}\omega B\left(l^2 - 2lx\right) = \frac{1}{2}\omega B l (l - 2x)$$

$$-\frac{2U_i}{\omega B l} + l = 2x$$
$$x = -\frac{U_i}{\omega B l} + \frac{l}{2}$$

$$x = 6.02\,\mathrm{cm}$$



$$U = \frac{1}{2}\omega B(r_2^2 - r_1^2)$$

15.04.2008

Induktivnost

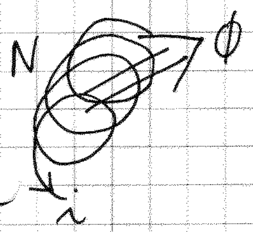
indukcija: $u_i = -\frac{d\phi}{dt}$

$\phi \approx B \cdot S$ ← i

$\phi = L i$ (če se da - linearna zveza)

$u_i = -\frac{d(L \cdot i)}{dt} = -L\frac{di}{dt}$

Lastna induktivnost

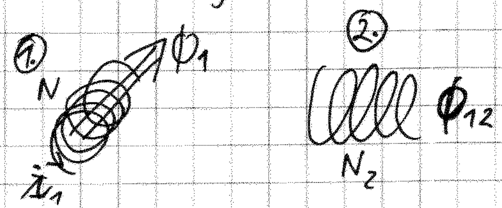


$L = \frac{N \cdot \phi}{i}$ $\quad N \cdot \phi = \Psi$

Faktor stresanja

$\sigma = 1 - k^2 \quad \sigma = [0,1)$

Medsebojna induktivnost

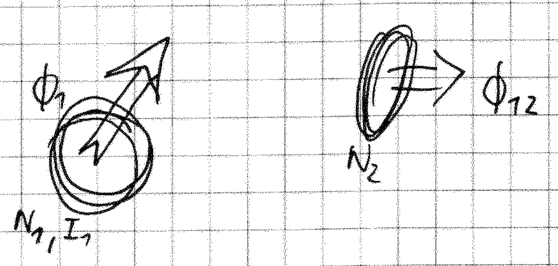


$L_{12} = \frac{N_2 \cdot \phi_{12}}{i_1} = L_{21}$

Faktor sklopa

$k = \frac{L_{12}}{\sqrt{L_1 L_2}} \quad k \in [0,1)$

Celotni pretok skozi krožno zanko ① z $N_1 = 100$ ovoji je pri toku $I_1 = 0,5A$ enak $\phi_1 = 0,4\ mVs$. V takem primeru je pretok skozi zanko ② z $N_2 = 20$ ovoji enak $\phi_{12} = 0,1\ mVs$. Določite medsebojno in obe lastni induktivnosti.



$L_{12} = \frac{N_2 \cdot \phi_{12}}{I_1} = 4\ mH$

$L_1 = \frac{N_1 \cdot \phi_1}{I_1} = = 80\ mH$

L_2 - ni podatkov

40% fluksa prve tuljave doseže drugo in 90% fluksa druge tuljave doseže prvo. Kolikšen je fluks sklopa?

$0.4\,\phi_1 = \phi_{12}$

$0.9\,\phi_2 = \phi_{21}$

$$k = \frac{L_{12}}{\sqrt{L_1 L_2}} = \frac{\frac{N_2 \Phi_{12}}{I_1}}{\sqrt{\frac{N_1 \phi_1}{I_1} \cdot \frac{N_2 \phi_2}{I_2}}} = \ldots \text{ ne gre}$$

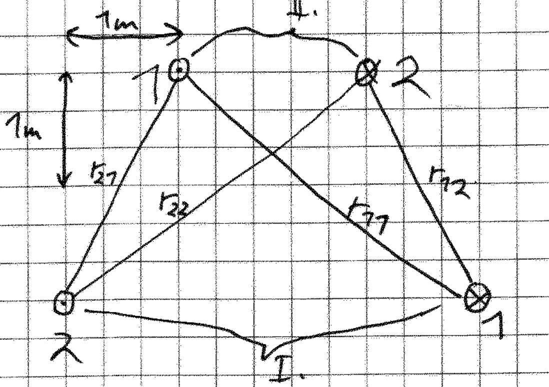
$$L_{12} = \sqrt{L_{12} \cdot L_{12}} = \sqrt{L_{12} \cdot L_{21}}$$

$$k = \frac{\sqrt{L_{12} L_{21}}}{\sqrt{L_1 L_2}} = \frac{\sqrt{\frac{\cancel{N_2} \phi_{12}}{\cancel{I_1}} \frac{\cancel{N_1} \phi_{21}}{\cancel{I_2}}}}{\sqrt{\frac{\cancel{N_1} \phi_1}{\cancel{I_1}} \frac{\cancel{N_2} \phi_2}{\cancel{I_2}}}} = \sqrt{\frac{\phi_{12}}{\phi_1} \cdot \frac{\phi_{21}}{\phi_2}} =$$

$$k = \sqrt{0.4 \cdot 0.9} = \sqrt{0{,}36} = 0{,}6$$

Za dvovod na sliki določi lastni, medsebojni induktivnosti, faktor sklopa in faktor stresanja mag. polja.

$\rho_0 = 1\,mm$
$l = 1\,km$



$r_{12} = \sqrt{5}$
$r_{21} = r_{12} = \sqrt{5}$
$r_{11} = \sqrt{13}$
$r_{22} = \sqrt{13}$

Formula za dvovode

$$L_1 = \frac{\mu_0 l}{\pi}\left[\frac{1}{4} + \ln\frac{d}{\rho_0}\right]$$

$$L_{12} = \frac{\mu_0 l}{2\pi} \ln\frac{r_{12} \cdot r_{21}}{r_{11} \cdot r_{22}}$$

$$L_1 = \frac{\mu_0 l}{\pi}\left[\frac{1}{4} + \ln\frac{d_1}{\rho_0}\right] = \frac{4\pi \cdot 10^{-7} \cdot 10^3}{\pi}\left[0{,}25 + \ln\frac{4}{10^{-3}}\right]$$

$$L_1 = 3.42 \cdot 10^{-3}\,H$$

$$L_2 = \frac{\mu_0 l}{\pi}\left[\frac{1}{4} + \ln\frac{d_2}{\rho_0}\right] = \frac{4 \cdot \pi \cdot 10^{3}}{\pi}\left[0{,}25 + \ln\frac{2}{10^{-3}}\right]$$

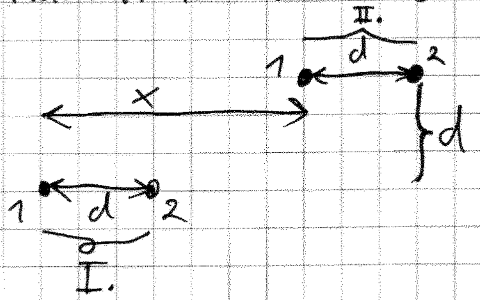
$$L_2 = 3.14 \cdot 10^{-3}\,H$$

$$L_{12} = \frac{\mu_0 l}{2\pi} \ln\frac{r_{12} \cdot r_{21}}{r_{11} \cdot r_{22}} = \frac{4 \cdot \pi \cdot 10^{-7} \cdot 10^3}{2\pi}\left|\ln\left|\frac{\sqrt{5}}{\sqrt{13}}\frac{\sqrt{5}}{\sqrt{13}}\right|\right| = 2 \cdot 10^{-4}\left|\ln\frac{5}{13}\right| = \underline{\underline{+0{,}191\,mH}}$$

$$k = \frac{L_{12}}{\sqrt{L_1 L_2}} = \frac{0.191}{\sqrt{3{,}42 \cdot 3.14}} = 0{,}06$$

$$\sigma = 1 - k^2 = 0.9964$$

Določite x med dvema enakima dvovodoma, da bo medsebojna induktivnost enaka $\emptyset$!

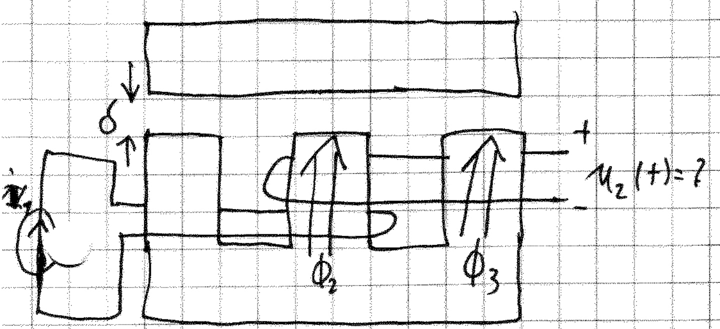


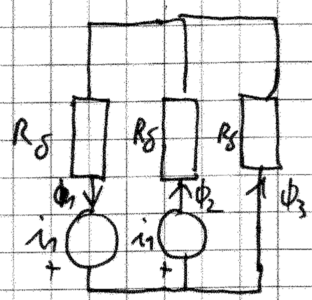
$$r_{11} = \sqrt{x^2+d^2}$$
$$r_{12} = \sqrt{(x+d)^2+d^2}$$
$$r_{21} = \sqrt{(x-d)^2+d^2}$$
$$r_{22} = \sqrt{x^2+d^2}$$

$$L_{12} = \frac{\mu_0 l}{2\pi} \ln \frac{r_{12} \cdot r_{21}}{r_{11} \cdot r_{22}} = \emptyset \Longrightarrow \frac{r_{12} \cdot r_{21}}{r_{11} \cdot r_{22}} = 1$$

$$\sqrt{(x+d)^2+d^2} \cdot \sqrt{(x-d)^2+d^2} = \sqrt{x^2+d^2}^{\,2}$$
$$x^2 = \frac{3}{2} d^2$$
$$x_{12} = \pm d\sqrt{\frac{3}{2}}$$

Izrazite napetost u_2 med sponkama drugega navitja, če je prvo vzbujeno s tokom $i_1(t) = I_0 \cos(\omega t)$, $I_0 = 10\,A$, $\omega = 400\,Hz$, magnetna upornost pa je zanemarljiva glede na upornost špranje širine $\delta = 1\,mm$.





$$u_2 = -\frac{d\phi}{dt} = \frac{d}{dt}(\phi_2 + \phi_3)$$

$$\phi_1 = \phi_2 + \phi_3$$

$$i_1 - i_1 = R_\delta \phi_2 + R_\delta \phi_1 \Rightarrow 0 = \phi_2 + \phi_1 \Rightarrow \phi_1 = -\phi_2$$

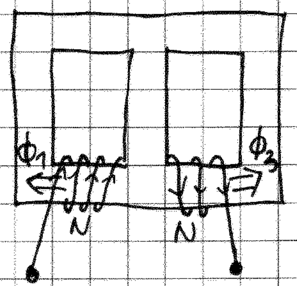
$$i_1 = R_\delta \phi_3 - R_\delta \phi_2 \Rightarrow \frac{i}{R_\delta} = \phi_3 - \phi_2 = -2\phi_2 - \phi_2 = -3\phi_2$$

$$\phi_2 = -\frac{i_1}{3R_\delta}$$
$$\phi_3 = \frac{2 i_1}{3 R_\delta}$$

$$u_2 = \frac{d}{dt}\left[\phi_2 + \phi_3\right] = -\frac{1}{3R_\delta}\, \omega I_0 \sin(\omega t) =$$
$$= -\frac{\omega \mu_0}{3} \frac{S \cdot I_0}{\delta} \sin(\omega \cdot t) = 6.4\,mV$$ 45

$$\phi_2 + \phi_3 = \frac{i_1}{3R_\delta}$$

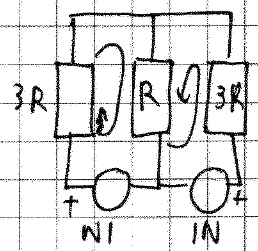
Izrazite induktivnost dvodelnega navitja, če je $R_1 = R_3 = 3R_2 = 3R$.



plan: predpostavimo I, izračunamo Φ

$$L = \frac{\overbrace{N\Phi_1 + N\Phi_3}^{\text{skupni pretok skozi navitje}}}{I_1} = \frac{2N\Phi_1}{I} = \frac{2N}{\cancel{I}}\frac{\cancel{I}N}{5R} = \frac{2N^2}{5R}$$

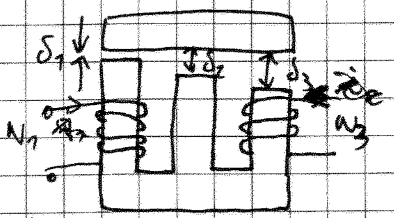
lastna induktivnost

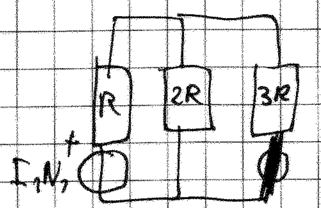


$$\Phi_2 = \Phi_1 + \Phi_3 = 2\Phi_1$$
$$IN = 3R\Phi_1 + R\Phi_2$$
$$IN = 3R\Phi_3 + R\Phi_2$$
$$\Phi_1 = \Phi_3$$
$$IN = 3R\Phi_1 + R\cdot 2\Phi_1 = 5R\Phi_1$$
$$\Phi_1 = \frac{IN}{5R}$$

Izračunajte medsebojno induktivnost navitij na železnem jedru. Mag. upornost železa zanemarite.

$\delta_1 = 0{,}1$ mm
$\delta_2 = 0{,}2$ mm
$\delta_3 = 0{,}3$ mm
$N_1 = 2000$
$N_3 = 3000$
$S = 10\ \mathrm{cm}^2$





$$L_{13} = \frac{N_3\Phi_{13}}{i_1} \quad \Phi_{13} = \Phi_3 \quad i_1 \text{ edini tok} = \frac{N_3\cdot 2\cdot I_1\cdot N_1}{I_1\ 11R} = \frac{2N_1N_3}{11R} =$$

$$R = \frac{\delta}{\mu_0 S} \qquad L_{13} = \frac{2N_1N_3\mu_0 S}{11\delta_1} = 13{,}7\ \mathrm{H}$$

$$\Phi_1 = \Phi_2 + \Phi_3$$
$$I_1N_1 = R_1\Phi_1 + R_2\Phi_2 = R\Phi_1 + 2R\Phi_2$$
$$0 = R_3\Phi_3 - R_2\Phi_2 = 3R\Phi_3 - 2R\Phi_2 = 0 \Rightarrow 3\Phi_3 = 2\Phi_2$$
$$\Phi_2 = \frac{3}{2}\Phi_3$$
$$\Downarrow$$
$$\Phi_3 = \frac{2I_1N_1}{11R} \qquad R = \frac{\delta_1}{\mu_0 S}$$

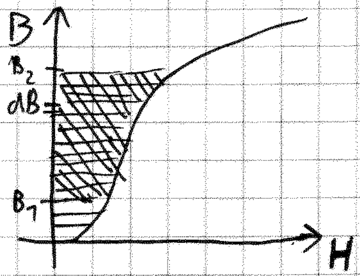
22.04.2008

Energija

gostota energije

$w_m = \int \vec{H}\cdot d\vec{B}$ linearni materiali: $B = \mu \cdot H \Rightarrow w_m = \int \frac{B}{\mu}\, dB = \frac{B^2}{2\mu}\Big|_{B_1}^{B_2}$

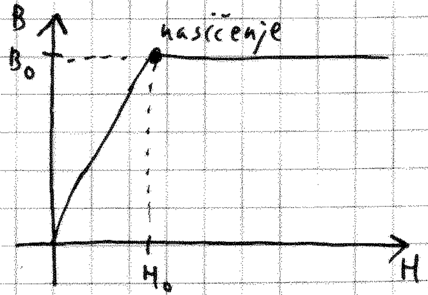
v splošnem:



Celotna energija:

$$W_m = \iiint w_m \, dv$$

Kolikšna energija je potrebna za namagnetenje jedra do nasičenja ($v_0 = 150\ dm^3$, glej sliko)?



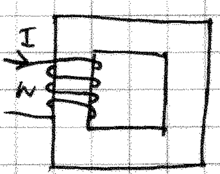
$$w_m = \int \vec{H}\cdot d\vec{B} = \frac{B_0 \cdot H_0}{2}$$

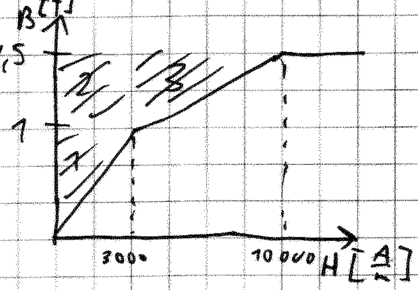
$$W_K = w_m \cdot v_0 = \frac{B_0 \cdot H_0}{2} \cdot v_0 = \frac{1}{2} \cdot 0{,}5 \cdot 50 \cdot 150 \cdot 10^{-3} =$$
$$= 1.875\ J$$

$B_0 = 0{,}5\ T$
$H_0 = 50\ A/m$

Kolikšna je akumulirana energija v jedru?

$N = 200$
$I = 10 A$
$l_s = 20 cm$
$S = 4 cm^2$





H_{max}: $I_{max} \cdot N = H_{max} \cdot l_s$

$$H_{max} = \frac{I_{max} \cdot N}{l} = \frac{10 \cdot 200}{0{,}2} = 10\,000\ \frac{A}{m}$$

$w_1 = 1500\ J/m^3$
$w_2 = 1500\ J/m^3$
$w_3 = 1750\ J/m^3$

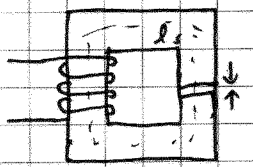
$\vec{w}_m = 4750\ J/m^3$

$$W_m = w_m \cdot v = w_m \cdot l_s \cdot S = 4750 \cdot 0{,}2 \cdot 4 \cdot 10^{-4}$$

$$W_m = 0.38\ J$$

Feromagnetno jedro $l_s = 30\,cm$, $S = 10\,cm^2$ z režo $\delta = 1\,mm$ namagnetimo do gostote $B_0 = 1\,T$. Kolikšen je energijski vložek v magnetenje, če je mag. krivulja materiala podana z izrazom $H = k \cdot B^2$?

$k = 900 \frac{A}{mT^2}$



v reži: $w_{m\delta} = \int_0^{B_0} \vec{H} \cdot d\vec{B} = \int_0^{B_0} \frac{B}{\mu_0} dB = \frac{B_0^2}{2\mu_0}$

$W_{m\delta} = w_m \cdot \delta \cdot S = \frac{B_0^2}{2\mu_0} \cdot \delta \cdot S$

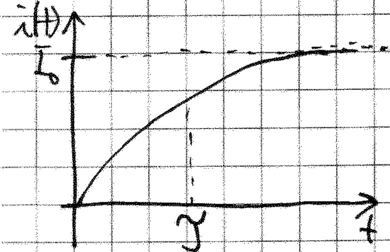
$W_{m\delta} = 0{,}4\,J$

v jedru: $w_{mj} = \int_0^{B_0} \vec{H} \cdot d\vec{B} = \int_0^{B_0} kB^2 \cdot dB = \frac{k}{3} \cdot B^3 \Big|_0^{B_0} = \frac{kB_0^3}{3}$

$W_{mj} = \frac{k \cdot B_0^3}{3} \cdot l_s S = 0{,}09\,J$

$W = W_{m\delta} + W_{mj} = 0{,}49\,J$

Na jedru $l_s = 0{,}5\,m$, $S = 30\,cm^2$, $N = 200$ je navitje s tokom $i(t) = I_0 \cdot (1 - e^{-t/\tau})$, $I_0 = 0{,}4\,A$ in $\tau = 1\,ms$. Magnetilna krivulja jedra $B = k\sqrt{H}$, $k = 0{,}05\,T\sqrt{m/A}$. Kolikšen je energijski vložek v časovnem intervalu od $[0, \tau]$?



$i(t) \rightarrow H(t) \rightarrow B(t)$

$H(t) = \frac{i \cdot N}{l_s} = \frac{N}{l_s} \cdot i(t)$

$B(t) = k \cdot \sqrt{H} = k \cdot \sqrt{\frac{N}{l_s} \cdot i(t)}$

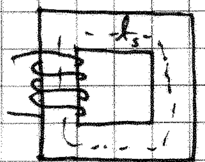
$w_m = \int \vec{H} \cdot d\vec{B} = \int \left(\frac{B}{k}\right)^2 dB$

$H = \left(\frac{B}{k}\right)^2$

$B = B(t)$

$\frac{dB}{dt} = \dot{B}$

$dB = \dot{B} \cdot dt$



$B(t) \Rightarrow B(t) = k\sqrt{\frac{N}{l_s}} \cdot i(t)^{\frac{1}{2}} =$

$= k\sqrt{\frac{N}{l_s}} \cdot I_0^{\frac{1}{2}} (1 - e^{-t/\tau})^{\frac{1}{2}}$

$\frac{dB}{dt} = k\sqrt{\frac{NI_0}{l_s}} \cdot \frac{1}{2} (1 - e^{-\frac{t}{\tau}})^{-\frac{1}{2}} (e^{-\frac{t}{\tau}}) \cdot \left(\frac{1}{\tau}\right)$

$= \frac{k}{2\tau} \sqrt{\frac{NI_0}{l_s}} \cdot \frac{e^{-\frac{t}{\tau}}}{\sqrt{1 - e^{-\frac{t}{\tau}}}}$; $w_m = \int_0^{\tau} \left(\frac{B}{k}\right)^2 \cdot dB$

SIZIFOVO DELO

$w_m = \frac{1}{3k^2} \cdot B^3 \Big|_0^{\tau}$

$w_m = \frac{1}{3k^2} \left[k\sqrt{\frac{NI_0}{l_s}} \cdot \sqrt{1 - e^{-\frac{t}{\tau}}}\right]^3 \Big|_0^{\tau}$

$w_m = \frac{1}{3k} \left[k\sqrt{\frac{NI_0}{l_s}} \cdot \sqrt{1 - e^{-\frac{t}{\tau}}}\right]^3 \Big|_0^{\tau}$

$w_m = \frac{1}{3k^2} k^3 \left(\frac{NI_0}{l_s}\right)^{3/2} \left(1 - \frac{1}{e}\right)^{3/2}$

$W_m = w_m \cdot S\, l_s = 26\,mJ$

$B = k\sqrt{\frac{B}{\mu_0}}$ $\gamma_0 B^2 = k^2 \cdot B$

$B = k^2$

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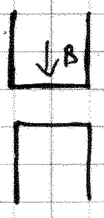
Sila med poloma magneta

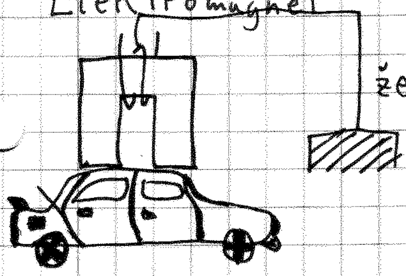
$$F_m \propto \left(\frac{1}{\mu_0\mu_r} + \frac{1}{\mu_0}\right) \ldots \propto \frac{1}{\mu_0}\left(\frac{1}{\mu_r} + 1\right) \ldots$$

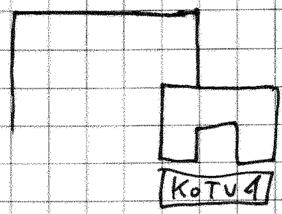


$$\boxed{F_m = \frac{B^2}{2\mu_0} \cdot S}$$

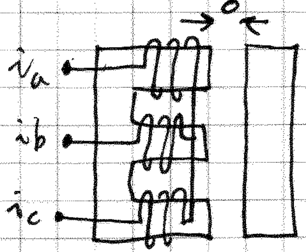
$$\frac{F_m}{S} = \frac{B^2}{2\mu_0}$$

Elektromagnet





Določite povprečno silo $\bar{F}$ na kotvo, če je jedro trifazno vzbujano, gostota v režah pa doseže $B_{max} = 1\,T$! ($S = 1\,cm^2$, $\delta = 0{,}5\,mm$).



$$B = B_0 \cdot \sin(\omega t)$$

$$B^2 = B_0^2 \cdot \sin^2(\omega t)$$

povprečni B^2: $\overline{B^2} = B_0^2 \cdot \frac{1}{2}$

$$F = F_a + F_B + F_c = \frac{\overline{B_A^2}}{2\mu_0} S + \frac{\overline{B_B^2}}{2\mu_0} S + \frac{\overline{B_C^2}}{2\mu_0} S = \frac{S}{2\mu_0} \cdot \left[\overline{B_A^2} + \overline{B_B^2} + \overline{B_C^2}\right]$$

$$F = \frac{3 \cdot S}{4\mu_0} B_0^2 \approx 62\,N$$

1. kol. OET II 5.5.2008

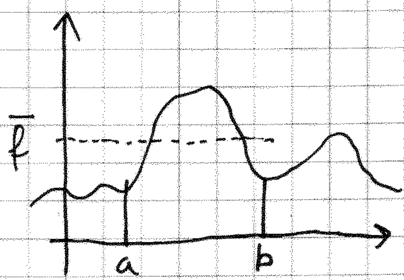
snov: - sila na vodnik v $\vec{B}$
- Biot-Savartov zakon ($\vec{B}$ v okolici tokovnih struktur)
(sestavljeni primeri)
→ (uporaba enačb)
- Amperov zakon za B, $\oint \vec{B} \cdot d\vec{l} = \mu_0 \cdot I$ (dodatne enačbe, polni vodniki itd...)
- magnetni pretok $\phi = \iint \vec{B} \cdot d\vec{s}$
- delo pri premiku zanke $A = I \cdot \Delta I$
- navor na zanko
- Lorentzova sila
- magnetni dipol
- snov v mag. polju, magnetizacija
- vektorji $\vec{M}, \vec{B}, \vec{H}$, $\vec{M} = \frac{\vec{B}}{\mu_0} - \vec{H}$
- mejni pogoji
- pravi Amperov zakon $\oint \vec{H} \cdot d\vec{l} = I_{zaob.}$
- skalarni mag. potencial
- magnetna vezja

06.05.2008

Signali

$f(t)$

povprečna vrednost:



$$\int_a^b f(t)\,dt = \bar{f}\cdot(b-a)$$

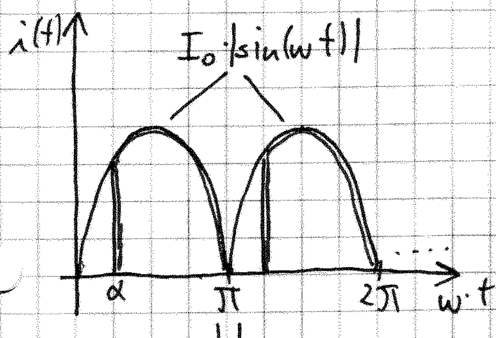
$$\boxed{\bar{f} = \frac{1}{b-a}\int_a^b f(t)\cdot dt}$$

efektivna vrednost:

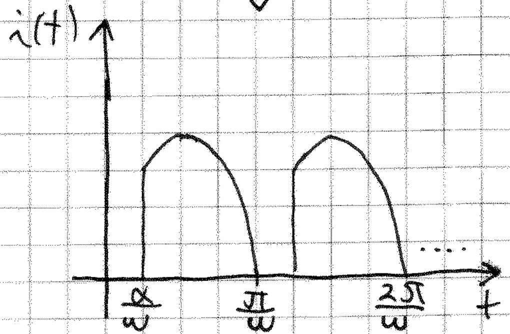
$$\boxed{f_{ef} = \sqrt{\frac{1}{b-a}\int_a^b [f(t)]^2\,dt}}$$

Za periodične signale, ki trajajo ∞ (dosti period) računamo $\bar{f}$ in f_{ef} v času ene periode.

Danemu toku določite povprečno (srednjo) in efektivno vrednost (v odvisnosti od kota α).



$\Downarrow$



$T = \frac{\pi}{\omega}$

$$\bar{I} = \frac{1}{T}\int_0^T i(t)\cdot dt$$

$$i(t) = \begin{cases} \emptyset\,; & t\in[0, \frac{\alpha}{\omega}] \\ I_0\sin(\omega t)\,; & t\in[\frac{\alpha}{\omega}, \frac{\pi}{\omega}] \end{cases}$$

$$\bar{I} = \frac{1}{\frac{\pi}{\omega}}\left[\int_0^{\alpha/\omega} 0\cdot dt + \int_{\alpha/\omega}^{\pi/\omega} I_0 \sin(\omega t)\,dt\right]$$

$$= \frac{1}{\frac{\pi}{\omega}} I_0 \int_{\alpha/\omega}^{\pi/\omega} \sin(\omega t)\cdot dt$$

$$= \frac{\cancel{\omega}}{\pi}\cdot I_0 \cdot \frac{1}{\cancel{\omega}}\left(-\cos(\omega t)\right)\Big|_{\alpha/\omega}^{\pi/\omega}$$

$$= \frac{I_0}{\pi}\left[-(-1) - (-\cos\alpha)\right]$$

α	$\bar{I}$
$\emptyset$	$\frac{2}{\pi}I_0 \cong 0{,}64\, I_0$
π	$\frac{I_0}{\pi}[1-1]=0$
$\frac{\pi}{2}$	$\frac{I_0}{\pi}[1+0] = \frac{I_0}{\pi}$

$$f_{ef}^2 = \frac{1}{T}\int_0^T |f(t)|^2 dt = \frac{1}{\frac{\pi}{\omega}} \cdot \left[\int_0^{\frac{\alpha}{\omega}} 0^2 dt + \int^{\frac{\pi}{\omega}} |I_0 \sin(\omega t)|^2 \cdot dt\right] =$$

$$= \frac{\omega}{\pi} I_0^2 \int_{\frac{\alpha}{\omega}}^{\frac{\pi}{\omega}} \sin^2(\omega t)\cdot dt = \frac{\omega}{\pi} I_0^2 \int_{\frac{\alpha}{\omega}}^{\frac{\pi}{\omega}} \frac{1}{2}\left[1-\cos(2\omega t)\right] dt$$

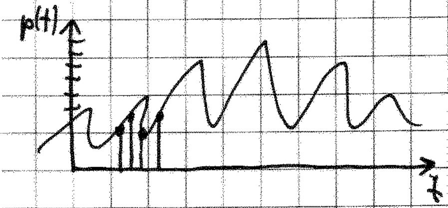
$$= \frac{\omega}{\pi} I_0^2 \frac{1}{2}\left[\int_{\frac{\alpha}{\omega}}^{\frac{\pi}{\omega}} dt - \int_{\frac{\alpha}{\omega}}^{\frac{\pi}{\omega}} \cos(2\omega t) dt\right] = \ldots\ldots$$

$$= \frac{I_0^2}{2\pi}\left[\pi - \alpha + \frac{1}{2}\sin 2\alpha\right] = \frac{I_0^2}{2}\left[1 - \frac{\alpha}{\pi} + \frac{1}{2\pi}\sin 2\alpha\right]$$

$$I_{ef} = \frac{I_0}{\sqrt{2}}\sqrt{1 - \frac{\alpha}{\pi} + \frac{1}{2\pi}\sin \alpha}$$

α	I_{ef}
0	$\frac{I_0}{\sqrt{2}}$
π	$\frac{I_0}{\sqrt{2}}\sqrt{\cancel{1-\frac{\pi}{\pi}}+\frac{1}{2\pi}\cancel{\sin 2\pi}} = 0$

zvok



vzorčna frekvenca (sampling rate) - v x smeri (bit rate) v y smeri

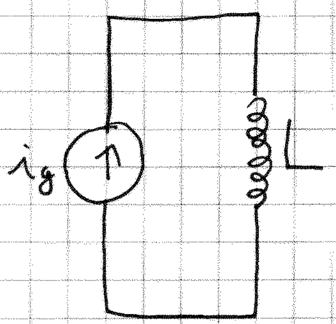
44,1 kHz

$2^3 = 8$ (3-bitni zapis)

16 bitni $= 2^{16}$

32 bitni $= 2^{32}$

Tuljava $L = 100\ mH$ je priključena na tokovni generator $i_g = I_0 \cdot \sin(\omega t)$, $I_0 = 100\ A$, $\omega = 10^3\ Hz$. Določite trenutno napetost na tuljavi $u(t)$ ter trenutno ($W_m(t)$) in povprečno energijo $\overline{W_m}$ v polju tuljave.



$$\boxed{u_L(t) = L \cdot \frac{di}{dt}}$$

$$u_L(t) = L \cdot \frac{d}{dt}\left(I_0 \sin(\omega t)\right) =$$

$$u_L(t) = L \cdot I_0 \cdot \omega \cdot \cos(\omega t) = 10^4 \cos(\omega t)\ V$$

$$u_0 = 10\ kV$$

$$\boxed{W_m(t) = \frac{1}{2} L \cdot i^2} = \frac{1}{2} \cdot L \left[I_0 \cdot \sin(\omega t)\right]^2 = \frac{1}{2} L \cdot I_0^2 \sin^2(\omega t) =$$

$$= \frac{1}{2} \cdot 0{,}1 \cdot 100 \cdot 100 \cdot \sin^2(\omega t) = 500 \sin^2(\omega t)\ J$$

$$\boxed{\overline{W_m} = \overline{\frac{1}{2} L \cdot I_0^2 \sin^2(\omega t)}} = \frac{1}{2} L I_0^2 \underbrace{\overline{\sin^2(\omega t)}}_{\frac{1}{2}} = \frac{1}{4} \cdot L \cdot I_0^2 = 250\ J$$

Prehodni pojavi

$$u_L(t) = L \cdot \frac{di}{dt}$$

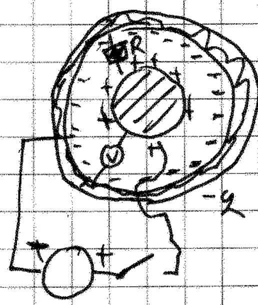
$$u_R(t) = R \cdot i_R(t)$$

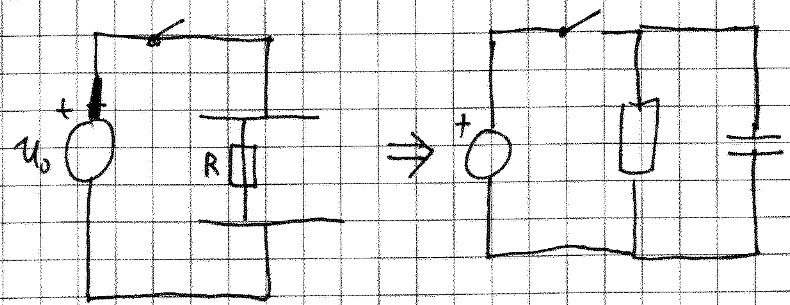
$$i_C(t) = C \cdot \frac{du}{dt}$$

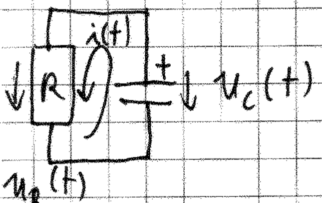
Veljata Kirchhofova zakona.

$$u_C(t) - u_C(0) = \frac{1}{C} \int_0^t i_C(t)\, dt$$

Enožilni kabel dolžine $l = 2$ km ima $C = 150\,\mu F$ in izgubno upornost $R = 1\ G\Omega$. Ob odklopu je bila napetost med žilama $u_0 = 60$ kV. V kolikšnem času napetost pade na 50 V?







$$u_c(t) = u_R(t) = u(t)$$

$$R \cdot i(t) \qquad u_c(0) + \frac{1}{C}\int_0^t i_c(t)\,dt$$

$$\dot{u}_c(t) = \dot{u}_r(t)$$

$$\dot{u}_c(t) = \frac{1}{C}\, i_c(t) = -\frac{1}{C} i \qquad i_c = -i_R$$

$$\dot{u}_R(t) = R \cdot \frac{di_R(t)}{dt} = R \cdot \frac{di}{dt}$$

$$\boxed{-\frac{1}{C} i = R \frac{di}{dt}} \Rightarrow -\frac{1}{RC} = \frac{1}{i}\frac{di}{dt}$$

$$-\frac{1}{RC}\, dt = \frac{di}{i(t)}$$

$$-\frac{1}{RC}\int_0^t dt = \int_{i(0)}^{i(t)} \frac{di}{i}$$

$$-\frac{1}{RC} \cdot t = \ln i \Big|_{i(0)}^{i(t)}$$

$$-\frac{1}{R \cdot C} \cdot t = \ln \frac{i(t)}{i(0)}$$

$$e^{-\frac{t}{RC}} = \frac{i(t)}{i(0)}$$

$$\boxed{i(t) = i(0) \cdot e^{-\frac{t}{RC}}}$$

$$\boxed{i(t) = i(0) \cdot e^{-t/RC}} \qquad RC = \tau \text{ - relaksacijski čas}$$

$$u_R = R \cdot i(t) = R \cdot i(0) \cdot e^{-t/RC} = \boxed{u_0 \cdot e^{-\frac{t}{\tau}} = u_R}$$

$$u_R(t=0) = u_0 = R \cdot i(0) \cdot e^{-0/\tau}$$

$$i(0) = \frac{u_0}{R}$$

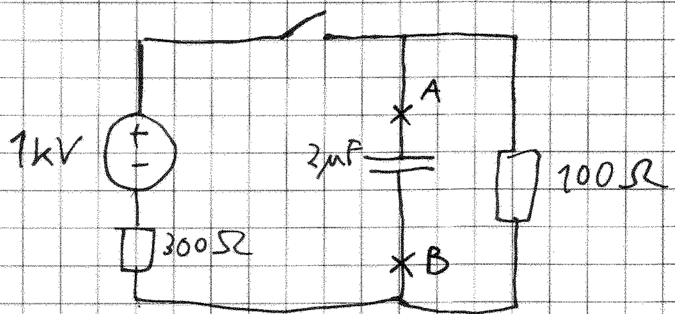
$$u(t) = u_{50} = u_0 \cdot e^{-\frac{t_{50}}{\tau}}$$

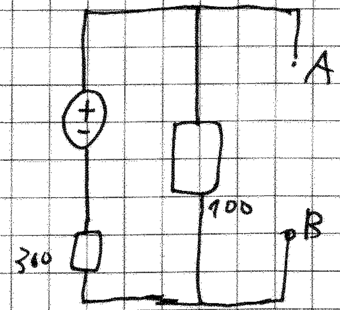
$$\frac{u_{50}}{u_0} = e^{-\frac{t_{50}}{\tau}}$$

$$-\tau \ln \frac{u_{50}}{u_0} = t_{50}$$

$$t_{50} = 1063{,}5\ s = 17\ \text{min}$$

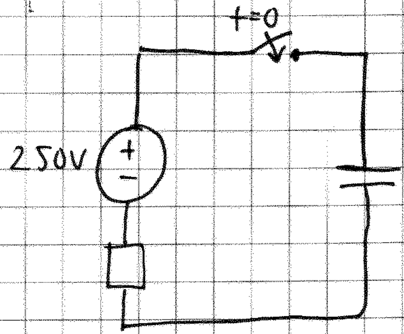
Kondenzator $C = 2\,\mu F$ ima med ploščama razmika $\delta = 0{,}1$ mm dielektrik prebojne trdnosti $E_p = 2\ {}^{mV}/_{m}$. Ob katerem času po vklopu stikala bo prišlo v kondenzatorju do preboja.





$$R_{Th} = \frac{100 \cdot 300}{100+300} = \frac{300}{4} = 75\,\Omega$$

$$U_{Th} = 100 \cdot \frac{1000}{300+100} = 250\ V$$



$$\tau = R \cdot C$$

polnjenje: $\boxed{U_c(t) = U_0\left(1 - e^{-\frac{t}{\tau}}\right)}$

preboj pri $U_p = E_p \cdot \delta$

$$E_p \cdot \delta = U_0\left(1 - e^{-\frac{t}{\tau}}\right)$$

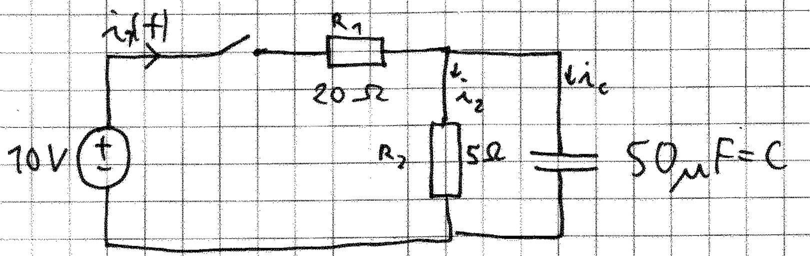
$$\frac{E_p \cdot \delta}{U_0} = 1 - e^{-\frac{t}{\tau}}$$

$$e^{-\frac{t}{\tau}} = 1 - \frac{E_p \cdot \delta}{U_0}$$

$$t = -\tau \ln\left(1 - \frac{E_p \cdot \delta}{U_0}\right)$$

$$t = 241{,}4\ \mu s$$

Določite $i(t)$ po vklopu stikala.



$i_1 = i_1(t)$
$i_2 = i_2(t)$
$i_3 = i_3(t)$

$\underline{i_1(t) = i_2(t) + i_c(t)}$ 3 enačbe za tri tokove

$\underline{U_g = R_1 i_1 + R_2 \cdot i_2}$

$\dot{U}_2 = \dot{U}_c$

$\underline{R_2 \frac{di_2}{dt} = \frac{1}{C} i_c} \Rightarrow i_c = R_2 C \frac{di_2}{dt}$

$$\boxed{\begin{aligned} i_1 &= i_2 + R_2 C \frac{di_2}{dt} \\ U_g &= R_1 i_1 + R_2 i_2 \end{aligned}}$$ 2 enačbi za 2 tokova

$$\boxed{U_g = R_1 \cdot \left(i_2 + R_2 \cdot C \frac{di_2}{dt}\right) + R_2 i_2}$$ 1 enačba za i_2

$i_2 = y$

$U_g = R_1 \left(y + R_2 C \frac{dy}{dt}\right) + R_2 y$

$U_g = (R_1 + R_2) y + R_1 R_2 C \frac{dy}{dt}$; Nastavek: $y = A + B e^{\lambda t}$

$\frac{dy}{dt} = \lambda B \cdot e^{\lambda t}$

začetni/končni pogoji:

$y(t=\infty) = \frac{U_g}{R_1 + R_2}$; $\lambda < 0$

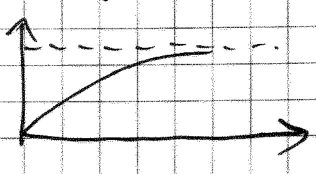
$A = \frac{U_g}{R_1 + R_2}$

$i_2(t=0) = y(t=0) = 0$ (ker je $U_c = 0$, U_c pa je enak $U_c = U_R$)

$y(t=0) \to A + B = 0$

$B = -A$

$y = A - A e^{\lambda t} = A(1 - e^{\lambda t}) = \underline{\frac{U_g}{R_1 + R_2}(1 - e^{\lambda t})}$ ker je $\lambda < 0$



13.05.2008

Harmonični viri in vezja

$R: u_R = R \cdot i_R$

$L: u_L = L \cdot \frac{di_L}{dt}$

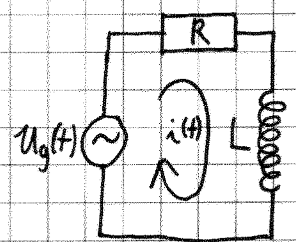
$C: u_C = C \frac{du_C}{dt}$

Kolikšna je napetost na uporu 10Ω v trenutku, ko je napetost na tuljavi $L = 5mH$ enaka 0.

$u_g(t) = U_g \sin(\omega t)$

$U_g = 10V$

$\omega = 1000\ Hz$



$u_R(t) = ?$

$u_L(t) = ?$

način ① (časovni prostor):

$$u_g(t) = u_R(t) + u_L(t)$$

$$u_g(t) = R \cdot i(t) + L \cdot \frac{di(t)}{dt}$$

$$U_g \cdot \sin(\omega t) = R \cdot i(t) + L \cdot \frac{di(t)}{dt}$$

nastavek za tok $i(t) = I_0 \cdot \sin(\omega t + \varphi)$

$$\frac{di(t)}{dt} = \omega \cdot I_0 \cdot \cos(\omega t + \varphi)$$

$$U_g \cdot \sin(\omega t) = R \cdot I_0 \cdot \sin(\omega t + \varphi) + \omega \cdot L \cdot \cos(\omega t + \varphi)$$

$$U_g \cdot \sin(\omega t) = R I_0 [\sin(\omega t)\cos\varphi + \cos(\omega t)\sin\varphi] + \omega L I_0 [\cos(\omega t)\cos\varphi - \sin(\omega t)\sin\varphi]$$

$\sin(\omega t)$: $U_g = R I_0 \cos\varphi - \omega L I_0 \sin\varphi$

$\cos(\omega t)$: $0 = R \cancel{I_0} \sin\varphi + \omega L \cancel{I_0} \cos\varphi \Rightarrow \frac{\sin\varphi}{\cos\varphi} = -\frac{\omega L}{R} = \tan\varphi$

$\varphi = -26,6°$

$$I_0 = \frac{U_g}{R\cos\varphi - \omega L \sin\varphi} = 0,894\ A$$

$$i(t) = 0,894 \cdot \sin(\omega t - 26,6°)\ A$$

$$u_L(t) = L \cdot \frac{di}{dt} = \omega L I_0 \cdot \cos(\omega t - 26,6°) = 4,47 \cdot \cos(\omega t - 26,6°) = 4,47 \sin(116,6 \omega t)$$

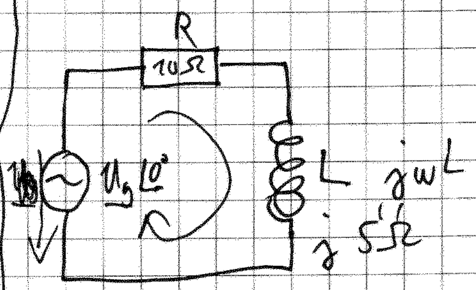
$$= -4,47 \cdot \sin(\omega t - 116,6)$$

$$u_R(t) = 10 \cdot 0,894 \cdot \sin(\omega t - 26,6°)$$

$$\omega t_0 = 116,6°$$

$$u_R(t_0) = 8,94 \sin(116,6° - 26,6°) = 8,94 \cdot \sin 90° = \underline{\underline{8,94\ V}}$$

način ② (kompleksni prostor):



$i_L = I_L \cdot e^{j\omega t}$

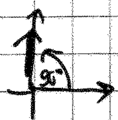
$u_L = L \cdot \frac{di}{dt} = L \cdot j\omega \cdot I_L \cdot e^{j\omega t}$

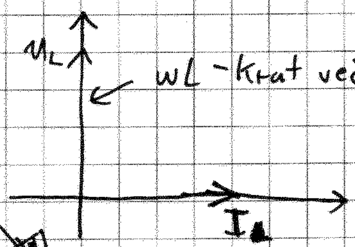
$U_L \cdot e^{j\omega t} = j\omega L I_L \cdot e^{j\omega t}$

$$\boxed{\frac{U_L}{I_L} = j\omega L}$$

$U_L = j \omega \cdot L \cdot I_L$

$= \omega \cdot L \cdot I_L \cdot e^{j90°}$

$j \Rightarrow$  $\to 1 \cdot e^{j90°}$



ωL - krat večja kot $|I_L|$

$u_L(t) = U_L \cdot \sin(\omega t)$

$$\left.\begin{array}{l}\sin(\omega t + \varphi) \\ \cos(\omega t + \varphi)\end{array}\right\} = e^{j(\omega t + \varphi)}$$

$\boxed{e^{i\pi} + 1 = 0}$

$e^{j\varphi} = \cos\varphi + j\sin\varphi$

$\frac{d}{dt}\left(e^{j\omega t}\right) = j\omega e^{j\omega t}$

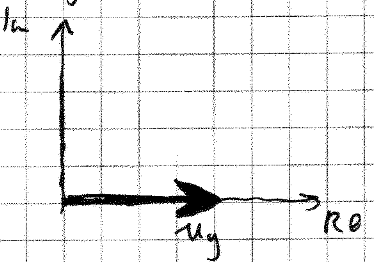
$e^{j63°} = \angle 63°$

$$\boxed{\begin{array}{l} X_L = j\omega L \\ X_C = -\frac{j}{\omega L}\end{array}}$$ impendance

$z = a + jb$

$z = \sqrt{a^2 + b^2}\, e^{j \arctan \frac{b}{a}}$

$\underline{U_g} = U_g \angle 0°$



$\underline{U_g} = (10 + j5) \cdot \underline{I}$

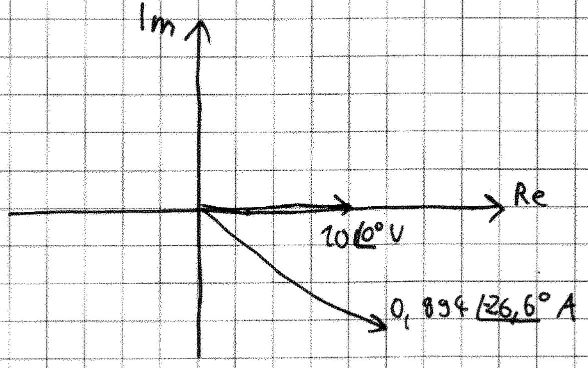
$\underline{I} = \frac{U_g}{10 + j5} = \frac{10 \angle 0°}{10 + j5} = \frac{10(10 - j5)}{(10 + j5)(10 - j5)} = \frac{100 - j50}{10^2 + 5^2}$

$\underline{I} = \frac{100 - j50}{125} = \frac{4 - 2j}{5} = (0{,}8 - j0{,}4)\ A$

$\underline{I} = \sqrt{0{,}8^2 + 0{,}4^2}\ \angle \arctan \frac{-0{,}4}{0{,}8}$

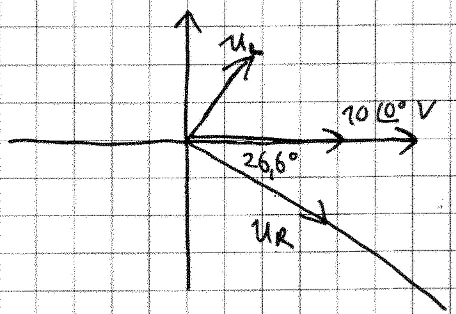
$\underline{I} = 0{,}894 \angle -26{,}6°\ A$

KAZALČNI DIAGRAM



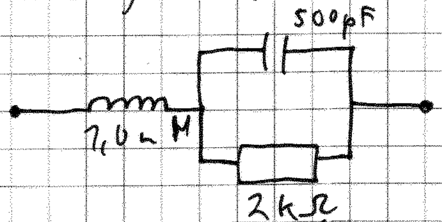
$$U_R = R \cdot I = 10 \cdot 0{,}894\angle 26{,}6^\circ$$
$$= \underline{\underline{8{,}94\angle 26{,}6^\circ\ V}}$$

$$U_L = j5 \cdot I = 5 \cdot \angle 90^\circ \cdot 0{,}894\angle -26{,}6^\circ = \underline{\underline{4{,}47\angle 63{,}4^\circ}}$$

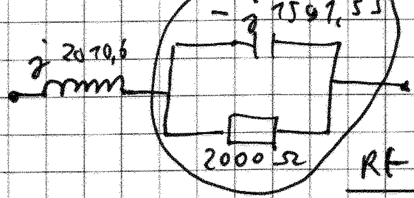


$$\boxed{\begin{aligned} \underline{Z} &= R + jX \\ \underline{Y} &= \frac{1}{\underline{Z}} \end{aligned}}$$

Izračunajte impendanco in admintanco dvopola pri $f = 200$ kHz.



$\Rightarrow$



$$\frac{R(-jX_c)}{R-jX_c}$$

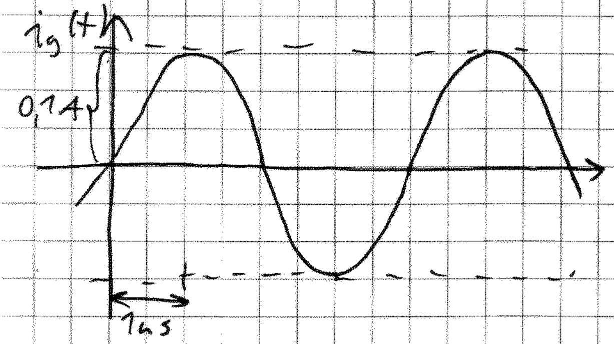
$$X_c = -\frac{1}{\omega C} = -\frac{1}{2\pi f \cdot C} = -1591{,}5\ \Omega$$

$$X_L = \omega \cdot L = 2\pi f \cdot L = 2010{,}6\ \Omega$$

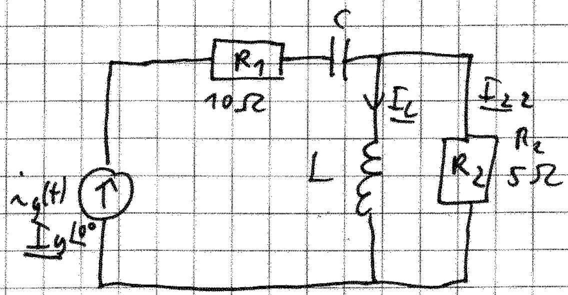
$$\underline{Z} = jX_L + \frac{R(-jX_c)}{R-jX_c} \cong (776 + j1036)\ \Omega$$

$$\underline{Y} = \frac{1}{\underline{Z}} = (463 - j618)\ \mu S$$

Določite potek napetosti na uporu R_2, ki je priključen na idealni tokovni vir, katerega signal je na sliki.



$R_1 = 10\ \Omega$
$R_2 = 5\ \Omega$
$L = 1\ mH$
$C = 1\ \mu F$



$i_g = I_g \sin(\omega t)$

$I_g = 0{,}1\ A$
$\omega = 2\pi f = 2\pi \cdot 250\ Hz$

$X_C = \frac{1}{j\omega C} = -j\,637\ \Omega$

$X_L = j\omega \cdot L = j\,1{,}57\ \Omega$

$\boxed{\underline{I}_g = \underline{I}_L + \underline{I}_2} = -j\frac{5}{1{,}57}\underline{I}_2 + \underline{I}_2 = \underline{I}_2\left[1 - j\frac{5}{1{,}57}\right]$

$\underline{U}_L = \underline{U}_2$

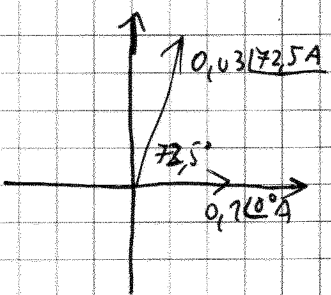
$\boxed{j1{,}57 \cdot \underline{I}_L = 5 \cdot \underline{I}_2}$

$\underline{I}_L = \frac{5}{j1{,}57}\underline{I}_2 = -j\frac{5}{1{,}57}\underline{I}_2$

$\underline{I}_2 = \frac{\underline{I}_g}{1 - j\frac{5}{1{,}57}} =$

$\underline{I}_2 = 9 \cdot 10^{-3} + j\,28{,}6 \cdot 10^{-3}$

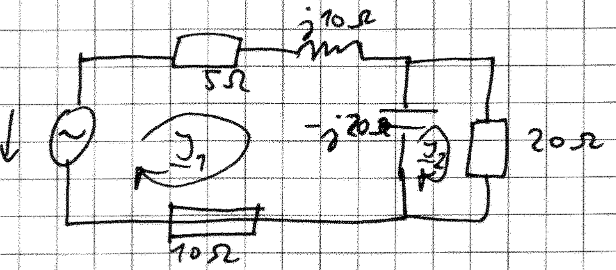
$\underline{I}_2 = 0{,}03\angle 72{,}5°\ A$



$i_2 = 0{,}03 \cdot \sin(\omega t + 72{,}5°)\ A$

$u_2 = 0{,}15 \cdot \sin(\omega t + 72{,}5°)\ V$

Določite delovno moč na bremenu:



moč na upora:

$p_R(t) = u_R \cdot i_R$

$\bar{P} = \int \frac{u^2}{R}\,dt = \frac{U_{max}^2}{2R}$ DELOVNA MOČ

~~moč na tuljavi~~

Q – jalova moč

S = celotna moč

$\underline{S} = P + jQ$

MOČ

$\underline{S} = P + jQ$

na elementu

$\underline{S} = \underline{U}_{ef}\,\underline{I}_{ef}^*$

$\underline{S} = \frac{1}{2}\underline{U}_m \cdot \underline{I}_m^* = \frac{1}{2}\underline{Z}\,\underline{I}_m \cdot \underline{I}_m^* =$

$\underline{S} = \frac{1}{2}\underline{Z}\,|\underline{I}|^2$

$100 = (5 + j10 - j20 + 10)\underline{J}_1 - (-j20)\underline{J}_2$

$0 = (-j20 + 20)\underline{J}_2 - (-j20)\underline{J}_1$

$100 = (15 - j10)\underline{J}_1 + j20\underline{J}_2$

$0 = +j20\,\underline{J}_1 + (20 - 20j)\underline{J}_1$

Matrični sistem

$$\begin{pmatrix} 15-j10 & j20 \\ j20 & 20-j20 \end{pmatrix} \begin{pmatrix} \underline{J}_1 \\ \underline{J}_2 \end{pmatrix} = \begin{pmatrix} 100 \\ 0 \end{pmatrix}$$

$j\underline{J}_1 + (1-j)\underline{J}_2 = 0$

$\underline{J}_1 = -\frac{1-j}{j}\cdot\underline{J}_2 = j\cdot(1-j)\underline{J}_2 = (1+j)\underline{J}_2$

$100 = (15 - j10)(1+j)\underline{J}_2 + j20\,\underline{J}_2$

$100 = [15 - j10 + j15 + 10 + j20]\,\underline{J}_2$

$100 = (25 + j25)\underline{J}_2$

$4 = (1 + j)\,\underline{J}_2$

~~$\underline{J}_2 = \frac{20}{5+j} \cdot \frac{5-j}{5-j} = \frac{20\cdot(5-j)}{25+1} = \frac{10}{13}(5-j)$~~

~~$\underline{J}_1 = (1+j)\frac{10}{13}(5-j)$~~

~~$\underline{J}_1 = \frac{10}{13}(6+j4)\,A$~~

$\underline{J}_2 = \frac{4}{1+j} = \frac{4\cdot(1-j)}{1^2+1^2} = 2\cdot(1-j)$

$\underline{J}_1 = (1+j)\cdot 2(1-j)$

$\underline{J}_1 = 4\,A$

Delovna moč se troši na R-ih:

$P_5 = 5\cdot|\underline{J}_1|^2 = 80\,W$

$P_{10} = 10\cdot|\underline{J}_1|^2 = 160\,W$

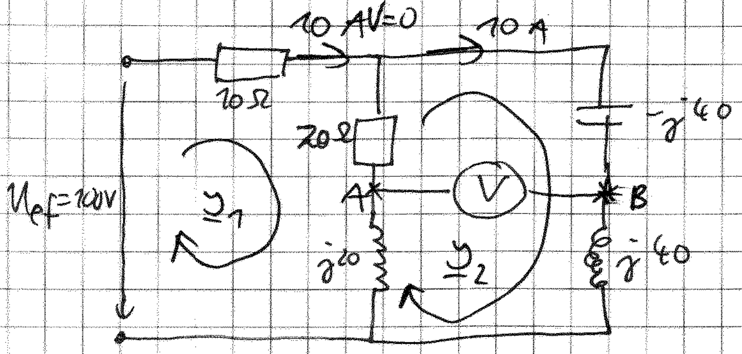
$P_{20} = 20\cdot|\underline{J}_2|^2 = 20\cdot 8 = 160\,W$

$\underline{S}_g = \underline{U}_g\cdot\underline{J}_1^* = 100\cdot 4 = 400 + j0$

$S_g = 400\,W + j0\,V{\cdot}A_r$

$[S] = V{\cdot}A$

Koliko kaže idealni V-meter?



$$U_V = |\underline{U}_{AB}| = |\underline{V}_A - \underline{V}_B|$$

$$100\angle 0° = (10+20+j20)\underline{I}_1 - (20+j20)\underline{I}_2$$

$$0 = (20+j20+j40-j40)\underline{I}_2 - (20+j20)\underline{I}_1$$

$$100 = (30+j20)\underline{I}_1 - (20+j20)\underline{I}_2$$

$$0 = (20+j20)\underline{I}_2 - (20+j20)\underline{I}_1$$

$$\underline{I}_2 = \underline{I}_1$$

$$100 = (30 \cancel{+j20} - 20 \cancel{-j20})\underline{I}_1$$

$$\underline{I}_1 = 10A = \underline{I}_2$$

$$V_A = 0V$$

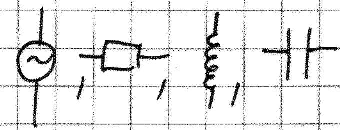
$$V_B: \underline{V}_0 - \underline{V}_B = -j40 \cdot \underline{I}_2 = -j40 \cdot 10$$

$$\underline{V}_A - \underline{V}_B = 0 - (-j400) = j400$$

$$|\underline{V}_A - \underline{V}_B| = \underline{\underline{400\,V}}$$

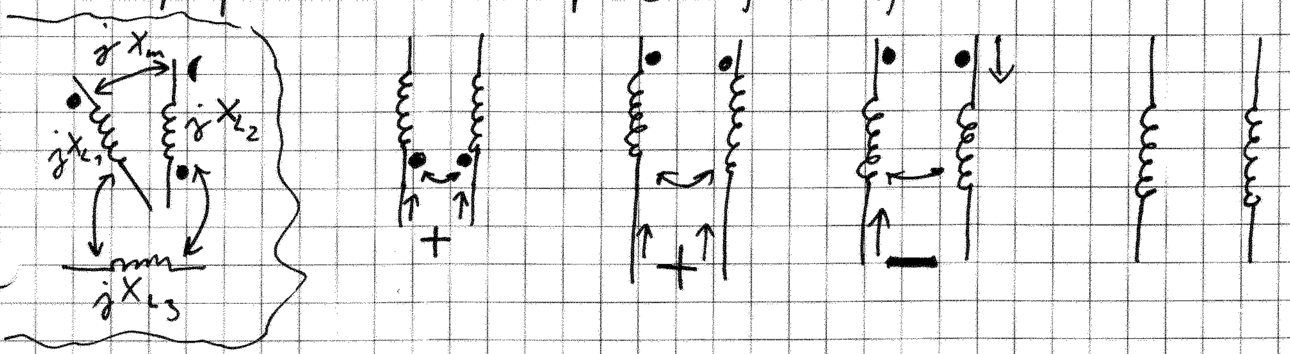
20.05.2008

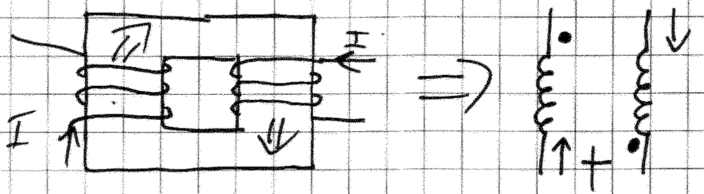
Vezja s harmoničnimi viri



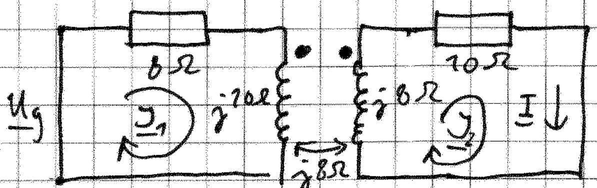
Medsebojna induktivnost

koncept pik - oznaka za pare tuljav ≡ M_{jk}





Določite $\underline{U}_g$, da bo $\underline{I} = j2$ A! $(\underline{I}_2 = 2\angle 90°$



$\underline{J}_2 = \underline{I}_2 = j2$

Zančni enačbi:

$$\underline{U}_g = (8 + j10)\cdot \underline{J}_1 - j8\,\underline{J}_2$$

$$0 = (10 + j8)\,\underline{J}_2 - j8\,\underline{J}_1 \Rightarrow (10 + j8)\,j2 = j8\,\underline{J}_1$$

$$j20 - 16 = j8\,\underline{J}_1$$

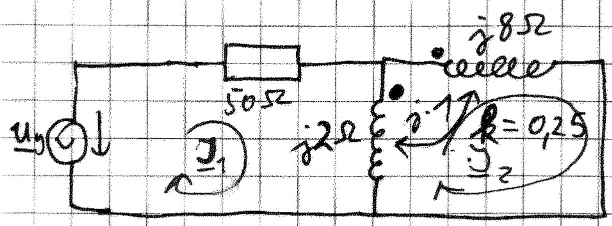
$$\underline{J}_1 = \frac{j20 - 16}{j8} = \frac{5 + j4}{2}\ \text{A}$$

$$\underline{U}_g = (8 + j10)\,\frac{5 + j4}{2} - j8\cdot j2$$

$$\underline{U}_g = (j41 + 16)\,\text{V} \cong 44\angle 68{,}7°\ \text{V}$$

Kaj pa, če zahtevamo $\underline{I}_2' = 2\angle 0°$? $\underline{U}_g' = 44\angle 68{,}7° - 90°\ \text{V}$

Določite vhodno impedanco vezja!



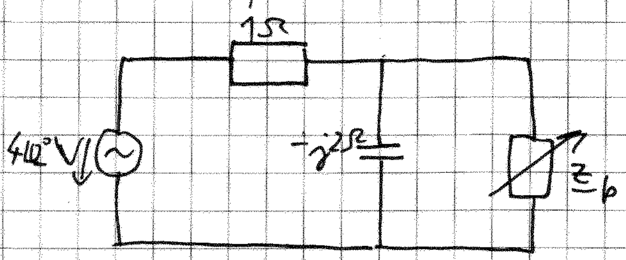
plan: - priključimo $\underline{U}_g$
- izračunamo tok skozi $\underline{U}_g$ ($\underline{I}_1$)
- $\underline{Z} = \frac{\underline{U}_g}{\underline{I}_1}$

$$X_M = k\sqrt{X_{L_1}\cdot X_{L_2}} = 0{,}25\cdot\sqrt{2\cdot 8} = \frac{1}{4}\cdot 4 = 1 \qquad j1$$

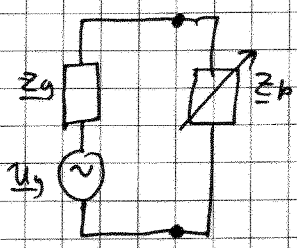
$$\underline{U}_g = (50\Omega + j2)\underline{I}_1 - j2\underline{I}_2 + j\underline{I}_2 \qquad -2j\underline{I}_2$$
$$0 = (j2+j8)\underline{I}_2 - j2\underline{I}_1 + j\underline{I}_1 - j\underline{I}_2 - j\underline{I}_2$$

$$\underline{U}_g = (50+j2)\underline{I}_1 - j\underline{I}_2$$
$$0 = j8\underline{I}_2 - j\underline{I}_1$$
$$8\underline{I}_2 = \underline{I}_1$$
$$\underline{I}_2 = \frac{1}{8}\underline{I}_1$$
$$\underline{U}_g = (50+j2)\underline{I}_1 - j\frac{1}{8}\underline{I}_1$$
$$\frac{\underline{U}_g}{\underline{I}_1} = \left(50+j2-j\frac{1}{8}\right) = \underline{\underline{50+j\frac{15}{18}\,\Omega}}$$

Pri kateri impedanci bremena, bo delovna moč na njem največja?
Kolikšna je ta moč?



TEOREM O MAX. MOČI



MAX MOČ NA $\underline{Z}_b$:

ko $\underline{Z}_b = \underline{Z}_g^*$ — konjugirano (resonanca $Im(z)=0$)

$$\boxed{P_{max} = \frac{|\underline{U}_g|^2}{8R}}$$ če je max.

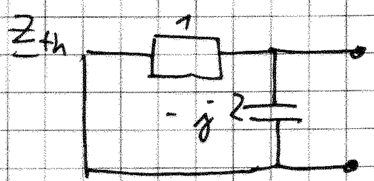
$\underline{Z}_g = R + jX$

če je $\underline{Z}_b = R_b$ (samo realno)

$R_b = |\underline{Z}_g|$

$$\boxed{P_{max} = \frac{|\underline{U}_g|^2}{4(R_g + |\underline{Z}_g|)}}$$

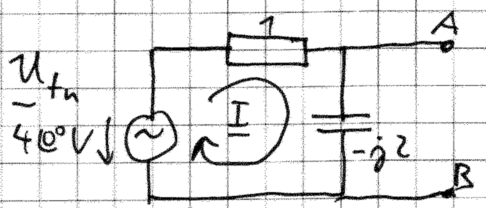
plan: - uporabimo Thevenina
- in P_{max}



$$\underline{Z}_{th} = \frac{1(-j2)}{1-j2} = \frac{4-j2}{5}\,\Omega = (0{,}8 - j0{,}4)\,\Omega$$

P_{max} pri: $\underline{Z}_b = (0{,}8 + 0{,}4j)\,\Omega$

$$P_{max} = \frac{|\underline{U}_g|^2}{8 \cdot R_{th}}$$

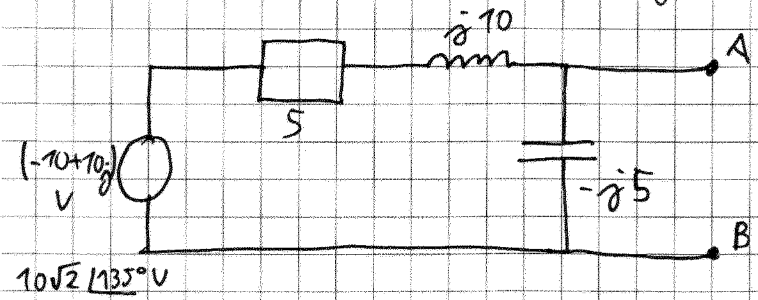


$$\underline{U}_{th} = -j2 \cdot \underline{I} = -j2 \cdot \frac{4}{1-j2} = 4 \cdot (0{,}8 - j0{,}4)$$

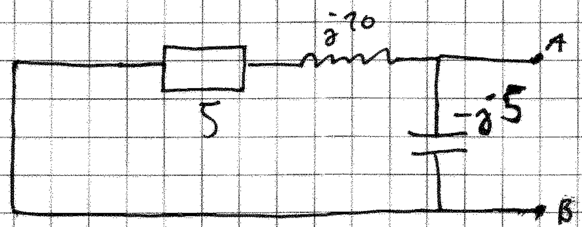
$$\underline{U}_{th} = (3{,}2 - j1{,}6)\,V = 3{,}58 \angle 26.56^\circ\,V$$

$$P_{max} = \frac{|\underline{U}_g|^2}{8 \cdot R_{th}} = \frac{3{,}58}{8 \cdot 0{,}8} = 2\,W$$

Določite elemente Theveninovega nadomestnega vezja med A in B.



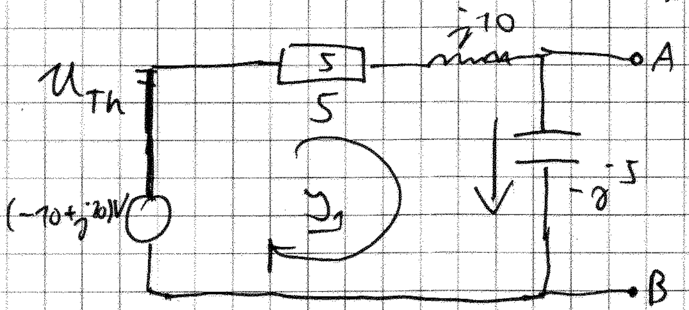
$\underline{Z}_{Th}$



$$\underline{Z}_{Th} = \frac{(5+j10)\cdot(-j5)}{(5+j10)+(-j5)}$$

$$\underline{Z}_{Th} = \frac{5}{2} - j\frac{15}{2}$$

U_{Th}

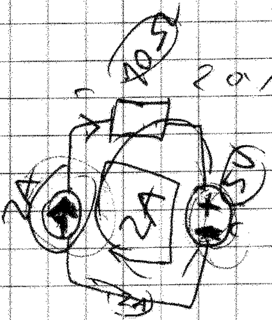


$$-10+j10 = (5+5j)\underline{J}_1$$

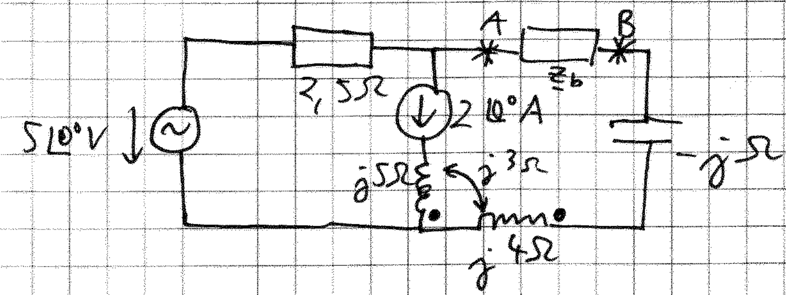
$$\underline{J}_1 = \frac{-10+j10}{5+5j}$$

$$\underline{J}_1 = 2j \text{ A}$$

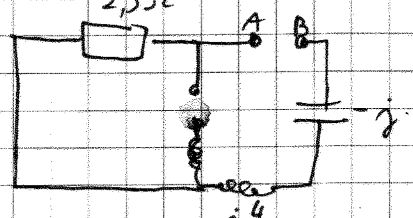
$$U_{Th} = -j5 \cdot 2j = \underline{10\,V}$$



Določite $\underline{Z}_b$ da bo na njem max. možna moč.



$\underline{Z}_b$

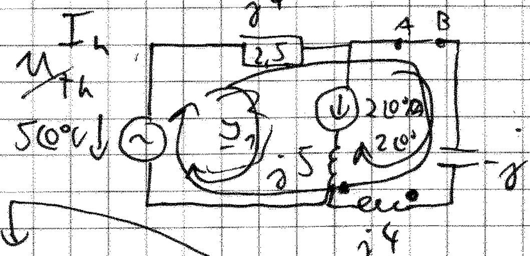


$Z_{Th} = (2{,}5 + j3)\,\Omega$ — konjugiran

P_{max} : $\underline{Z}_b = (2{,}5 - j3)\,\Omega$

I_n

U_{Th}



$$\underline{I}_n = \underline{J}_1 - 2$$

$$5 = (2{,}5 - j + j4)\underline{J}_1 - (j4 - j)2 - j3\cdot 2$$

$$5 = (2{,}5 + j3)\underline{J}_1 - j12$$

$$5 + j12 = (2{,}5 + j3)\underline{J}_1$$

$$\underline{J}_1 = (3{,}180 - j0{,}984)\text{ A}$$

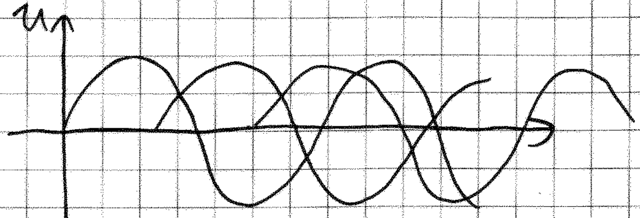
$$\underline{I}_n = (1{,}180 + j0{,}984)\text{ A}$$

66 $U_{Th} = Z_{Th} \cdot I_n$

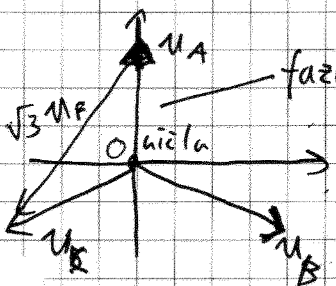
$U_{Th} = j6\,V = 6\,\angle 90^\circ$ V

$$P_{max} = \frac{6^2}{8\cdot 2{,}5} = 1{,}8\,W$$

Večfazni sistemi (trofazni)

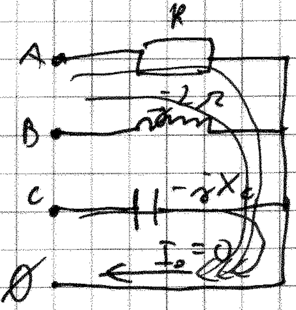


Trije u_y zamaknjeni za 120°



400/230 V
380/220 V

Pozitivni simetrični trif. sistem 400/230 V_{ef}. je priključen na breme na sliki. Kolikšna morata biti R in X_{C}, da bo tok v povratnem vodniku enak 0?

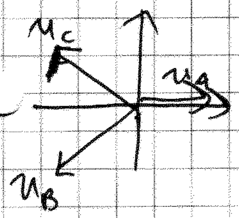


$\underline{V}_0 = 0\,V$

$\underline{V}_A = 230\,V$

$\underline{V}_B = 230\,\angle{-120°}\,V$

$\underline{V}_C = 230\,\angle{120°}\,V$



$\underline{I}_A + \underline{I}_B + \underline{I}_C = 0$

$\underline{I}_A = \frac{\underline{V}_A - \underline{V}_0}{R} = \frac{230\angle 0°}{R}$

$\underline{I}_B = \frac{\underline{V}_B - \underline{V}_0}{j2} = \frac{230\angle{-120°}}{j2} = 115\angle{-210°}\,A$

$\underline{I}_C = \frac{\underline{V}_C - \underline{V}_0}{-jX_c} = \frac{230\angle 120°}{X_c\angle{-90°}} = \frac{230}{X_c}\angle 210°$

$$\underline{I}_A + \underline{I}_B + \underline{I}_C = 0 = \frac{230}{R} + 115\angle{-210°} + \frac{230}{X_c}\angle 210° = 0 + j0$$

Re in Im $\Rightarrow$ dve enačbi, iz obeh R in X_c

$R = \frac{2}{\sqrt{3}}\,\Omega$

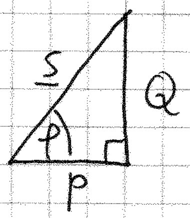
$X_c = 2\,\Omega$

27.05.2008

Kompenzacija moči

$\underline{S} = P + jQ$

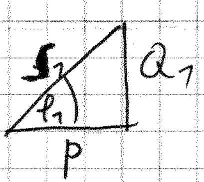
močnostni trikotnik



faktor delovnosti moči: $p = \cos\varphi$

A 500 kVA transformer is at full load with an overall power factor of 0,6 lagging. The power factor is improved by adding capacitors until the overall power factor becomes 0,9 lagging. Determine the kVAr of capacitors required! After the correction of the power factor, what percentage of full load is the transformer carrying.

before



$p_1 = \cos\varphi_1 = 0,6$

$P = S_1 \cdot \cos\varphi_1 = 0,6 \cdot S_1$

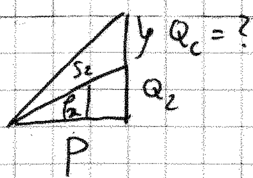
$P = 300\ kW$

$Q_1 = \sqrt{S_1^2 - P^2}$

$= \sqrt{500^2 - 300^2}$

$= 400\ kVAr$

after



$p_2 = \cos\varphi_2 = 0,9, \quad P = 300\ kW$

$Q_2 = P \cdot \tan\varphi_2 \Rightarrow \varphi_2 = \arccos p_2 = 26°$

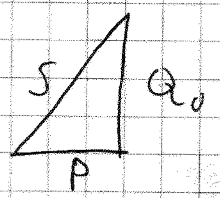
$Q_2 = 300 \cdot 0,48 = 146\ kVAr$

$Q_c = Q_1 - Q_2 = 400 - 146 = 254\ kVAr$

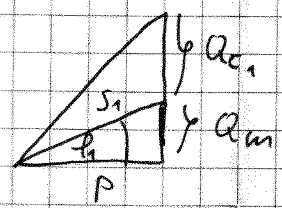
percentage of full load: $\eta = \frac{S_2}{S_1} \cong \frac{333}{500} = 66,7\%$

Aleta®

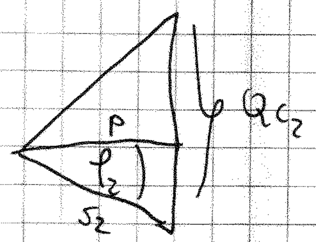
Določite mejni vrednosti kompenzacijskega kondenzatorja $[C_1, C_2]$, znotraj katerega bo ta motorju s podatki $P = 2{,}2\ kW$, $\cos\varphi = 0{,}75$, $U_g = 220\ V$, $f = 50\ Hz$. popravil faktor delavnosti nad 0,95!



1.)



2.)



$\varphi_2 = -\varphi_1$

$Q_m = P \cdot tg\,\varphi_1$

$\varphi_1 = \arccos 0{,}95 = 18{,}2^\circ$

$\underline{\underline{Q_m = 723\ VAr}}$

$Q_0 = P \cdot tg\,\varphi_0$

$\varphi_0 = \arccos(0{,}75) = 41{,}4^\circ$

$\underline{\underline{Q_0 = 1940\ VAr}}$

$Q_{c_1} = Q_0 - Q_m = 1940 - 723 = 1217\ VAr$

$Q_{c_2} = Q_0 + Q_m = 1940 + 723 = 2663\ VAr$

$\underline{S} = \underline{U} \cdot \underline{I}^* \Rightarrow \underline{S} = \underline{Z}|\underline{I}|^2$ $\qquad \underline{I}_c = \frac{\underline{U}_c}{\underline{X}_c}$

na C:

$jQ_c = j \cdot X_c \cdot |\underline{I}_c|^2 = j X_c \left|\frac{\underline{U}}{\underline{X}_c}\right|^2 = j\frac{|\underline{U}|^2}{X_c} = j\frac{U_{ef}^2}{X_c}$

$Q_c = \frac{U_{ef}^2}{X_c} = \omega \cdot C \cdot U_g^2 = 2\pi f \cdot C \cdot U_{ef}^2$

$\boxed{C = \frac{Q_c}{2\pi f \cdot U_{ef}^2}}$ $\qquad C_1 = \frac{Q_{c_1}}{2\pi \cdot 50 \cdot 220^2} = 80\ \mu F$

$C_2 = \frac{Q_{c_2}}{2\pi \cdot 50 \cdot 220^2} = 175\ \mu F$

$C = [80, 175]$