OSNOVE ELEKTROTEHNIKE II
zapiski z avditornih vaj

Šolsko leto	2007 / 2008
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Avtor dokumenta	Blaž Potočnik
Sodelavci	Blaž Potočnik, Aljoša Praznik

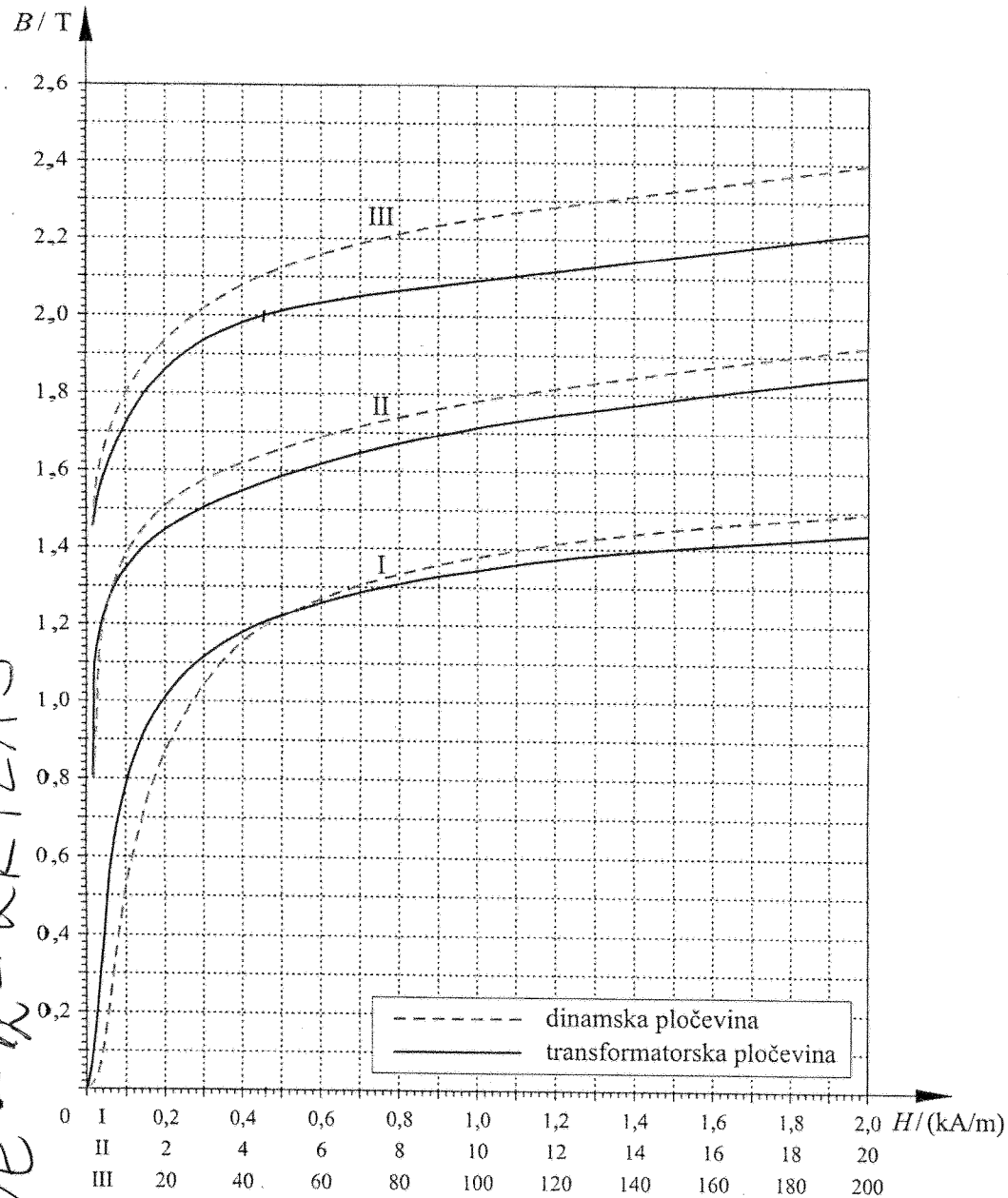
UREJANJE DOKUMENTA

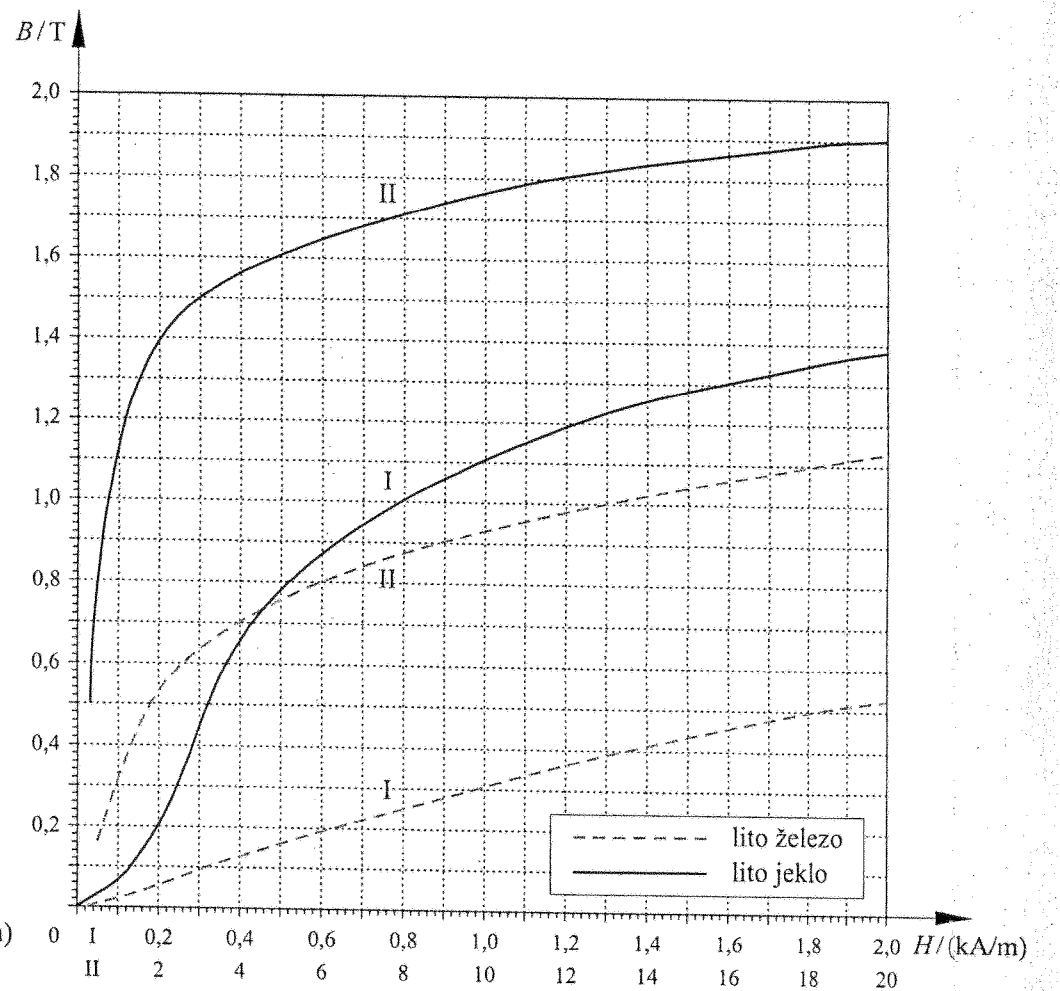
VERZIJA	01	**REVIZIJA**	01
DATUM	24. 2. 2009		
ZADNJI POPRAVLJAL	/		
PREGLEDAL	Blaž Potočnik, Aljoša Praznik		

OPOMBE

POPRAVKI

Srednje krivulje magnetiziranja za mehke magnetne materiale



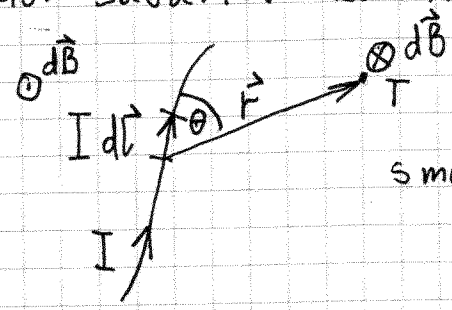


OE - N - KRIŽAJ

Osnove elektrotehnike II - vaje

20.2.08

Biot-Savartov zakon



$$d\vec{B} = \frac{\mu_0 I\, d\vec{l} \times \vec{r}}{4\pi r^3}$$

smer: $d\vec{l} \times \vec{r}$

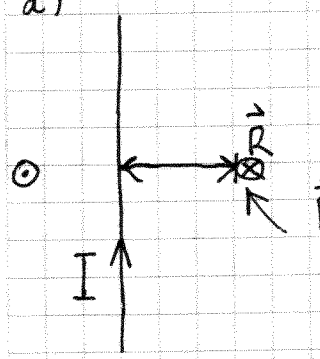
$\rightarrow d\vec{B} \otimes$ (notri)

$$|d\vec{l} \times \vec{r}| = dl \cdot r \cdot \sin\theta$$

$$dB = \frac{\mu_0 I\, dl \sin\theta}{4\pi r^2}$$ ← abs. vrednost.

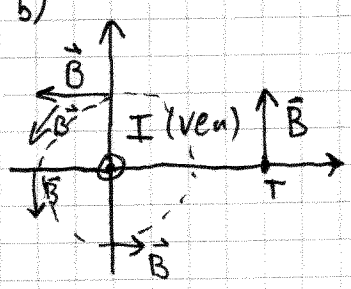
Tokovna premica

a)

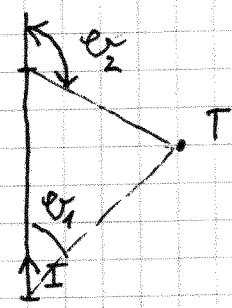


$$\vec{B} = \frac{\mu_0 I}{2\pi R}\, \vec{e}_\varphi$$

b)

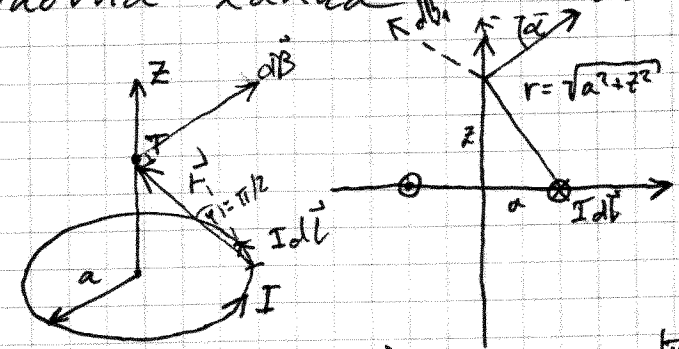


Tokovna daljica



$$\vec{B} = \frac{\mu_0 I}{4\pi R}\left(\cos\theta_1 - \cos\theta_2\right)$$

Tokovna zanka



$$d\vec{B} = \frac{\mu_0 I\, a\, d\varphi \sin\left(\frac{\pi}{2}\right)}{4\pi\left(a^2+z^2\right)}$$

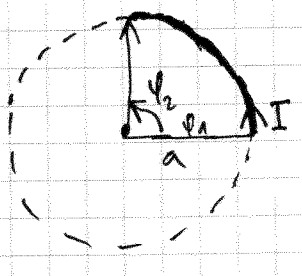
$$dB_z = dB \cdot \sin\alpha = \frac{\mu_0 a\, d\varphi\, I}{4\pi\left(a^2+z^2\right)} \cdot \frac{a}{\left(a^2+z^2\right)^{1/2}}$$

$$B_z = \frac{\mu_0 I a^2}{4\pi\left(a^2+z^2\right)^{3/2}} \int_0^{2\pi} d\varphi$$

$$= \boxed{\vec{e}_z \frac{\mu_0 \cdot I a^2}{2\left(a^2+z^2\right)^{3/2}}}$$

B pri $z=0$: $\boxed{\vec{B}(z=0) = \vec{e_z}\,\frac{\mu_0 I}{2a}}$

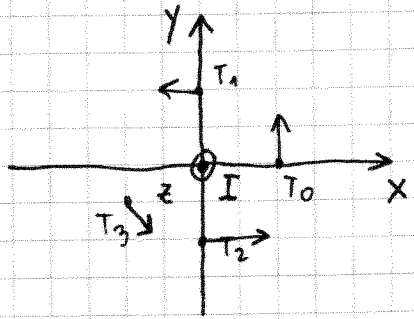
Del tokovne zanke



$$dB = \frac{\mu_0 I\, d\vec{l} \times d\varphi}{4\pi a^2} \quad / \int \rightarrow \vec{B} = \frac{\mu_0 I}{4\pi a}\int_{\varphi_1}^{\varphi_2} d\varphi =$$

$$= \frac{\mu_0 I}{4\pi a}(\varphi_2 - \varphi_1)$$

- Vzdolž Z-osi neskončna tanka žica s tokom $I = 15\,A$ v smeri Z-osi. Določite gostoto m. pretoka v točkah $T_0(1,0,0)\,m$, $T_1(0,2,2)\,m$, $T_2(0,-2,0)\,m$, $T_3(-2,-1,0)\,m$



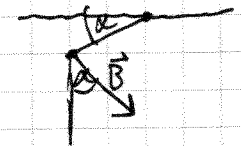
$\mu_0 = 4\pi\,10^{-7}\ Vs/Am$

$B(T_1) = \frac{\mu_0 I}{2\pi R}$ $\quad B(T_1) = \frac{2\cdot 4\pi\cdot 10^{-7}\cdot 15}{2\pi\cdot 1} = 3{,}0\cdot 10^{-6}\,T\,\vec{e_y}$

$B(T_2) = \frac{4\pi\,10^{-7}\,15}{2\pi\cdot 2} = 1{,}5\,T\,\vec{e_x}$

$B(T_3) = \frac{4\pi\,10^{-7}\cdot 15}{2\pi\cdot 2\cdot \sqrt{5}} = 1{,}5\,T\,\vec{e_x}$ $\quad |B(T_4)| = \frac{4\pi\,10^{-7}\cdot 15}{2\pi\cdot\sqrt{5}} = 1{,}35\cdot 10^{-6}\,T$

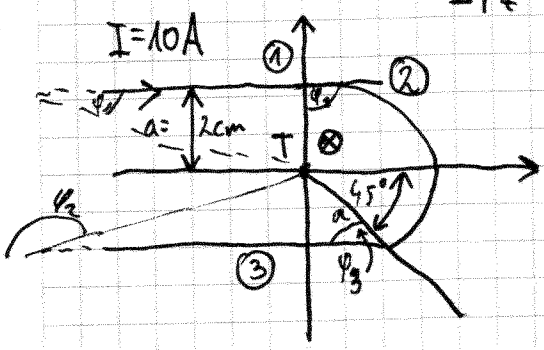
smeri T_3:



$\vec{B_3} = 1{,}34\cdot 10^{-6}\,T\,(\vec{e_x}\sin\alpha - \vec{e_y}\cos\alpha)$

$= 1{,}34\,\mu T\,(\vec{e_x}(1/\sqrt{5}) - \vec{e_y}(2/\sqrt{5})) = \vec{e_x}\,0{,}6\,\mu T - \vec{e_y}\,1{,}2\,\mu T$

- Določite gostoto magnetnega pretoka v točki T za tokovodnik na sliki



– iz treh delov

$$B_1 = \frac{\mu_0 I}{4\pi R}(\cos\varphi_1 - \cos\varphi_2) = \frac{\mu_0 I}{4\pi a}\cdot(\cos 0 - \cos\tfrac{\pi}{2})$$

$$B_1 = \frac{4\pi\cdot 10^{-7}\,\frac{Vs}{Am}\cdot 10A}{4\pi\cdot 2\cdot 10^{-2}m} = \boxed{5\cdot 10^{-5}\,T}$$

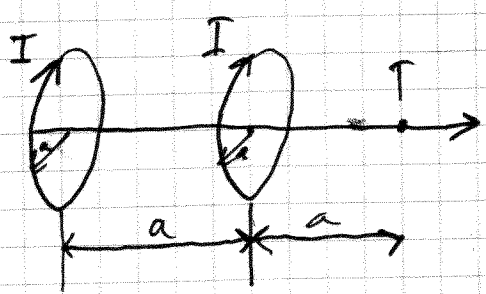
↑ tokovna daljica (poltrak), $\varphi_1 \to 0$.

$$B_2 = \frac{\mu_0 I}{4\pi a}\beta = \frac{4\pi\cdot 10^{-7}\,Vs\cdot 10A}{Am\,4\pi\cdot 2\cdot 10^{-2}m}\cdot\frac{3\pi}{4} = \boxed{5\cdot 10^{-5}\cdot 3\pi/4\,\frac{Vs}{m^2}}$$

$$B_3 = \frac{\mu_0 I}{4\pi(a/\sqrt{2})}\left(\cos\frac{\pi}{4} - \cos\pi\right) = \frac{\mu_0 I}{4\pi(a/\sqrt{2})}\cdot\left(\frac{\sqrt{2}}{2}+1\right) = 5\cdot 10^{-5}\cdot\frac{\sqrt{2}(\sqrt{2}+2)}{2}\,T$$

$$B = B_1 + B_2 + B_3 = 5\cdot 10^{-5}\,(1 + 3\pi/4 + \sqrt{2}(\sqrt{2}+2)/2)\,T(-\vec{e}_z) = 288\,\mu T(-\vec{e}_z)$$

- Skozi soosni tuljavi polmerov in medsebojne razdalje $a = 11{,}0\,mm$ teče tok $I = 4A$ v nadhazanih smereh! Določite $\vec{B}$ v točki $T(0,0,a)$



(superpozicija polj)

$\vec{B}$ v osi tokovne zanke

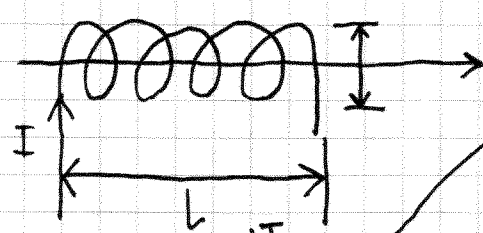
$$\vec{B} = \frac{\mu_0 I a^2}{2(a^2+z^2)^{3/2}}\vec{e}_z$$

$$B(T) = \frac{\mu_0 I a^2}{2(a^2+a^2)^{3/2}} + \frac{\mu_0 I a^2}{2(a^2+4a^2)^{3/2}} =$$

$$= \frac{\mu_0 I a^2}{2}\left(\frac{1}{(2a)^{3/2}} + \frac{1}{(5a)^{3/2}}\right) = \frac{\mu_0 I a^2}{2}\left(\frac{1}{2\sqrt{2}\sqrt{a^3}} + \frac{1}{5\sqrt{5}\sqrt{a^3}}\right)$$

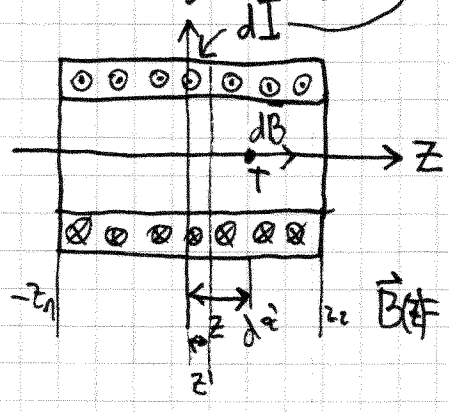
$$= \frac{4\pi\,4\cdot 10^{-7}}{a}\left(5^{-3/2} + 2^{-3/2}\right) =$$

• Izpelji formulo za polje v osi solenoida



$$dI = \frac{N I}{L} \cdot dz'$$

$$\int dI = \int_{-L/2}^{L/2} \frac{N \cdot I}{L} dz' = \frac{N \cdot I}{L} \cdot L = N \cdot I$$

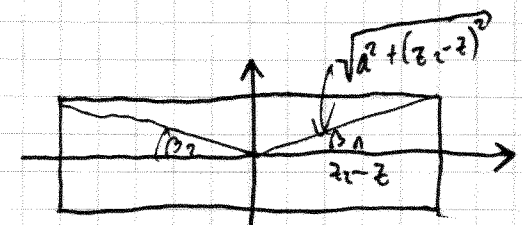


$$d\vec{B} = \frac{\mu_0 dI\, a^2}{2(a^2 + (z-z')^2)^{3/2}}$$

$$d\vec{B} = \vec{e_z} \frac{\mu_0 N \cdot I\, a^2}{L\, 2(a^2+(z-z')^2)^{3/2}}$$

$$\vec{B} = \vec{e_z} \frac{\mu_0 N \cdot I\, a^2}{2L} \int_{-z_1}^{z_2} \frac{dz'}{(a^2+(z-z')^2)^{3/2}} =$$

$$= \vec{e_z} \frac{\mu_0 N I a^2}{2L} \left. \frac{z'-z}{a^2\sqrt{a^2+(z-z')^2}} \right|_{-z_1}^{z_2} = \vec{e_z} \frac{\mu_0 N I}{2L} \left[\frac{z_2 - z}{\sqrt{a^2+(z-z_2)^2}} + \frac{z_1+z}{\sqrt{a^2+(z_1+z)^2}} \right]$$



$$\boxed{\vec{B} = \vec{e_z} \frac{\mu_0 N I}{2L} (\cos\beta_1 + \cos\beta_2)}$$

lepša formula

Poenostavitve:

- Dolga tuljava $L \gg a$

B(v sredini) – B(v notranjosti, ne pa na robovih):

$\beta_1 = \beta_2 \to 0°$

$$\boxed{B = \frac{\mu_0 N I}{L}}$$

- na robu dolge tuljave: $\beta_1 = 0°$, $\beta_2 = 90°$

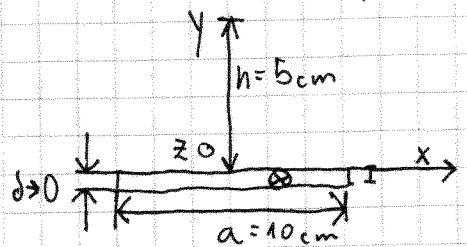
$$B = \frac{\mu_0 N I}{2L}$$

5.3.08

Primer:

- Določite polje v središču in na robu tuljave z $N = 1000$ ovoji, $l = 10\,cm$ in $a = 4\,cm$. Tok $I = 1\,A$.
  - izračunaj s formulo $B = \frac{\mu_0 N I}{2l}(\cos\beta_1 + \cos\beta_2)$

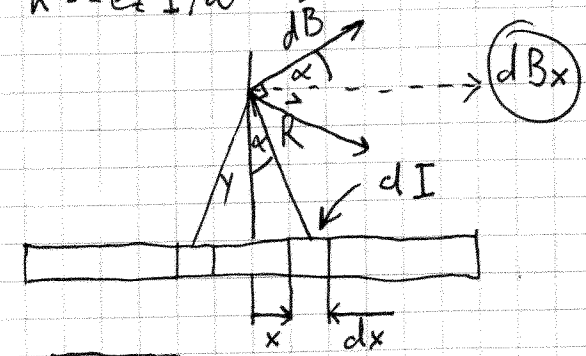
Izračunajte gostoto magnetnega pretoka nad središčem prevodnega traku. Tok je 100 A



površinski tok

tokovna obloga $K = \frac{I}{a}$

$\vec{K} = -\vec{e}_z I/a$



$dB = \frac{\mu_0 dI}{2\pi R}$ $\quad R = \sqrt{x^2+y^2}$

$dB_x = \frac{\mu_0 dI}{2\pi R}\cos\alpha$ $\quad \cos\alpha = y/R$

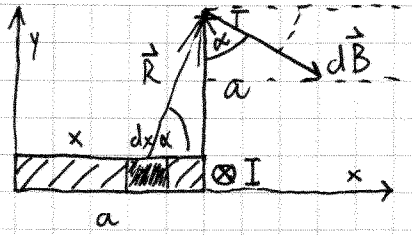
$dI = J \cdot dA = \frac{I}{\delta \cdot a} \cdot \delta \cdot dx = K\,dx$

$dB_x = \frac{\mu_0 K\,dx}{2\pi}\frac{y}{x^2+y^2}$

$B_x = \int_{-a/2}^{a/2} \frac{\mu_0 K y}{2\pi(x^2+y^2)}\,dx = \frac{\mu_0 K y}{2\pi}\cdot 2\int_0^{a/2}\frac{dx}{x^2+y^2} = \frac{\mu_0 K}{2\pi}\frac{1}{y}\operatorname{atg}\frac{x}{y}\Big|_0^{a/2} =$

$= \boxed{\frac{\mu_0 K}{\pi}\operatorname{atg}\frac{a/2}{y}}$

$\vec{B} = \vec{e}_x \frac{4\pi\,10^{-7}\ Vs/Am\cdot K}{\pi}\operatorname{atg}\frac{1012\ cm}{5\ cm} = 4\cdot 10^{-4}\operatorname{atg}(1)\cdot\vec{e}_y = \vec{e}_x\, 10^{-4}\,\pi\ T$



$$dB = \frac{\mu_0\, dI}{2\pi R}$$

$$R = \sqrt{x^2+y^2} = \sqrt{a^2+(a-x)^2}$$

$$dI = K\,dx$$

$$dB_x = \frac{\mu_0 dI}{2\pi\sqrt{a^2+(a-x)^2}} \cdot \sin\alpha \qquad \sin\alpha = \frac{a}{R}$$

$$= \frac{\mu_0 K dx \cdot a}{2\pi\left(a^2+(a-x)^2\right)} *$$

$$B_x = \int_0^a \frac{\mu_0 I\, dx}{2\pi\left(a^2+(a-x)^2\right)}$$

integral: $a-x = t$, $-dx = dt$

$$B_x = \frac{\mu_0 I}{2\pi}\left[\int \frac{1}{a^2+(a-x)^2}(-dt)\right] + C$$

$$= \frac{\mu_0 I}{2\pi}\,\frac{1}{a}\,\operatorname{atg}\frac{t}{a} = \frac{\mu_0 I}{2\pi}\,\frac{1}{a}\,\operatorname{atg}\frac{a-x}{a} =$$

$$= \frac{-\mu_0 I K}{2\pi}\left(-\frac{\pi}{4}\right) = \boxed{\frac{\mu_0 K}{8}}$$

išćemo še B_y

$$dB_y = \frac{\mp\mu_0\, dI}{2\pi\sqrt{a^2+(a-x)^2}}\cos\alpha \qquad \cos\alpha =$$

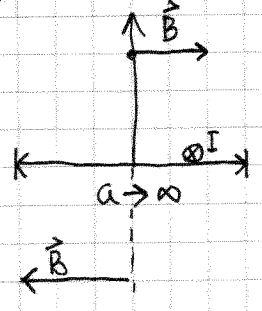
$$B_y = -\frac{\mu_0 K}{2\pi}\int_0^a \frac{(a-x)\,dx}{a^2+(a-x)^2} = \qquad \left\| \begin{array}{l} a-x = t \\ dx = dt \end{array} \right.$$

$$= \pm\frac{\mu_0 K}{2\pi}\int \frac{t\,dt}{a^2+t^2} + \text{Konst.} \qquad \left\| \begin{array}{l} a^2+t^2 = z \\ dz = 2t\,dt \end{array} \right.$$

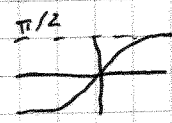
$$= \pm\frac{\mu_0 k}{2\pi}\frac{1}{2}\int \frac{dz}{z} = \frac{\pm\mu_0 k}{4\pi}\log\left(a^2+(a-x)^2\right)\Big|_0^a = \boxed{\frac{-\mu_0 I}{4\pi}\log\frac{2a^2}{a^2}}$$

$$\vec{B} = \frac{\mu_0 K}{4}\left(\frac{\vec{e_x}}{2} - \frac{\vec{e_y}\,\log 2}{\pi}\right)$$

! Polje v okolici tokovne obloge (neskončna ravnina s površinskim tokom K)

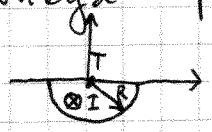


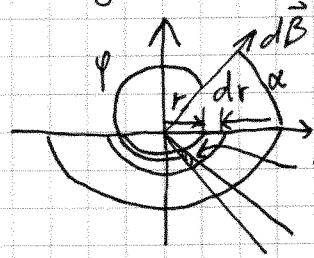
$$B_x = \lim_{a\to\infty} \frac{\mu_0 K}{\pi} \underbrace{\operatorname{atg}\frac{a/2}{y}}_{\to \pi/2} =$$



$$= \boxed{\frac{\mu_0 K}{2}}$$

Izračunajte gostoto magnetnega pretoka ravnega polkrožnega vodnika s tokom I v točki T.





$$dI = J \cdot dA = \frac{I}{A}\,dA = \frac{I}{\pi R^2/2}\, r\,d\varphi\,dr$$

$$dA = r\,d\varphi\,dr$$

$$dB = \frac{\mu_0\, dI}{2\pi r} \rightarrow \begin{matrix} dB_x = dB\cos\alpha \\ dB_y = dB\sin\alpha \end{matrix}$$

$$\varphi - \alpha = 3\pi/2 \rightarrow \alpha = \varphi - 3\pi/2$$

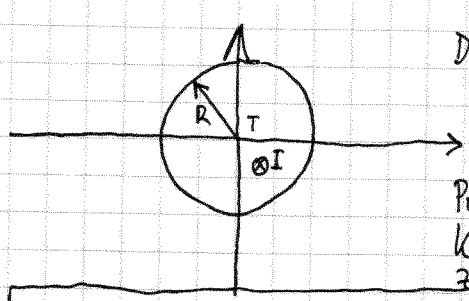
$$dB_x = \frac{\mu_0 I}{\pi R^2/2}\,\frac{r\,d\varphi\,dr}{2\pi r}\cos(\varphi - 3\pi/2)$$

$$B_x = \frac{\mu_0 I}{\pi^2 R^2}\int_{\pi}^{2\pi}\int_{0}^{R}\cos(\varphi - 3\pi/2)\,dr\,d\varphi = \frac{\mu_0 I}{\pi^2 R^2}\, r\Big|_0^R \cdot \sin(\varphi - 3\pi/2)\Big|_{\pi}^{2\pi} =$$

$$= \frac{\mu_0 I}{\pi^2 R}(1+1) = \frac{2\mu I}{\pi^2 R}$$

za y komponento lahko vidimo, da ne obstaja.

$$\vec{B} = \frac{2\mu_0 I}{\pi^2 R}$$



Določite gostoto $\vec{B}$ v središču.

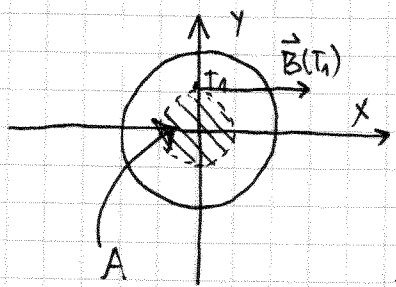
$B(T) = ?$

Prestavimo točko T_1 v $R/2$

Kolikor toka zajamemo (amperov zakon) z Biot Savartovim pravec dela.

$$\oint_{\mathcal{L}} \vec{B} \cdot d\vec{l} = \mu_0 \int_A \vec{J}\, d\vec{A} = \mu_0 \cdot I_{\text{ohlenjen}}$$

AMPEROV ZAKON

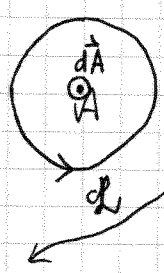


$\vec{B} = -e_\varphi B(r)$

$d\vec{l} = e_\varphi r\, d\varphi$

$\vec{B}\, d\vec{l} = -B(r)\, r\, d\varphi$

$$-\oint_{\mathcal{L}} B(r)\, r\, d\varphi = -B(r) \int_0^{2\pi} r\, d\varphi = -B(r) \cdot 2\pi r$$



$$\int_A \vec{J}\, d\vec{A} = -J \cdot A(r) = \text{ker je polje homogeno}$$

$$= \frac{I}{\pi R^2}\, \pi r^2$$

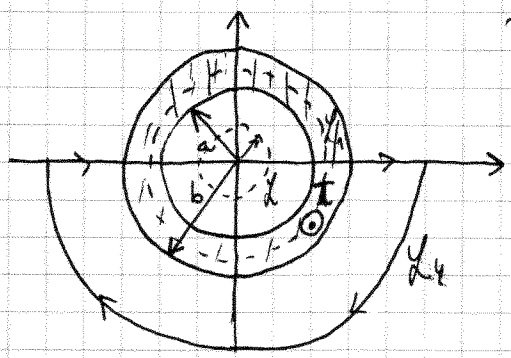
$$B(r) \cdot 2\pi r = \frac{\mu_0 I}{\pi R^2}\, \pi r^2$$

$$B(r) = \frac{\mu_0 I}{2\pi R^2}\, r$$

$$\vec{B} = -e_\varphi \frac{\mu_0 I}{2\pi R^2}\, r$$

$$\vec{B}(T_1) = e_x \frac{\mu_0 I}{2\pi R^2} \frac{R}{2} = \boxed{e_x \frac{\mu_0 I}{4\pi R}}$$

Določite polje znotraj votlega vodnika s polmerom a in b s tokom I.



3 področja: $r \leq a$, $b \geq r \geq a$, $r \geq b$

$B(r \leq a)$

$\oint_\gamma \vec{B} \cdot d\vec{l} = \vec{B} \cdot 2\pi r = \mu_0 \cdot 0$ nismo ohlenili toka

$B(r < a) = 0$ znotraj je polje enako 0.

$B(b \geq r \geq a)$

$\oint_{\gamma_1} \vec{B} d\vec{l} = \vec{B} \cdot 2\pi r = \mu_0 \overbrace{J \cdot A}^{I \text{ ohlenjen}} = \mu_0 \cdot J \cdot (\pi r^2 - \pi a^2) = \mu_0 J \pi (r^2 - a^2)$

$J = \dfrac{I}{\pi (b^2 - a^2)}$

$B \cdot 2\pi r = \mu_0 \dfrac{I}{\pi (b^2 - a^2)} \cdot \pi (r^2 - a^2)$

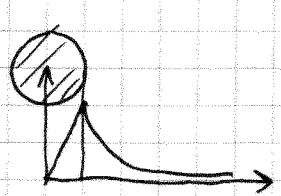
$B = \dfrac{\mu_0 I}{2\pi r} \dfrac{(r^2 - a^2)}{(b^2 - a^2)}$

$B(r \geq b)$

$B \cdot 2\pi r = \mu_0 \cdot I$

$$\boxed{B = \frac{\mu_0 \cdot I}{2\pi r}}$$

Poln vodnik:



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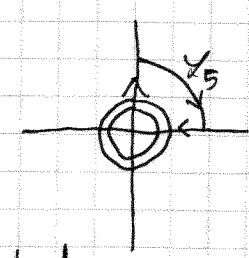
Določite $\oint_{\mathcal{L}_4} \vec{B}\,d\vec{l}$ | Določite po $\mathcal{L}_5$

$\oint_{\mathcal{L}_4} \vec{B}\,d\vec{l} = -\mu_0 I/2$ | $\oint_{\mathcal{L}_5} \vec{B}\,d\vec{l} = -\mu_0 \frac{I}{4}$



smer obhoda

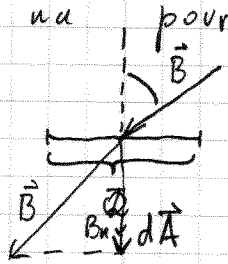
Magnetni pretok - fluks

$I = \int_A \vec{j}\,dA$ $\quad \Phi_{el.} = \int_A \vec{E}\,d\vec{A}$ (gaussov zakon)

$$\boxed{\Phi = \int_A \vec{B}\,d\vec{A}}$$

- Določimo magnetni pretok

$\vec{B} = 5\,mT$ skozi ravno pravokotno zanko $A = 5 \times 4\ cm^2$ pri čemer polje upada pod kotom $\alpha = 60°$ na normalo na površino.

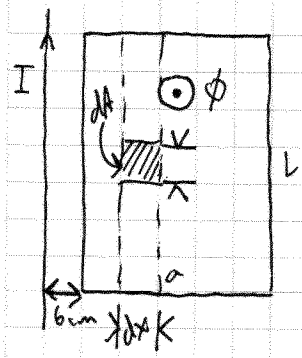


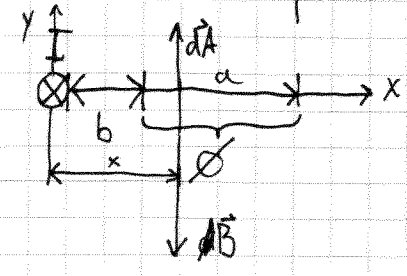
$$\Phi = \int_A \vec{B}\,d\vec{A} = \int_A \vec{B}\cdot\vec{e_n}\,dA = \int_A \underbrace{B\cdot\cos\alpha}_{B_n}\cdot dA =$$

$$= \int_A B_n\,dA$$

$$\Phi = B\cdot\cos\alpha\cdot A = 5\,mT\cdot\cos 60\cdot 20\cdot 10^{-4}\ m^2 = 5\,\mu T m^2 = \underline{5\,nWb} = \underline{5\,nVs}$$

- V ravnini x,y leži zanka $a = 5\,cm$, $L = 10\,cm$. Določite fluks skozi zanko, ki ga povzroča vodnik s tokom $I = 100A$. Vodnik leži v ravnini zanke in je vzporeden z zanko.





$$\vec{B} = \frac{\mu_0 I}{2\pi x}(-\vec{e_y})$$

$d\vec{A} = dx\,dz\,\vec{e_y}$ (kaže v smeri fluksa)

12.3.08

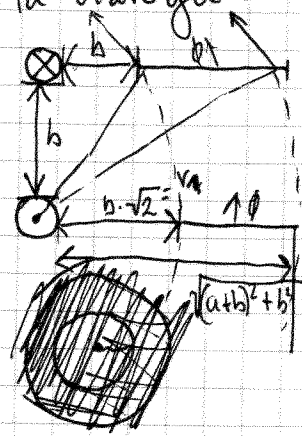
$$\vec{B}\,d\vec{A} = -\frac{\mu_0 I}{2\pi x}\,dx\,dz$$

$$\phi = \int_A \vec{B}\,d\vec{A} = \int_b^{a+b}\int_0^l \frac{\mu_0 I}{2\pi}\,\frac{dx\,dz}{x} = \boxed{-\frac{\mu_0 I}{2\pi}\cdot l \log\frac{a+b}{b}}$$

$$\phi = \frac{-4\cdot\pi\cdot 10^{-7}\,Vs\cdot 100A\cdot 0{,}1m}{Am}\log\frac{11}{6}$$

$$= -1{,}2\,\mu Wb$$

Ista naloga: Določimo fluks, ki jo dvovod na sliki povzroča skozi zanko.



Za en tok že imamo. $I_{zgornji} \rightarrow \phi_{zgornji} = -1{,}2\,\mu Wb$

Rabimo B_y.

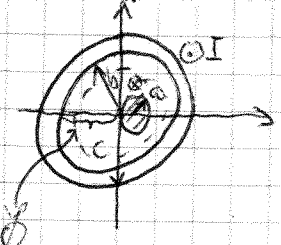
$$\phi = \int_A \vec{B}\,d\vec{A} = \frac{\mu_0 I}{2\pi}\int_{r_1}^{r_2}\int_0^l \frac{dx\,dz}{x} = \frac{\mu_0 I\, l}{2\pi}\cdot\log\frac{\sqrt{(a+b)^2+b^2}}{b\sqrt{2}}$$

pretoki enaki na istih radijih.

$\phi = 0{,}78\,\mu Wb \rightarrow \phi_{spodnji}$

$\phi_{skupni} = \phi_{zgornji} + \phi_{spodnji} = \underline{-0{,}43\,\mu Wb}$

Kolikšen je pretok na dolžino 1m od izhodišča do točke $-b$.

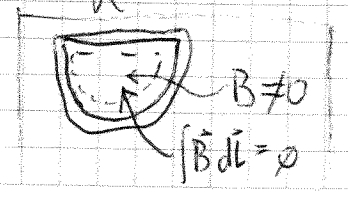


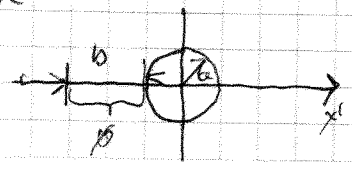
$a = 0{,}6$ cm
$b = 2$ cm
$c = c = 2{,}5$ cm
$I = 50A$

$\phi = \phi_{not} + \phi_{zun}$

$\phi = \int_A \vec{B}\,d\vec{a}$

$\oint_\lambda \vec{B}\,d\vec{l} = 0 \Rightarrow \vec{B} = 0$ (znotraj od zunanjega)





$d\vec{A} = -e_y\,dx'dz$

$\vec{B} = e_y\,\mu_0 I/2\pi(x')$

$\Phi = \int\int (e_y \mu_0 I/2\pi(x'))\,dx'dz$

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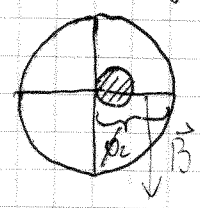
Aleta

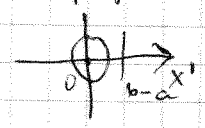
$$\Phi = \int_0^l \int_a^{a+b} (-e_y) \frac{\mu_0 I}{2\pi (x')} e_y \, dx \, dz = \boxed{-\frac{\mu_0 I}{2\pi} \ln \frac{a+b}{a} \cdot l}$$

$$-\frac{4\pi \cdot 10^{-7} \frac{Vs}{Am} \cdot 50A \cdot 1m}{2\pi} \ln \frac{2,6}{0,6} = 14,66 \mu Wb$$

Izračunajmo še Φ_2 (ni fluksa skozi vodnik - polji se odštejeta)

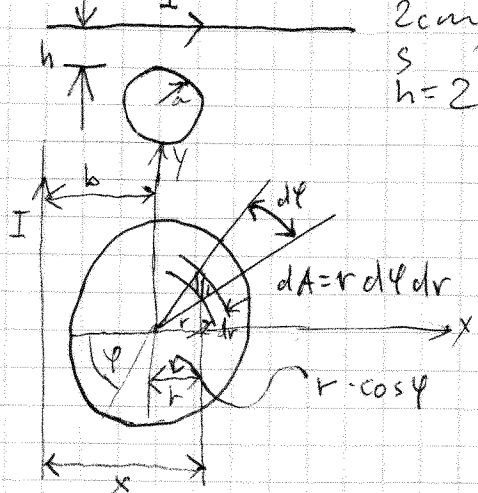


$$\Phi_2 = \int_0^l \int_0^{b-a} (-e_y) \frac{\mu_0 I}{2\pi (x')} dx' \, dz =$$



$$= \boxed{\frac{\mu_0 I}{2\pi} l \cdot \log \frac{b-a}{a}}$$ Paziti na koor. sis.

- Položite fluks skozi tanko polmera 2cm, ki je od ravnega vodnika s tokom $I = 10$ A oddaljena $h = 2$ cm



$$B = \frac{\mu_0 I}{2\pi x} = \text{(v tej } dA\text{)}$$

$$= \frac{\mu_0 I}{2\pi (b + r\cos\varphi)}$$

$$dA = r \, d\varphi \, dr$$

$$\Phi = \int_0^{2\pi} \int_0^a \frac{\mu_0 I}{2\pi (b + r\cos\varphi)} r \, dr \, d\varphi =$$

$$= \frac{\mu_0 I}{2\pi} \int_0^a r \, dr \int_0^{\pi} \frac{2 \, d\varphi}{b + r\cos\varphi} = \frac{\mu_0 I}{2\pi} \int_0^a r \, dr \, \frac{2\pi}{\sqrt{b^2 - r^2}} =$$

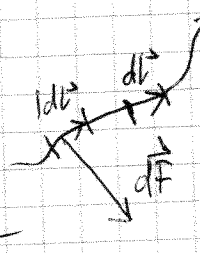
$$= \mu_0 I \int_0^a \frac{r \, dr}{\sqrt{b^2 - r^2}} = \mu_0 I \left(-\sqrt{b^2 - r^2} \right) \Big|_0^a =$$

$$= \boxed{\mu_0 I \left(b - \sqrt{b^2 - a^2} \right)}$$

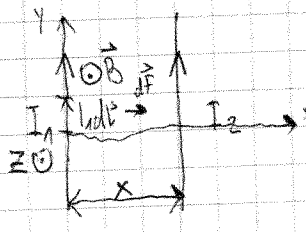
$\phi = 4\pi \cdot 10^{-7}\ Vs/Am \cdot 10A\ (6-\sqrt{36-4})\cdot 10^{-2} m \simeq \underline{431\ nVs}$

Sila na vodnik v mag. polju

$$\boxed{d\vec{F} = I\, d\vec{l} \times \vec{B}}$$



sila med vzporednima ravnima vodnikoma



$I_1 d\vec{l} = I_1 e_y dy$

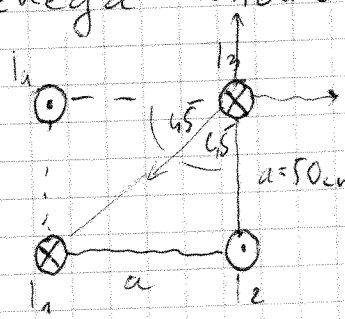
$\vec{B} = (\mu_0 I_2 / 2\pi x)\, e_z$

$d\vec{F} = I_1 e_y dy \times e_z\ (\mu_0 I_2 / 2\pi x)$

$d\vec{F} = e_x \frac{\mu_0 I_1 I_2}{2\pi x} dy$

$\vec{F} = \int_0^l e_x \frac{\mu_0 I_1 I_2}{2\pi x} dy = \boxed{\frac{\mu_0 I_1 I_2}{2\pi x} \cdot l \cdot \vec{e_x}}$

Določite silo na zgornji desni vodnik na dolžini enega kilometra.



$I_1 = 10A \quad I_2 = 20A \quad I_3 = 30A \quad I_4 = 20A$

$\vec{F}^{(2)} = e_y = \frac{\mu_0 I_2 I_3}{2\pi a}$

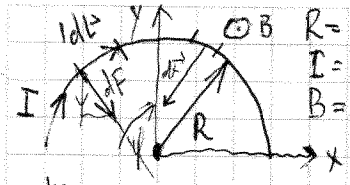
$\vec{F}^{(4)} = e_x \frac{\mu_0 I_4 I_3}{2\pi a}$

$\vec{F}^{(1)} = \left(\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right) \frac{\mu_0 I_1 I_3}{2\pi a \sqrt{2}}$

$F_x = l \frac{\mu_0 I_3}{2\pi}\left(\frac{I_4}{a} - \frac{I_1}{a\sqrt{2}\sqrt{2}}\right) = \frac{\mu_0 I_3 \cdot l}{2\pi a}\left(\frac{2I_4 - I_1}{2a}\right) = \frac{4\pi 10^{-7} \cdot 10^3}{2\pi \cdot 0{,}5} \cdot (600 - 150) =$

$= 4 \cdot 10^{-4} \cdot 450 = \underline{0{,}18\ N}$

$F_y = \frac{l \cdot \mu_0 I_3}{2\pi a}\left(I_2 - 0{,}5 I_1\right) = \frac{10^3 \cdot 4\pi \cdot 10^{-7} \cdot 30A \cdot 15A}{2\pi \cdot 0{,}5} = 0{,}18\ N$



$R = 4\,cm$
$I = 6\,A$
$B = 0{,}5\,T$

$dF = I\,dl\,B$

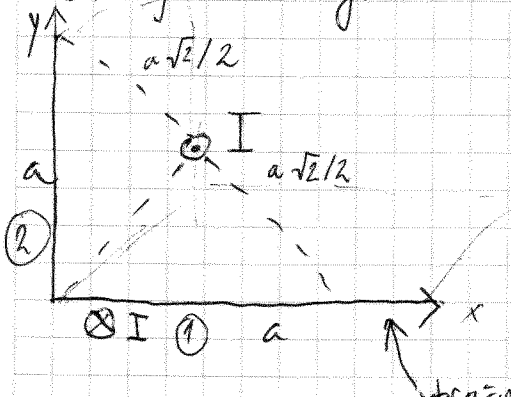
$dl = R\,d\varphi$

$dF_y = dF \sin\varphi$

$$F_y = \int_0^{\pi} IBR\,d\varphi \sin\varphi = I \cdot B R (\cos\pi - \cos 0) = -2IBR$$

$$\vec{F} = -e_y \vec{2}IBR = -e_y \cdot 2 \cdot 6\,A \cdot 0{,}5\,T \cdot 4 \cdot 10^{-2}\,m = \boxed{-e_y\, 24 \cdot 10^{-2}\,N}$$

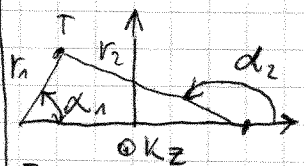
• Linijski vogalni tokovodnik s tokom I in linijski vodnik tvorita dvovod. Določite F_{mag} med njima na dolžini 10 m.



$I = 400\,A \quad l = 10\,cm \quad a = 10\,cm$

tračni vodnik

TRAČNI VODNIK



$$B_x = \frac{\mu_0 k_z}{2\pi}(\alpha_1 - \alpha_2)$$

$$B_y = \frac{\mu_0 k_z}{2\pi} \log \frac{r_1}{r_2}$$

$$B_{x1} = \frac{\mu_0 k_z}{2\pi}\left(\frac{\pi}{4} - \frac{3\pi}{4}\right) = -\frac{\mu_0 k_z}{2\pi}\left(-\frac{\pi}{2}\right)$$

$B_{x2} = 0$ (kot $r_1 = r_2$)

$B_{y1} = 0$

$$B_{y2} = +\frac{\mu_0 k_z}{2\pi}\left(-\frac{\pi}{2}\right)$$

$$\vec{B} = B\vec{x}_1 + B\vec{y}_2 = \frac{\mu_0 k_z}{2\pi}\left(-\frac{\pi}{2}\right)(\vec{e_x} - \vec{e_y})$$

$$\vec{F} = I\,d\vec{l} \times \vec{B} = I\vec{e_z} \cdot l \times \frac{\mu_0 k_z}{2\pi}\left(\frac{\pi}{2}\right)(\vec{e_x} - \vec{e_y})$$

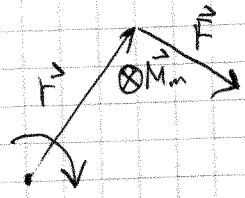
$\vec{F} = I l \frac{\mu_0 k}{4} (\vec{e}_y + \vec{e}_x) =$ $\quad K = 1/2a$

$= \frac{\mu_0}{4} I l \frac{1}{2a} (\vec{e}_x + \vec{e}_y)$

$\vec{F} = \frac{4\pi 10^{-7} \frac{Vs}{Am} (400A)^2 \, 10m}{8 \cdot 0{,}1m} (\vec{e}_x + \vec{e}_y) = 2{,}51\,N(\vec{e}_x + \vec{e}_y)$

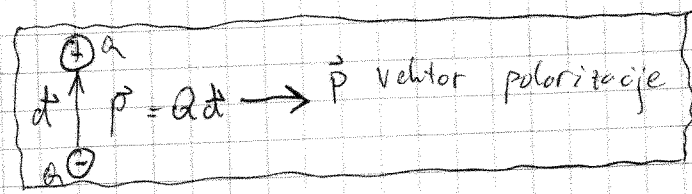
NAVOR

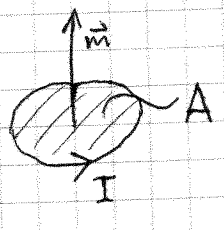
$\vec{M}_m = \vec{r} \times \vec{F}$



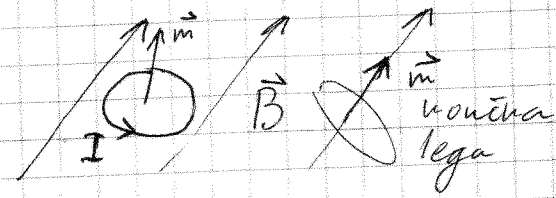
$\vec{M}_m = \vec{m} \times \vec{B}$

magnetni dipolni moment

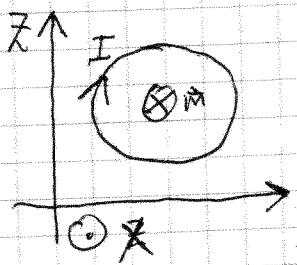




$\vec{m} = \vec{e}_n I A$



V ravnini $x = 0$ leži zanka $A = 1\,cm^2$ s tokom $I = 1{,}5\,mA$. Kolikšen navor deluje na zanko, če jo iz postavimo polju $\vec{B} = (1'5, 5'2, 3'4)\,mT$.



$\vec{m} = -\vec{e}_x I A = -\vec{e}_x \, 10^{-4} m^2 \cdot 1{,}5 \cdot 10^{-3} A =$

$= 1{,}5 \cdot 10^{-7} Am^2 (-\vec{e}_x)$

$\vec{M}_m = \vec{m} \times \vec{B} = 1{,}5 \cdot 10^{-7} Am^2 (-e_x) \times (1{,}5, 5'2, 3'4) \cdot 10^{-3} T$

$= 1{,}5 \cdot 10^{-7} (1'5 \cdot \emptyset - \vec{e}_z \cdot 5{,}2 + \vec{e}_y \cdot 3{,}4) \cdot 10^{-3} Nm$

$= \boxed{1{,}5 \cdot 10^{-10} (\emptyset, 3'4, -5'2)\,Nm}$

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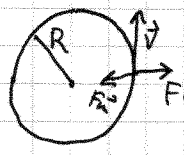
19.3.2008

LORENTZOVA SILA

$\vec{F} = Q\vec{E} + Q\vec{v} \times \vec{B}$

enakomerno kroženje

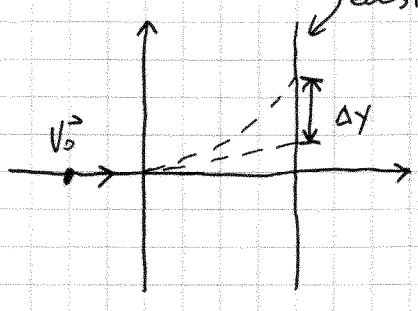


$|\vec{F_c}| = |\vec{F_m}|$

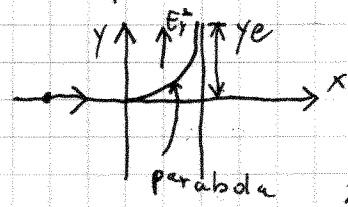
$\frac{mv^2}{2} = QvB$ $\qquad \boxed{R = \frac{mv}{QB}}$

$v/R = \omega = QB/m$ ciklotronska frekvenca

- 1. OE 14.4.1999
- 1. Delce ($Q = 1.6 \cdot 10^{-18}$ As, $m = 2 \cdot 10^{-21}$ kg) vpihujemo v prostor $x > 0$ s hitrostjo $v_0 = e_x 4$ m/s
  Določite razdaljo na $l = 20$ cm oddaljenem zaslonu, če v prvem primeru vklopimo le el polje $E = 5$ V/m, v drugem pa le magnetno (homogeno) v z smeri $B = e_z 0{,}25$ T



① vklopimo $\vec{E}$:



$x \propto t$ $\qquad y \propto t^2$

$X = V_0 t$

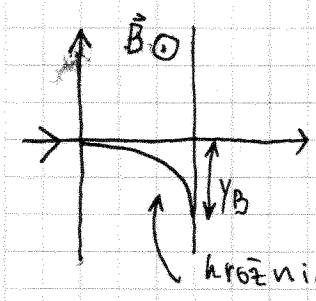
$y = y_0 + V_0 y t + at^2/2 = 1/2 \cdot (QE/m) \cdot t^2$

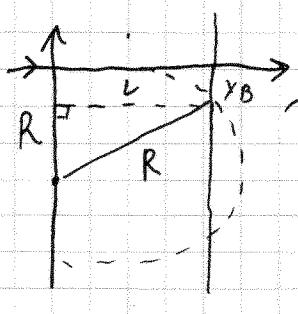
$ma_y = QE \rightarrow a_y = QE/m$

$y = \frac{QE}{2m}\left(\frac{x}{v_0}\right)^2 \Rightarrow y(x=l) = y_e = \frac{QE}{2m}\left(\frac{l}{v_0}\right)^2 =$

$= \frac{1.6 \cdot 10^{-18} \cdot 5 \cdot 0.2^2}{2 \cdot 2 \cdot 10^{-21} \quad 4^2} = \boxed{5\ \text{m}}$

② B (samo B)





$(R - y_B)^2 + l^2 = R^2$

$(R - y_B)^2 = R^2 - l^2$

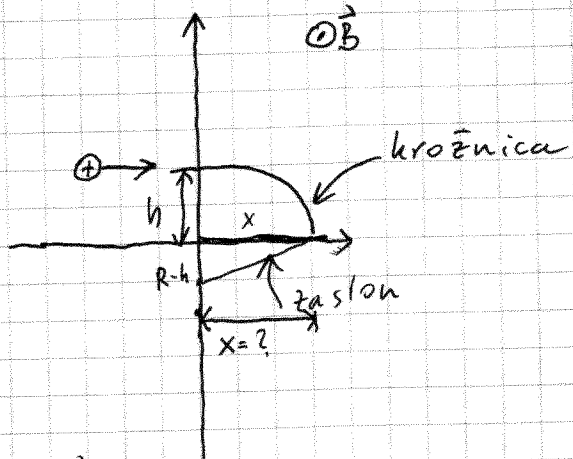
$R - y_B = \pm\sqrt{R^2 - l^2}$ $\qquad y_B = R \mp \sqrt{R^2 - l^2}$

$$R = \frac{m \cdot v}{QB} = \frac{2 \cdot 10^{-27} \cdot 10^{3}}{1.6 \cdot 10^{-18} \cdot 0,025} = 0,02\,m$$

$$y_B = 0,2 - \sqrt{(0,2)^2 - (0,2)^2} = 0.2\,m \qquad \Delta y = y_E + y_B = \boxed{5,2\,m}$$

• 1.0 EII, 1994

Delec ($Q = 1,6 \cdot 10^{-19}\,As$, $m = 1.66 \cdot 10^{-26}\,kg$) vstopi v mag polje gostote pretoka $B = 1mT$ s hitrostjo $v = 6265\,m/s$ na višini $0,2\,m$ nad zaslonom. Slika. Kje je zadane zaslon.



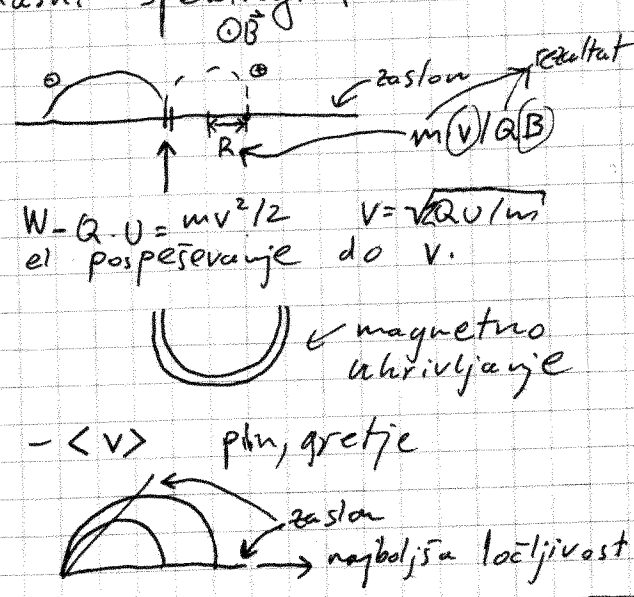
$$(R-h)^2 + x^2 = R^2$$
$$R^2 - 2Rh + h^2 = R^2 - x^2$$
$$x^2 = 2Rh - h^2$$
$$x = \pm\sqrt{h(2R-h)}$$

rešljivo: $2R \geq h$, drugače se ne dotakne zaslona

$$R = \frac{m \cdot v}{Q \cdot B} = \frac{1.66 \cdot 10^{-26}\ 6265}{1.6 \cdot 10^{-19}\ \ 10^{-3}} = \boxed{0.63\,m}$$

$$x\sqrt{0.2 \cdot (2 \cdot 0.6^3 - 0.2)} = \sqrt{0.212} \simeq \boxed{0.45\,m}$$

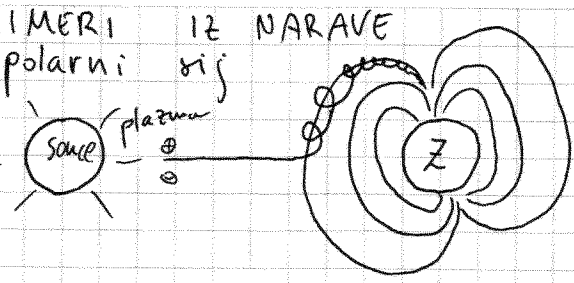
Masni spektrograf



18

PRIMERI IZ NARAVE

- polarni sij

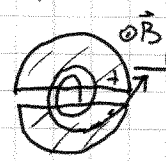


$V = \frac{150 \cdot 10^9}{10 \cdot 3600} = 4000\ km/s$

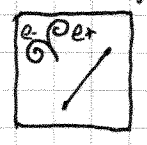
- pospeševalnih

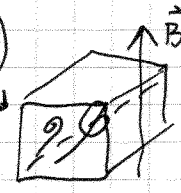
pospeševanje osnovnih delcev → - v mirujočo tarčo
- v druge delce z nasprotno hitrostjo

- linearni
- ciklotroni
- betatroni



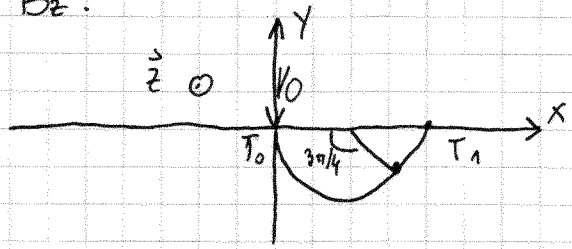
detektorji → • meglična celica
• mehurčna celica





- 1. OE II, 3. 5. 04

Elektron vstopi z vodoravne točke T_0 v polprostor $y < 0$, v katerem je homogeno magnetno polje $\vec{B} = (0, 0, B_z)$ in po času $t_1 = 25\ ns$ doseže T_1. Določite B_z.



kotna hitrost (iz podatkov)

$\omega = \frac{\varphi_1}{t_1}$

$R = mv/QB$

$\frac{QB}{m} = \frac{v}{R} = \omega$

$\omega = \frac{Q}{m} B$

$B_z = \omega \frac{m}{Q} = \frac{\varphi_1 m}{t_1 Q} =$

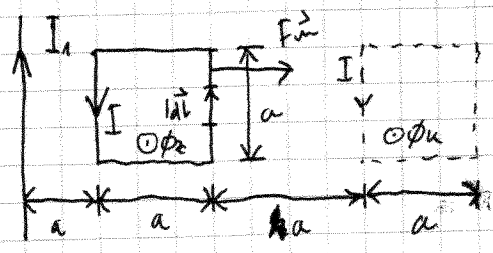
$= \frac{3\pi \cdot 9.1 \cdot 10^{-31}}{4 \cdot 25 \cdot 10^{-9} \cdot 1.6 \cdot 10^{-19}} \approx 60 \cdot 10^{-5}\ T =$

$= \boxed{0{,}6\ mT}$

po kakšnem radiu se giblje, če je $V = 0.1c$

$V = 3 \cdot 10^7\ m/s$

$R = \frac{v}{\omega} = \frac{3 \cdot 10^7\ m \cdot 4 \cdot 25 \cdot 10^{-9}}{s\ 3\pi} = \frac{1}{\pi}\ m \approx 0{,}3\ m$



$A_m = ?$

$A_m = I \cdot (\phi_k - \phi_z)$

$\phi_z = \phi_{z,I_1} + \phi_{z,I}$

$\phi_k = \phi_{k,I_1} + \phi_{k,I}$

$\phi_{zI} = \phi_{kI}$

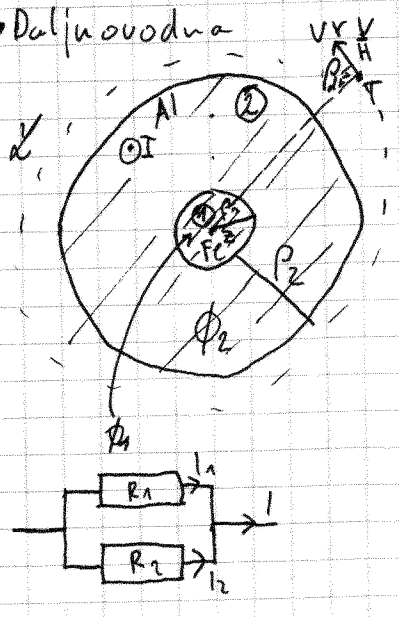
$\phi_k - \phi_z = \phi_{k,I_1} - \phi_{z,I_1}$

$\phi_{z,I_1} = \frac{-\mu_0 I_1 \cdot a}{2\pi} \log \frac{2a}{a}$ ← referenčna smer

$\phi_{k,I_1} = \frac{-\mu_0 I_1}{2\pi} \frac{a}{\text{vzporedna dolžina}} \log \frac{4a}{3a}$

$A_m = (-I)\left(-\frac{\mu_0 I_1 a}{2\pi}\right) \log \frac{3}{2} = \frac{\mu_0 I I_1 a}{2\pi} \log \frac{3}{2} > 0$

• Daljnovodna



(jeklo, aluminij) jeklo - feromagnetik

$I = 400A$ ima

$A_1 = 20\,mm^2 \Rightarrow P_1 = \sqrt{A_1/\pi} \simeq 2{,}52\,mm$

$A_2 = 120\,mm^2 \Rightarrow P_2 = \sqrt{(A_1+A_2)/\pi} \simeq 6{,}68\,mm$

$\gamma_2/\gamma_1 = 4$

$\mu_1/\mu_2 = 500$ $\quad \mu_2 \simeq \mu_0$

$\varphi_1 = \frac{\phi_1}{l} = ?$ $\quad \varphi_2 = \frac{\phi_2}{l}$

$I_1 + I_2 = I$

$I_1 R_1 = I_2 R_2$

$\frac{I_2}{I_1} = \frac{R_1}{R_2} = \frac{\overbrace{\gamma_2}^{4}}{\gamma_1} \frac{\overbrace{A_2}^{6}}{A_1} = 24$

$R_1 = \frac{1}{\gamma_1} \cdot \frac{l}{A_1}$ $\quad R_2 = \frac{1}{\gamma_2} \frac{l}{A_2}$

$I_2 = 24 I_2$ $\quad I_1 + 24 I_1 = 400A \rightarrow I_1 = 16A,\ I_2 = 384A$

$\oint_{\mathcal{L}} \underbrace{\vec{H}\cdot d\vec{l}}_{\text{vpraševa } H\cdot dl} = I_{prosti}$ $\qquad H\,dl = H_\varphi(\rho)\,dl$

$$\oint_{\mathcal{L}} H_\varphi(\rho)\,dl = H_\varphi(\rho)\oint_{\mathcal{L}} dl = 2\pi\rho\, H_\varphi(\rho)$$

a) $\rho < \rho_2$: $I_{prosti} = I$

b) $\rho < \rho_1$: $I_{prosti} = (I/\pi\rho_1^2)\cdot\pi\rho^2$

c) $\rho_1 < \rho < \rho_1$: $I_{prosti} = I_1 + (I_2/\pi(\rho_2^2-\rho_1^2))\,(\rho^2-\rho_1^2)\pi$

$$2\pi\rho\, H_\varphi(\rho) = \begin{cases} (I_1/\rho_1^2)\cdot\rho^2 & \rho < \rho_1 \\ I_1 + (I_2/(\rho_2^2-\rho_1^2))(\rho^2-\rho_1^2) & \rho_1 < \rho < \rho_2 \\ I & \rho > \rho_2 \end{cases} \Big/ :2\pi\rho \Big/ \mu(\rho)$$

$$\mu(\rho) = \begin{cases} \mu_1 = 500\,\mu_0 ; & \rho < \rho_1 \\ \mu_0 ; & \rho > \rho_1 \end{cases}$$

$$B_\varphi = \begin{cases} \frac{N_1 I_1}{2\pi\rho_1^2}\rho ; & \rho < \rho_1 \\ \frac{\mu_0 I_1}{2\pi\rho} + \left(\frac{\mu_0 I_2}{2\pi(\rho_2^2-\rho_1^2)}(\rho-\rho_1^2)\rho\right) ; & \rho_1 < \rho < \rho_2 \\ \frac{\mu_0 I}{2\pi\rho} ; & \rho > \rho_2 \end{cases}$$

mi enačbej, polje nervemo

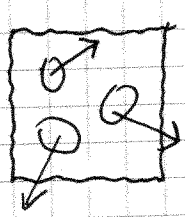
$$\Phi_1 = \int_0^{\rho_1} B_\varphi(\rho)\,d\rho = \ldots = \mu_1 I_1/4\pi = 9000\,\mu\,Wb/m$$

$$\Phi_2 = \int_{\rho_1}^{\rho_2} B_\varphi(\rho)\,d\rho = \ldots = \frac{\mu_0}{2\pi}\left[\frac{I_2}{2} + \left(I_1 - \frac{\rho_1^2 I_2}{\rho_2^2-\rho_1^2}\right)\log\frac{\rho_2}{\rho_1}\right] \simeq 25{,}9\,\mu\,Wb/m$$

26.3.2008

Magnetni materiali

$\vec{M}$ vektor magnetizacije

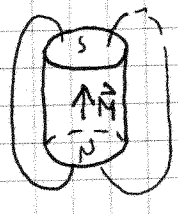


vsota vseh dip. momentov na volumen.

$\vec{m}$ magnetni dipolni moment

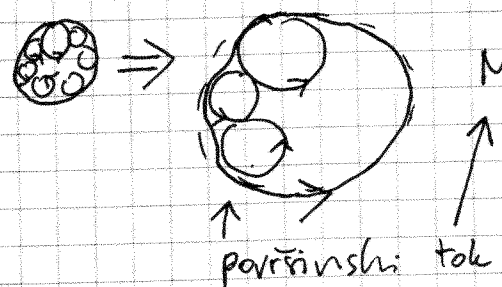
$$\vec{M} = \sum \vec{m}_i / \Delta V \ [A/m]$$

- Magnet valj $2r = 1cm$, $l = 5cm$ z $M = 5{,}3 \cdot 10^{-3}$ A/m
  Kolikšen je dipolni moment?



$m = ?$

$$m = M \cdot V = M \cdot \pi r^2 \cdot l = 5{,}3 \cdot 10^3 \, A/m \cdot \pi \cdot (0{,}5 \cdot 10^{-2} m)^2 \cdot 5 \cdot 10^{-2} m = 2{,}08 \cdot 10^{-2} \, Am^2$$



Feromagnetiki Ferimagnetiki (feriti)

$\vec{H}$ - magnetna poljska jakost

$$\oint_A \vec{H} d\vec{l} = I_{objeti} = \int_A \vec{j} \, d\vec{A}$$ Amperov zakon

$$\vec{M} = \chi \vec{H}$$

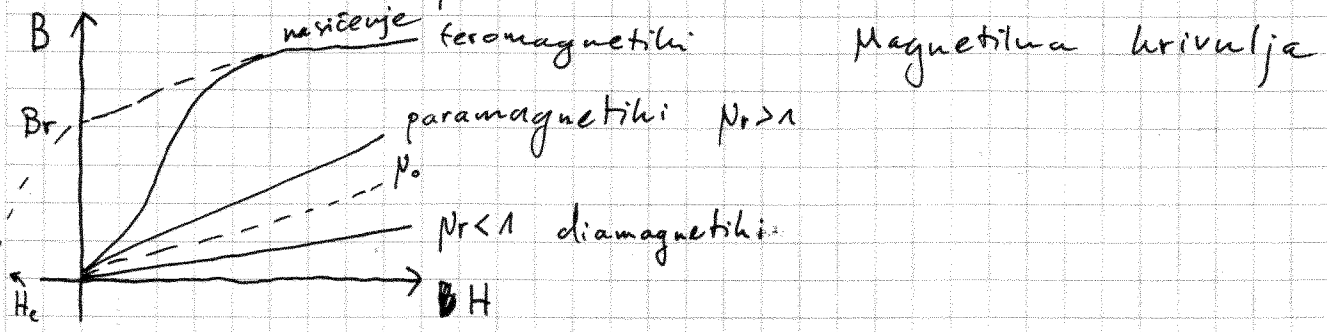
χ magnetna susceptibilnost

$$\vec{B} = \mu_0 \vec{H} + \mu_0 \vec{M}$$

$$\vec{B} = \mu_0 \vec{H} + \mu_0 \chi \vec{H} = (1+\chi)\mu_0 \vec{H}$$

$$\boxed{\vec{B} = \mu_r \mu_0 \vec{H}}$$

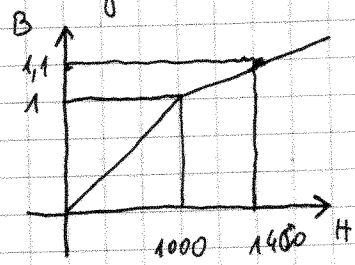
μ_r ← relativna permeabilnost $= \chi + 1$



Magnetilna krivulja

$$\mu_r = \frac{B}{H \mu_0}$$ statična relativna permeabilnost

- $B = 0{,}5\,T$, $B = 1\,T$, $B = 1{,}1\,T$ iz linearizirane magnetilne krivulje določite μ_r. (H pri $B = 1\,T$)



$$\mu_R = \frac{B}{\mu_0 H}$$

$$\mu_r(1\,T) = \frac{1\,T}{4\pi \cdot 10^{-7}\,Vs/Akm \cdot 1000\,A/m} = 795$$

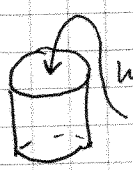
$\mu_r(0{,}5\,T) = 795$ konstanten v linearnem območju

$\mu_r(1{,}1\,T) = 625$

(M) $1\,T = \mu_0 1000\,A/m + \mu_0 M$

$$M = \frac{1\,T}{\mu_0} - 1000\,A/m = M$$ M!

$M = 795 \cdot 10^3\,A/m$.

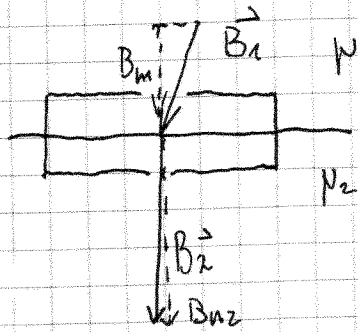


$B \approx \frac{\mu_0 NI}{l}$ v sredini kot tuljava

$B \approx \frac{1}{2}\frac{\mu_0 NI}{l}$ na robu

$\Rightarrow K = M \rightarrow B = \frac{1}{2}\mu_0 M$

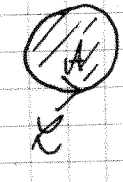
Mejni pogoji

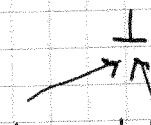


$B = \mu_r \mu_0 H = \mu H$

$\oint_A \vec{B} d\vec{A} = 0 \Rightarrow B_{n1} = B_{n2}$

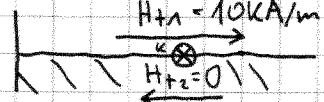
$\oint_{\partial A} \vec{H} d\vec{l} = \int_A \vec{j}\, d\vec{A}$





Če $K = 0$, je $H_{t1} = H_{t2}$. Če $K \neq 0$, je $H_{t1} - H_{t2} = \pm K$

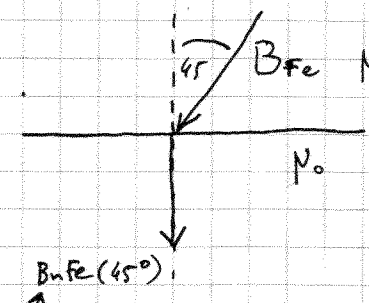
$\vec{n} \times (\vec{H}_1 - \vec{H}_2) = \vec{K}_{prosti}$



$H_{oblage} = 10 kA/m = K$

Primer

Primer: V feromagnetiku s $\mu_{Fe} = 1200 \mu_0$ je homogeno polje $B_{Fe} = 1,2 T$, ki ga z normalo oklepa $\alpha_{Fe} = 45°$! Določite absolutno vrednost B v zraku.



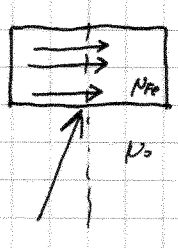
$B_{nFe} = B_{n\,zrak}$

$B_{nFe} = B_{Fe} \cdot \sin 45 = 0,848\ T$

$\frac{B_{tFe}}{\mu_{Fe}} = \frac{B_{to}}{\mu_0}$

$B_{to} = \frac{B_{tFe}\,\mu_0}{\mu_{Fe}} = \frac{0,85\,T \cdot \mu_0}{1200\,\mu_0} = 7 \cdot 10^{-4}\,T$ ← v zraku praktično ni tang. komponente

$B_{v\,zraku} \simeq 0,85\,T$

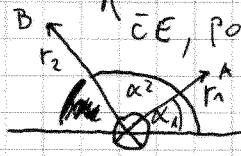


Magnetni potencial (napetost)

$\oint \vec{E}\,d\vec{l} = 0 \leftrightarrow V_e = \int_T^{T(V=0)} \vec{E}\,d\vec{l}$

$\oint \vec{H}\,d\vec{l} = 0 \qquad V_m = \int_T^{T(V_m=0)} \vec{H}\,d\vec{l}$

↑ če E, potem



$\Theta = V_B - V_A$

$V_m = 0$ pri $\alpha = 0$

$I = 10A$

$\alpha_2 = 3\pi/4$

$\alpha_1 = \pi/4$

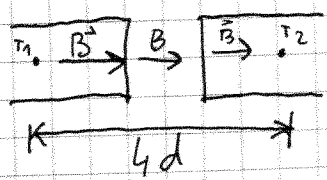
$H = B/\mu_0 = \frac{\mu_0 I}{2\pi r}\frac{1}{\mu_0} = \frac{I}{2\pi r}$

$V_A = \int_T^{T(V=0)} \vec{H}\,d\vec{l} = \int_{\alpha_1}^{0} e_\varphi \frac{I}{2\pi r}\, r\,d\varphi\, e_\varphi$

$= -\frac{I}{2\pi}\int_{\alpha_1}^{0} d\varphi = \boxed{\frac{I}{2\pi} \cdot \alpha_1}$

$$V_B = \frac{I}{2\pi}\alpha_2 \qquad \Theta = \frac{1}{2\pi}(\alpha_2 - \alpha_1) = \frac{1}{2\pi}\left(\frac{2\pi}{4}\right) = \boxed{\frac{I}{4}}$$

$\mu = 10^{-4}$ Vs/Am feromagnetik, pojavi se razpoka $d = 1$ mm (razpoka). Polje v feromagnetiku 1T, $\perp$ na razpoko. Kolikšna je magnetna napetost med T_1 in T_2.



$$\Theta_{T_1T_2} = \int_{T_1}^{T_2} \vec{H}\,d\vec{l} = H_{Fe}\cdot l_{Fe} + H_0\,d = H_{Fe}\cdot 3d + H_0 d$$

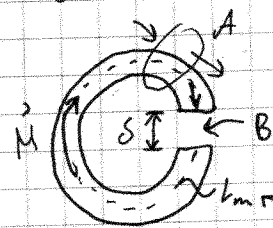
($=$: homogeno polje)

$H_{Fe} = B_{Fe}/\mu_{Fe}$ $\quad H_0 = B_0/\mu_0$ $\quad B_0 = B_{Fe}$ (mejni pogoji)

$$\frac{B_{Fe}}{\mu_{Fe}}\cdot 3d + \frac{B_0}{\mu_0}d$$

$$\Theta = \left(\frac{1\,T}{10^{-4}\,\frac{Vs}{Am}}\cdot 3 + \frac{1\,T}{4\pi\cdot 10^{-7}\,\frac{Vs}{Am}}\right)\cdot 10^{-3}\,m = 826\,A$$

- Kolikšna je magnetizacija permanentnega magneta z zračno režo, če izmerimo $B_\delta = 0{,}5\,T$!



$a = 2{,}5$ cm
$b = 3{,}5$ cm
$A = 1\,cm^2$
$\delta = 0{,}5$ cm
l_m ← srednja dolžina gorotvnice

Predpostavimo homogeno polje v jedru in zračni reži in računamo po sredini jedra. Zanemarimo stresanje polja v reži.

$$\oint \vec{H}\,d\vec{l} = \emptyset \Rightarrow \sum_i H_i l_i = \emptyset$$

$$H_m l_m + H_\delta\cdot\delta = \emptyset$$

$$\int_A \vec{B}\,d\vec{A} = \emptyset \Rightarrow \sum_j \Phi_j = \emptyset$$

Abitur

1.4.2008

$\vec{\Phi}_m = \vec{\Phi}_\delta$

$B_m A_m = B_\delta A_\delta$

zanemaritev stresanja polja

$B_m = \mu_0 H_m + \mu_0 M = B_\delta$ $\quad H_m = \frac{B_\delta}{\mu_0} - M$

$\left(\frac{B_\delta}{\mu_0} - M\right) l_m + \frac{B_\delta}{\mu_0}\delta = \Phi \Rightarrow M$

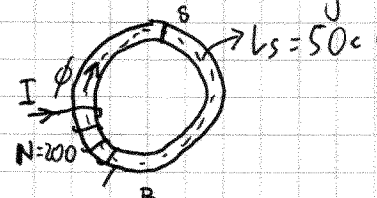
$\frac{B_\delta}{\mu_0}\left(l_m + \delta\right) - M l_m = \Phi$

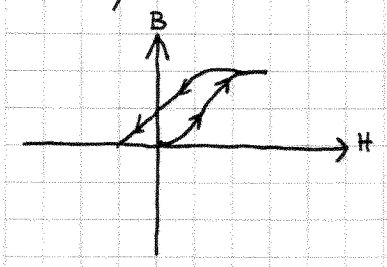
$M = \frac{B_\delta}{\mu_0}\left(1 + \frac{\delta}{l_m}\right)$

$$M = \frac{0{,}5\,T}{4\pi\,10^{-7}\,\frac{Vs}{Am}}\left(1 + \frac{0{,}5\,cm}{6\pi\,cm - 0{,}5\,cm}\right)$$

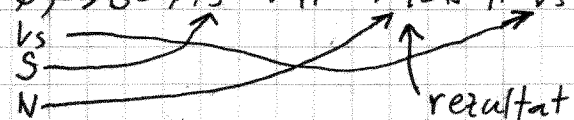
Magnetilna krivulja

V enozančnem jedru iz transformatorske pločevine preseka 5 cm² in srednje dolžine 50 cm je magnetni pretok 0,5 mVs. Kolikšen tok teče skozi navitje z 200 ovoji?





plan: podatki: $\Phi \rightarrow B = \Phi/S \rightarrow H \rightarrow I = \frac{1}{N} \cdot H \cdot l_s$

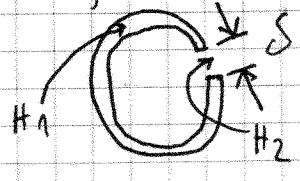


$B = \frac{\Phi}{S} = \frac{0{,}5\,mVs}{5 \cdot 10^{-4}\,m^2} = 1\,T$

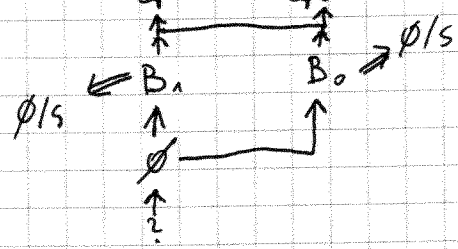
$B = 1\,T \xrightarrow[\text{krivulja}]{\text{mag}} H = 0{,}2\,kA/m = 200\,A/m$

$I = \frac{H \cdot l_s}{N} = \frac{200 \cdot 0{,}5}{200} = 0{,}5\,A$

② Kolikšen mora biti I (za prejšnjo nalogo), če je v jedru reža širine $\delta = 1\,mm$.



$$N I = H_1(l_s - \delta) + H_0 \delta$$



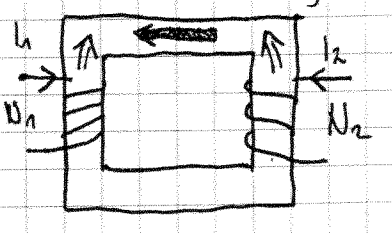
$B_1 = \emptyset/S = 1\,T = B_0$

$H_1 \approx 200\,A/m$ $\qquad H_0 = \frac{B_0}{\mu_0} = \frac{1\,T}{4\pi \cdot 10^{-7}} \simeq \frac{100 \cdot 10^5}{4\pi} \approx 8 \cdot 10^5 \frac{kA}{m}$

$$I = \frac{1}{N}\left[H_1(l_s - \delta) + H_0 \delta\right] = \frac{1}{200}\left[200 \cdot (0{,}5 - 0{,}001) + 800 \cdot 10^3 \cdot 10^{-3}\right] =$$

$$= \frac{9}{2} = 4{,}5\,A$$

- 1. kol OE II, 23.4.2002: Na feromagnetnem jedru iz transformatorske pločevine, s tema navitji $I_1 = 5A$, $N_1 = 100$, $I_2 = 10A$, $N_2 = 200$. Določite srednjo gostoto magnetnega pretoka v jedru, če je $l_s = 30\,cm$ in $S = 20\,cm^2$.



$$B \leftarrow H \leftarrow IN = H \cdot l_s$$

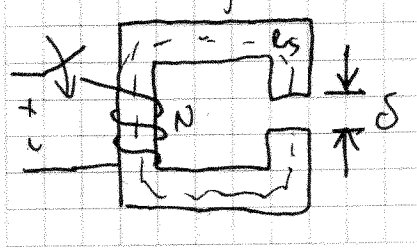
$$H \cdot l_s = I_2 N_2 - I_1 N_1$$

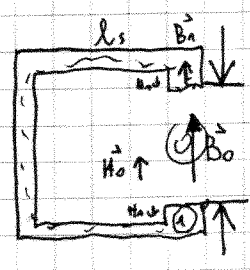
$$H = \frac{I_2 N_2 - I_1 N_1}{l_s} = \frac{2000 - 500}{0{,}3} = \frac{1500}{3} \cdot 10 = 5000\,A/m$$

$$H = 5000\,A/m \xrightarrow[krivulja]{magnetna} B \simeq 1{,}58\,T$$

- 1 kol OE II, 4.5.1999: Feromagnetno jedro z režo magnetimo predhodno z vklopom stikala. Določite povprečno magnetizacijo v jedru, če je v reži $B_0 = 0{,}6\,T$! Srednja dolžina = 0,3 m, širina reže je $\delta = 1\,mm$.

$$\vec{M} = \frac{\vec{B}}{\mu_0} - \vec{H}$$





$$\oint \vec{H}\, d\vec{l} = I N$$

$$H_1 l_s + H_0 \delta = \emptyset$$

$$H_1 = -\frac{H_0 \delta}{l_s}$$

$B_1 = B_0$ se ohranja
H_0 v nasprotni smeri

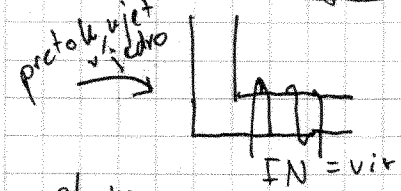
Abeta

$\vec{B}_0 = \mu_0 \cdot \vec{H}_0$

$M = \frac{B_1 = B_0}{\mu_0} - H_1 \qquad M = \frac{B_0}{\mu_0} - \left(-\underbrace{H_0}_{\frac{B_0}{\mu_0}} \frac{\delta}{l_s}\right) = \frac{B_0}{\mu_0} + \frac{B_0}{\mu_0}\frac{\delta}{l_s} = \boxed{\frac{B_0}{\mu_0}\left(1 + \frac{\delta}{l_s}\right)} =$

$= \frac{0.6 \cdot 10^7}{4\pi}\left(1 + \frac{1}{300}\right) = 5 \cdot 10^5 \ A/m$

Modelna vezja



analogija el. vezij:

el. vez.
- I
- U_g
- U
- $R = U/I$

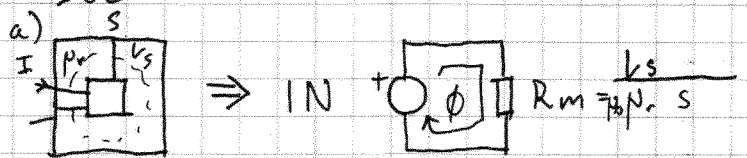
mag. vezje
- $\Phi = \int B_n / S_n$
- IN
- $H_n l_n$ (padci mag. napetosti)
- $R_m = H_n l_n / \Phi = H_n l_n / B_n S_n$

Če velja $B_k = \mu_k \cdot H_k$:

$$R_m = \frac{l_k}{\mu_k \cdot S_k}$$

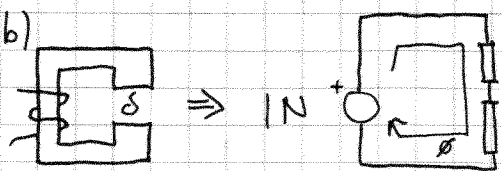
MODELI

a)



$R_m = \frac{l_s}{\mu_0 N_r \cdot S}$

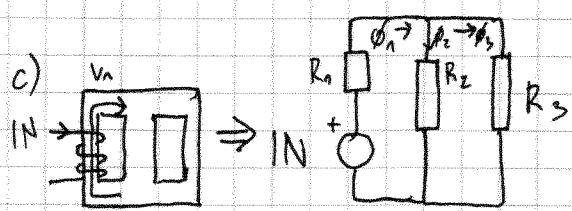
b)



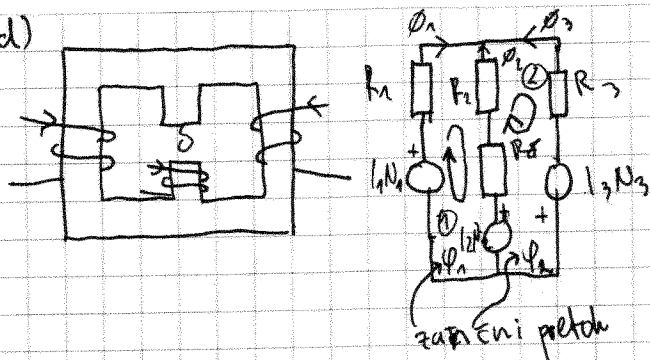
$R_m = l_s / \mu_0 N_r S$

$R_0 = \delta / \mu_0 S$

c)



d)

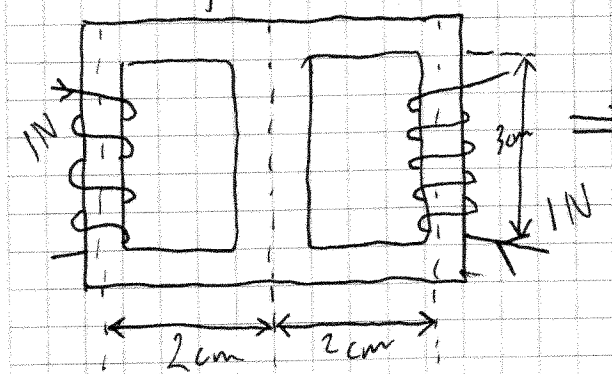


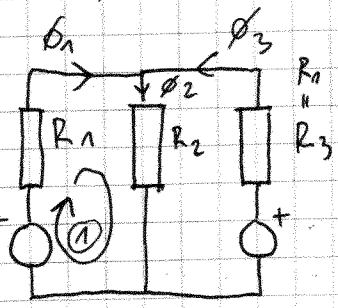
① $I_1N_1 - I_2N_2 = R_1\Phi_1 - R_2\Phi_2 - R_\delta\Phi_2$

② $I_2N_2 + I_3N_3 = (R_\delta + R_2)\Phi_2 - R_3\Phi_3$

Φ_1: $I_1N_1 - I_2N_2 = (R_1 + R_2 + R_\delta)\Phi_1 - (R_2 + R_\delta)\Phi_2$

- izpit 14. 6. 2006: Simetrično jedro $S = 1\,cm^2$ in $\mu_r = 10^4$ ima v srednjem stebru $B_2 = 0{,}8\,T$. Kolikšen je I skozi dvodelno navitje z $N = 150$ ovoj: (2×150)





$R_1 = R_3 = \dfrac{a}{\mu_r S \mu_0}$

$a = 7\,cm$

$b = 3\,cm$

$R_2 = \dfrac{b}{\mu_r S \mu_0}$

$\Phi_1 + \Phi_3 = \Phi_2 = 2\Phi_1 \rightarrow \Phi_1 = \Phi_2/2$

① $IN = R_1\Phi_1 + R_2\Phi_2$ } $\Phi_3 = \Phi_1$

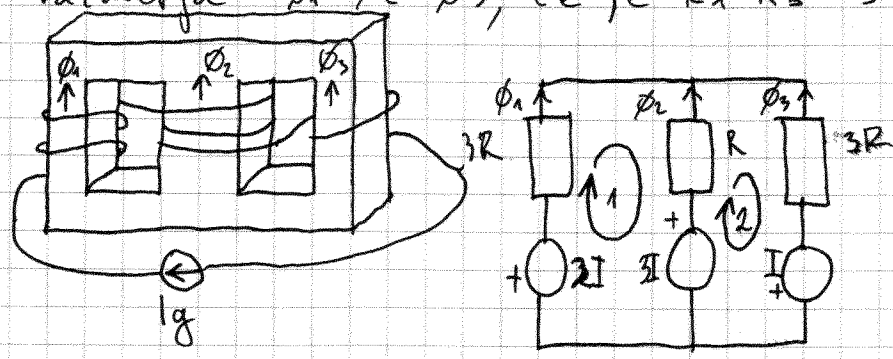
② $IN = R_1\Phi_3 + R_2\Phi_2$ }

$IN = R_1\Phi_2/2 + R_2\Phi_2$

$$I = \frac{\Phi_2}{N}\left[\frac{R_1}{2} + R_2\right] = \frac{B_2 S}{N}\left[\frac{a}{2N_1\mu_0 S} + \frac{b}{\mu_r \mu_0 S}\right]$$

$$= \frac{B_2}{N\mu_0\mu_r}\left[\frac{a}{2} + b\right] = \frac{0{,}8 \cdot 10^7}{150\ 10^4\ 4\pi}(3{,}5 + 3)\cdot 10^{-2} = \boxed{27{,}6\,mA}$$

- Tristebrno jedro ovijemo s tokovno pentljo. Določite razmerja $\Phi_1 : \Phi_2 : \Phi_3$, če je $R_1 = R_3 = 3R_2 = 3R$



$\Phi_1 : \Phi_2 = \alpha \qquad \Phi_3 : \Phi_2 = \beta$

① $-2I - 3I = 3R\Phi_1 - R\Phi_2$

$-5I = R \cdot (3\Phi_1 - \Phi_2)$

$\boxed{3\Phi_1 - \Phi_2 = -5I/R}$

② $3I + I = R\Phi_2 - 3R\Phi_3$

$4I = R(\Phi_2 - 3\Phi_3)$

$\boxed{(\Phi_2 - 3\Phi_3) = 4I/R}$

$1/R = c$

$\Phi_1 + \Phi_2 + \Phi_3 = 0$

$3\Phi_1 - \Phi_2 = -5c \longrightarrow$

$\Phi_2 - 3\Phi_3 = 4c \longrightarrow$

$\Phi_1 + \Phi_2 + \Phi_3 = \Phi$

$\Phi_1 = 1/3\,[\Phi_2 - 5c]$

$\Phi_3 = 1/3\,[\Phi_2 - 4c]$

$$\frac{1}{3}\left[\Phi_2 - 5c\right] + \Phi_2 + \frac{1}{3}\left[\Phi_2 - 4c\right] = \Phi$$

$$\frac{5}{3}\Phi_2 - \frac{5}{3}c - \frac{4}{3}c = 0$$

$$\Phi_2 = \frac{9}{5}c$$

$$\Phi_1 = 1/3\left(\frac{9}{5}c - 5c\right) = -\frac{16}{15}c$$

$$\Phi_3 = 1/3\left(\frac{9}{5}c - 4c\right) = -\frac{11}{15}c$$

$$\frac{\Phi_1}{\Phi_3} = \frac{16}{11} \qquad \frac{\Phi_2}{\Phi_3} = -\frac{27}{11}$$

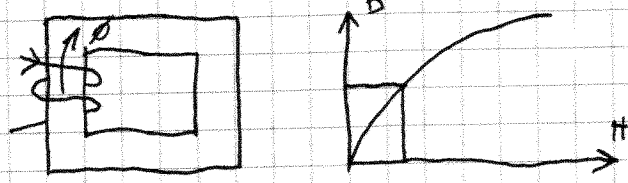
$\Phi_1 : \Phi_2 = 16 : 11$

$\Phi_2 : \Phi_3 = -27 : 11$

$\Phi_1 : \Phi_2 : \Phi_3 = 16 : -27 : 11$

9. 4. 2008

- Namagnetpolan jedro $l_s = 0{,}5\ m$, $S = 30\ cm^2$ je navitje $N = 2000$. Magnetilna krivulja: $B = k \cdot \sqrt{H}$, kjer $k = 0{,}05\ T\sqrt{m/A}$. Pri kolišnem toku je $\phi = 3 \cdot 10^{-3}\ Vs$?)

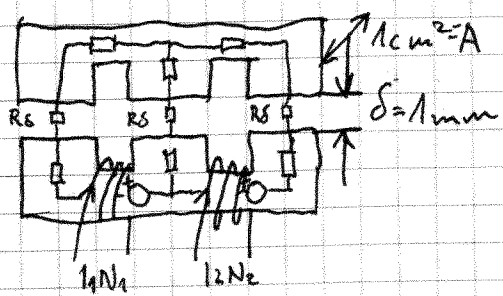


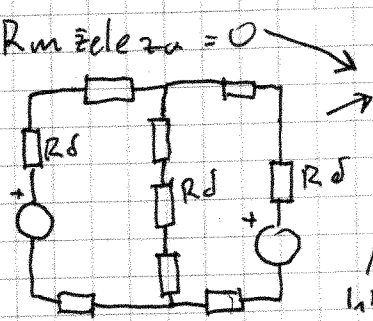
$\phi \rightarrow B \xrightarrow{k_s} H \rightarrow I$

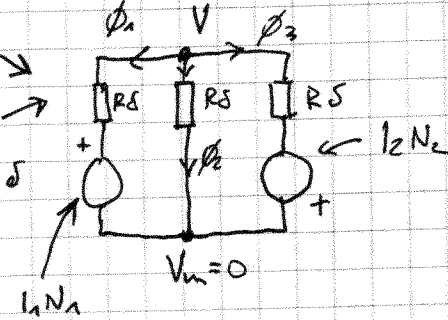
$$I = \frac{H \cdot l_s}{N} \qquad H = \left(\frac{B}{k}\right)$$

$$I = \left(\frac{B}{k}\right)^2 \frac{l_s}{N} = \frac{l_s}{N}\left(\frac{\phi}{kS}\right)^2$$

$$I = \frac{0.5}{2000}\left(\frac{3 \cdot 10^{-3}}{0.05 \cdot 30 \cdot 10^{-4}}\right)^2 = 0.1\ A$$

- 





$I_1N_1 = 200 \qquad I_2N_2 = 400$

$\phi_1 + \phi_2 + \phi_3 = 0$

$$\frac{V - I_1N_1}{R_S}\left(= \phi_1\right) + \frac{V}{R_S}\left(= \phi_2\right) + \frac{V + I_2N_2}{R_S} = 0$$

$3V = I_1N_1 - I_2N_2 = \underline{-200\ A}$ ← amperski ovoj

$V = -200/3\ A$

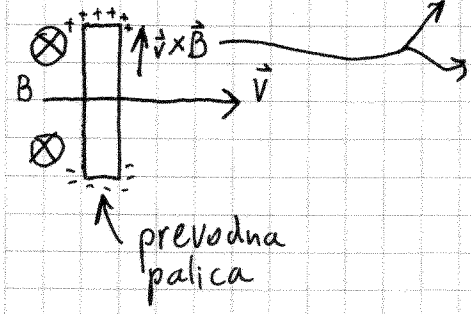
$$\phi_2 = \frac{V}{R_S} = \frac{V}{\frac{\delta}{\mu_0 A}} = -8{,}4\ \mu Vs$$

← magnetna uporaba

32

Inducirana napetost

$\vec{F_m} = Q \cdot \vec{v} \times \vec{B}$ $\qquad \vec{E_i} = \frac{\vec{F_m}}{Q} = \vec{v} \times \vec{B}$ inducirano polje

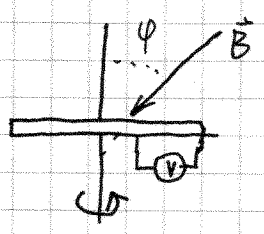


$\Delta U_i = \vec{E_i} \Delta \vec{l}$

$U_i = \oint \vec{E_i} d\vec{l}$ $\qquad U_i = \oint \vec{E} d\vec{l}$ ($\vec{E} = \vec{E_i} + \vec{E_s}$)

$\left.\begin{array}{l} \vec{v} = 5\,m/s \\ B = 20\,mT \\ l = 10\,cm \end{array}\right\}$ $U_i = vBl = 5\,m/s \cdot 20 \cdot 10^{-3}\,Vs/m^2 \cdot 0{,}1\,m = 10\,mV$

• Bakren disk premera $2R = 40\,cm$ se vrti z $n = 2000/min$ v $B = 10\,mT$ (homogeno polje). Smer polja oklepa z osjo $\alpha = 60°$. Disha se dotikata dva drsnika prvi na $r_1 = 10\,cm$, drugi na $r_2 = 20\,cm$. Kolikšna abs. vrednost napetosti meri idealni voltmeter?



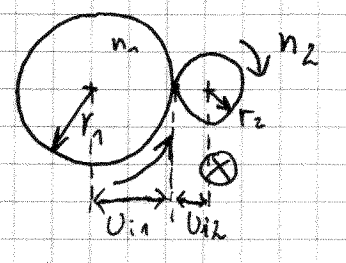
$E_i = vB\cos\varphi$

$v = \omega r$

$dU_i = B\cos\varphi\,\omega r\,dr$ $\qquad \omega = 3000 \cdot 2\pi/60$

$U_i = B\omega\cos\varphi \int_{r_1}^{r_2} r\,dr = 23\,mV$

•



$r_2 = a/2$
$r_1 = a$

Izrazite U_i med osema, če se nitka v homogenem magnetnem polju gostote B, ki vpada pravokotno.

$$U_i = \int (v \times B) \cdot d\vec{l}$$

$$U_i = \int_0^a \omega_1 B r \, dr + \int_0^{a/2} \omega_2 B r \, dr = \frac{\omega_1 B a^2}{2} + \frac{\omega_2 B a^2}{8}$$

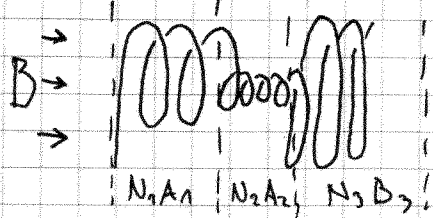
$$\omega = \frac{2\pi n}{60} \qquad n_1 = \frac{n_2}{2}$$

$$U_i = \frac{2\pi n_2}{60} B \frac{a^2}{2} + \frac{2\pi n_2}{60} B \frac{a^2}{8} = \frac{B a^2 \pi n_2}{60}\left(\frac{1}{2} + \frac{1}{4}\right) = \boxed{\frac{3 B a^2 \pi n_2}{4 \cdot 60}}$$

TRANSFORMATORSKA INDUCIRANA NAPETOST

$u_i = -\frac{d\Psi}{dt}$ $\quad \Psi$ = magnetni sklep

Za N zank, skozi katere gre isti fluks Φ, je $\Psi = N \cdot \Phi$.

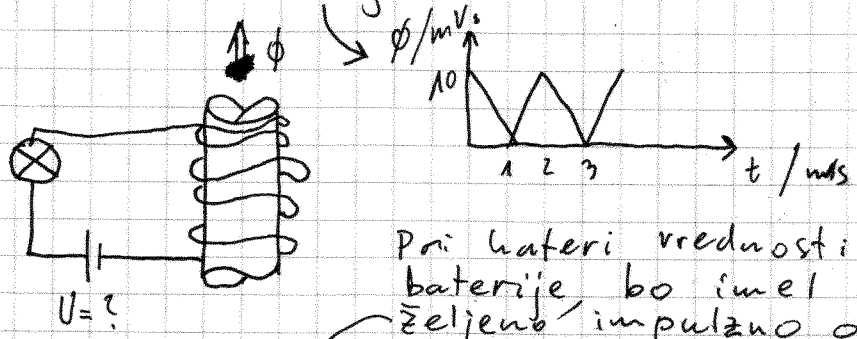


$$\Psi = B(N_1 A_1 + N_2 A_2 + N_3 A_3) =$$
$$= N_1 \Phi_1 + N_2 \Phi_2 + N_3 \Phi_3$$

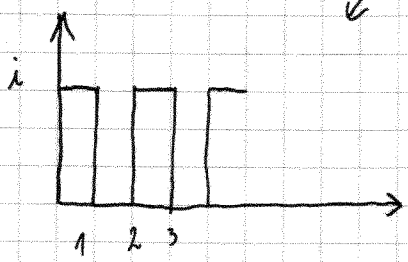
V splošnem: $\boxed{\Psi = \sum_i N_i \Phi_i}$

Ahita

Magnetni pretok v feromagnetnem jedru podaja časovna funkcija.



Pri kateri vrednosti napetosti U baterije bo imel tok I v zanki želeno impulzno obliko.
Pojav samoindukcije je zanemarljiv, navitje ima šest navojev.



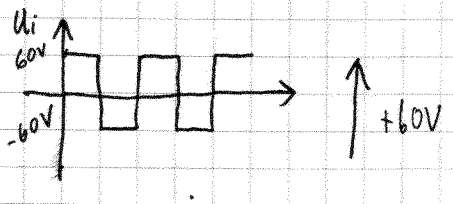
$$u_i = -\frac{d\psi}{dt} = -N\frac{d\Phi}{dt}$$ (Lenz)

Ker gre za odsekoma linearen signal, lahko to napetost v linearnih odsekih smatramo kot

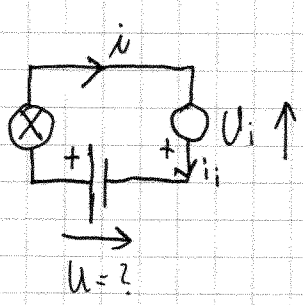
$$u_i = -N\frac{\Delta\Phi}{\Delta t} \qquad u_i(0s < t < 1ms) =$$

$$= 6\frac{10\ mVs - 10mVs}{1\ ms - 0s} = 60V$$

$$u_i(1ms < t < 2ms) = -60V$$

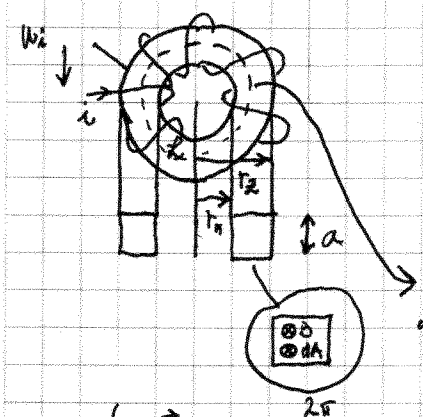


$U = 60V$



16.4.2008

- V zračni toroidni tuljavi pravokotnega prereza je tok $i = I_0 \sin(\omega t)$, $I_0 = 0{,}1\,A$, $\omega = 10^3\,s^{-1}$, $N = 1000$. $R_n = 3\,cm$, $R_z = 4\,cm$, $a = 1\,cm$. Določita u_i!



$$u_i = \frac{-d\Psi}{dt} = -N\frac{d\Phi}{dt} \qquad \omega = 2\pi f$$

$$\Psi = N\cdot\Phi \qquad \Phi = \int_A \vec{B}\,d\vec{A}$$

$$\oint_{\mathcal{L}} \vec{B}\,d\vec{l} = \mu_0 N I$$

$$\oint_{\mathcal{L}} \vec{B}\,d\vec{l} = \int_0^{2\pi} B\cdot r\,d\varphi = B\cdot 2\pi r$$

$$B\cdot 2\pi r = \mu_0 N I$$

$$B = \mu_0 N I / 2\pi r$$

$$\Phi = \int_A \frac{\mu_0 N I}{2\pi r}\,dA = \int_{r_n}^{r_z}\int_0^a \frac{\mu_0 N I}{2\pi r}\,dr\,dz = \frac{\mu_0 N I}{2\pi}\int_{r_n}^{r_z}\frac{dr}{r}\int_0^a dz =$$

$$= \frac{\mu_0 N I}{2\pi}\log\frac{r_z}{r_n}\cdot a$$

$$\Phi(t) = \frac{\mu_0 N I_0 \sin(\omega t)\,a}{2\pi}\log\frac{r_z}{r_n}$$

$$u_i(t) = -\frac{N^2\mu_0 I_0 \sin(\omega t)\,a}{2\pi}\log\frac{r_z}{r_n}\cdot\frac{d}{dt} = -\frac{L\cdot I_0\sin(\omega t)}{dt}$$

$$\boxed{L = \frac{\mu_0 N^2 a}{2\pi}\log\frac{r_z}{r_n}}$$ induktivnost toroidnega navitja

$$L = \frac{4\pi\cdot 10^{-7}\,\frac{Vs}{Am}\cdot 1000^2\cdot 10^{-2}\,m\log\frac{4}{3}}{2\pi}$$

$$L = 2\cdot 10^{-3}\,\frac{Vs}{A}\cdot\log\frac{4}{3} =$$

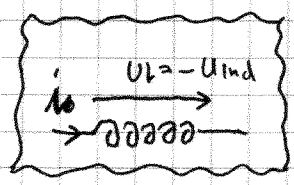
$$L = 5{,}75\cdot 10^{-4}\,Vs/A =$$

$$= 5{,}75\cdot 10^{-4}\,H$$

$$u_i = -L\cdot I_0\,\omega\cdot\cos(\omega t)$$

$$U_{max} = L\cdot I_0\cdot\omega = \omega L\cdot I_0$$

ωL → reaktanca: upornost pri izmeničnih signalih

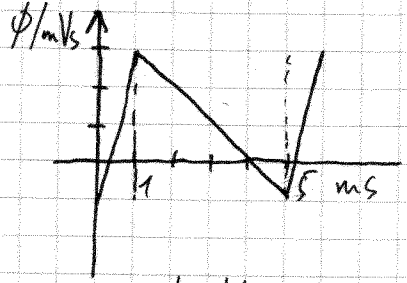


$$U_{max} = \frac{\mu_0 N^2 a\, I_0\,\omega\,\log(r_z/r_n)}{2\pi} =$$

$$= 5{,}75\cdot 10^{-4}\,\frac{Vs}{A}\cdot 0{,}1\,A\cdot 10^3\,s^{-1} = \boxed{5{,}75\cdot 10^{-2}\,V}$$

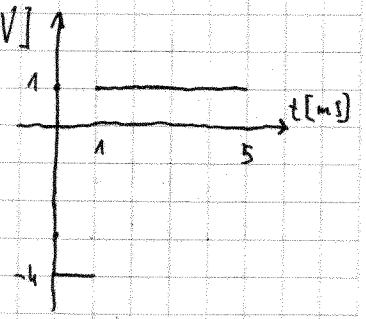
Abita

• Določite inducirano napetost v zanki, če se magnetni pretok v tej zanki spreminja po narisani krivulji



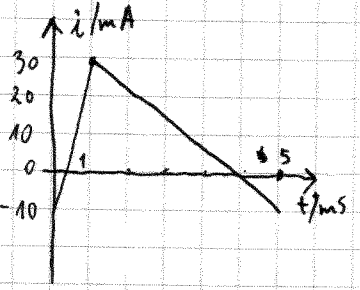
$u_i = -d\Phi/dt$

$u_i = \begin{cases} -4\,V & ; \ 0 < t < 1\ ms \\ 1\,V & ; \ 1 < t < 5\ ms \end{cases}$

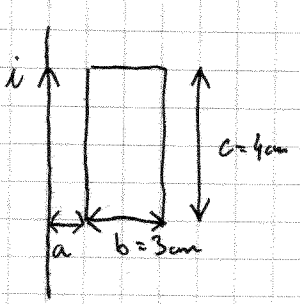


$u_i = \frac{-4\,mVs}{1 \cdot 10^{-3}\,s} = -4\,V$ $\quad u_i = \frac{-(-4\,mVs)}{4 \cdot 10^{-3}\,s} = 1\,V$

• V ravnem vodniku je tok, kot prikazuje diagram. Določite u_i v zanki, ki je od vodnika oddaljena a = 2 cm. Vodnik leži v ravnini zanke.



Za medsebojno induktivnost isto, samo ne rabimo toka.



$B = \frac{\mu_0 i}{2\pi r}$

$\Phi = \int_A \vec{B}\,d\vec{A} = \int_0^c \int_a^{a+b} \frac{\mu_0 i}{2\pi r}\,dr\,dz = \frac{\mu_0 i c}{2\pi} \log\left(\frac{a+b}{a}\right)$

M

medsebojna induktivnost

$u_i = \frac{-d\Phi}{dt} = -M\frac{di}{dt}$

$M = \frac{\mu_0 c}{2\pi} \log\left(\frac{a+b}{a}\right)$

$= \begin{cases} -7{,}33 \cdot 10^{-9} \cdot 40\ V & * \ 0 < t < 1\ ms \\ -7{,}33 \cdot 10^{-9} (-1) \cdot 10\ V & 1 < t < 5\ ms \end{cases}$

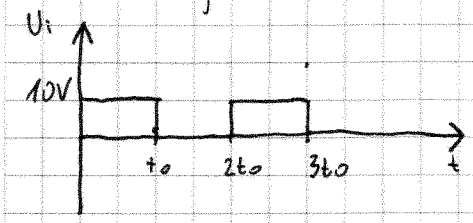
$M = \frac{4\pi \cdot 10^{-7} \frac{Vs}{Am}}{2\pi} \log\left(\frac{a+b}{a}\right) \cdot 0{,}04 =$

$= 2 \cdot 10^{-7} \frac{Vs}{A} \cdot 4 \cdot 10^{-2} \log 2 =$

$= \begin{cases} -2{,}932 \cdot 10^{-7}\ V & 0 < t < 1\ ms \\ 7{,}33 \cdot 10^{-8}\ V & 1\ ms < t < 5\ ms \end{cases}$

$= 7{,}33 \cdot 10^{-9}\ [Vs/A = H].$

• Kako se mora spremeniti magnetni pretok v zanki z $n = 10$ ovoji, da se v njej inducirajo impulzi naslednjih oblik:



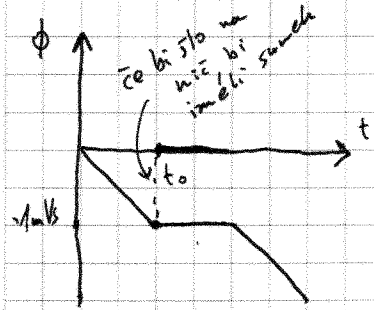
$$U_i = -N \frac{d\phi}{dt}$$

$$\frac{1}{N} U_i \, dt = -d\phi$$

$$\phi = -\frac{1}{N} \int_0^{t_0} U_i \, dt$$

$$\phi(0 < t < t_0) = -\frac{1}{N} \int_0^t 10V \, dt$$

$$= -\frac{1}{N} \cdot 10V \cdot t = \frac{-1}{10} \cdot 10V \cdot t = -1V \cdot t$$



$$\phi(t_0 < t < 2t_0) = -\frac{1}{10} \left[\int_0^{t_0} 10V \, dt + \int_{t_0}^{t} 0 \, V \, dt \right]$$

Izračunajte medsebojno induktivnost na enoto dolžine in faktor sklopa:

2.

$r = 0{,}05\,a$

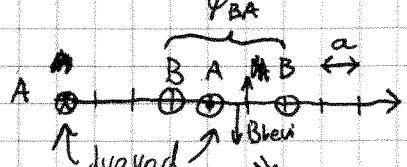
$$\phi_{BA} = \phi_{BA\,levi} + \phi_{BA\,desni}$$

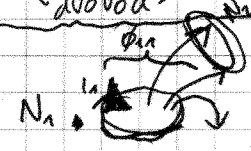
$$\phi_{BA\,l} = \int \vec{B} \, d\vec{A} = -\int_0^L \frac{\mu_0 I_A}{2\pi r} \, dr \cdot dL =$$

$$= -\frac{\mu_0 I_A \cdot L}{2\pi} \log \frac{6a}{3a}$$

$$\phi_{BA\,d} = \int_0^a \int_r^{2a} \frac{\mu_0 I_A}{2\pi r} \, dr \, dL =$$

$$= \frac{\mu_0 I_A}{2\pi} \log \frac{2a}{a}$$





$$L_1 = \frac{N_1 \phi_{11}}{I_1} \quad \text{ali} \quad \psi_{11} = N_1 \phi_{11} = L_1 I_1$$

$$M_{21} = \frac{N_2 \phi_{21}}{I_1} \qquad M_{12} = \frac{N_1 \phi_{12}}{I_2}$$

$$L_2 = \frac{N_2 \phi_{22}}{I_2}$$

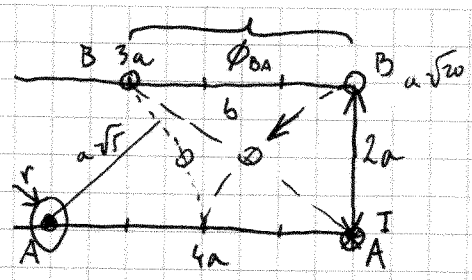
faktor sklopa

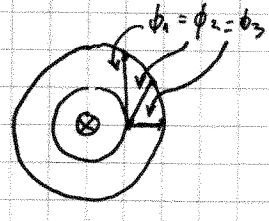
če ni feromagnetikov, potem $M_{12} = M_{21} = M = k\sqrt{L_1 L_2}$

$\phi_{BA} = 0$

$M = 0$

38





! gostotne cevke

$$b = \sqrt{(3a)^2 + (2a)^2} = \sqrt{13}\,a$$

D: $\phi_{BA} = \int B\,dA = \int_0^{2a}\int_{a\sqrt{5}}^{b\sqrt{13}a} \frac{\mu_0 I}{2\pi r}\,dr\,dl = \frac{\mu_0 I b}{2\pi}\log(\sqrt{13}/2)$

L: $\phi_{BA} = \frac{\mu_0 I}{2\pi}\int_{a\sqrt{5}}^{a\sqrt{20}} \frac{dr}{r}\,dl = \frac{\mu_0 I b}{2\pi}\log\frac{\sqrt{20}}{\sqrt{5}}$

$$\phi_{BA} - \phi_{BA\,levi} + \phi_{BA\,desni} = \frac{\mu_0 I b}{2\pi}\log\frac{\sqrt{13}\sqrt{20}}{2\sqrt{5}}$$

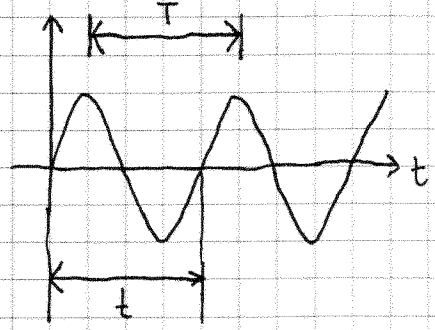
$$\boxed{M = \frac{\mu_0 b}{2\pi}\log\frac{\sqrt{260}}{2\sqrt{5}}}$$

$$L_A \approx \frac{\phi_{AA}}{I_A} = \frac{1}{I_A}\int_0^{4a-r}\int_r \frac{\mu_0 I_A}{2\pi r}\,dr\,dl = \frac{\mu_0 b}{2\pi}\log\frac{4a-r}{r}$$

7.5.2008

Izmenični signali

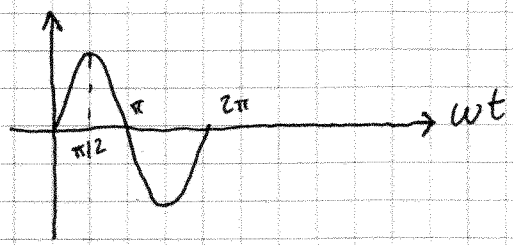
Periodični signali

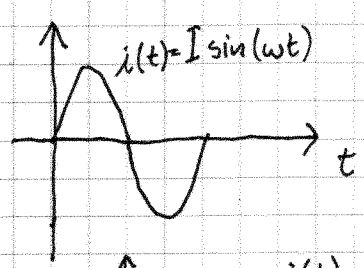


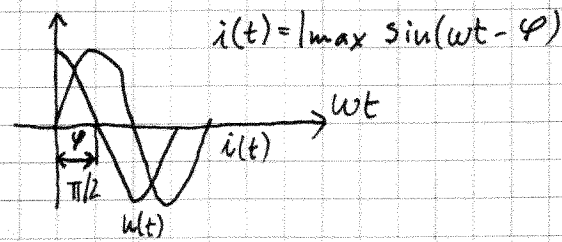
$$f = \frac{1}{t}\,[Hz]$$

$$\omega = 2\pi f\ [s^{-1}]$$

Harmonični signali





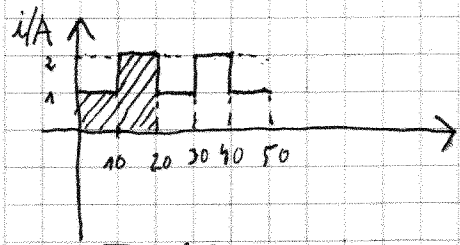


Srednja vrednost signala (povprečna)

$$I_{sredn\,ja} = \frac{1}{T}\int_0^T i(t)\,dt$$

Efektivna vrednost

$$I_{ef} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt}$$



$i(t)$, I_{sr}, I_{ef} ?

$T = 20\,ms$

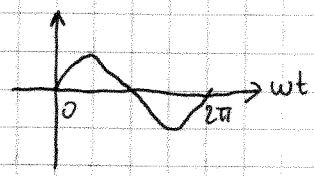
$$I_{sr} = \frac{1}{20\,ms}\int_0^{20\,ms} i(t)\,dt = \frac{1}{20\,ms}\left[\int_0^{10\,ms} 1A\,dt + \int_{10\,ms}^{20\,ms} 2A\,dt\right] = \frac{1}{20\,ms}\left[1A\cdot 10\,ms + 2A\cdot(20\,ms - 10\,ms)\right] =$$

$$= \frac{1}{20ms}\left(3A \cdot 10ms\right) = \frac{3}{2}\,A$$

$$I_{ef} = \sqrt{\frac{1}{20ms}\left[\int_0^{10ms}(1A)^2\,dt + \int_{10ms}^{20ms}(2A)^2\,dt\right]} = \sqrt{\frac{1}{20ms}\left[1A^2 \cdot 10ms + 4A^2 \cdot 10ms\right]} =$$

$$= \sqrt{\frac{5}{2}}\,A$$

- Trihoten signal



$i(t) = I_0 \sin(\omega t)$

$$I_{sr} = \frac{1}{2\pi}\int_0^{2\pi} I_0 \sin(\omega t)\,d(\omega t)$$

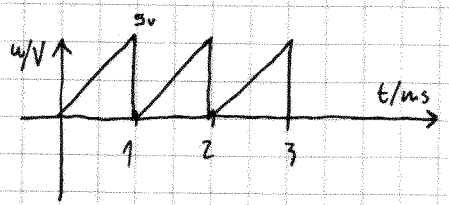
$$I_{ef}^2 = \frac{1}{2\pi}\int_0^{2\pi} I_0^2 \sin^2(\omega t)\,d(\omega t)$$

$$\boxed{\sin^2\alpha = \frac{1}{2}\left(1-\cos 2\alpha\right)}$$

$$I_{ef}^2 = \frac{I_0^2}{2\pi}\int\left(\frac{1}{2}(1-\cos(2\omega t))\right)d(\omega t) = \frac{I_0^2}{2\pi}\left[\int_0^{2\pi}\frac{1}{2}\,d(\omega t) - \int_0^{2\pi}\frac{1}{2}\cos(2\omega t)\,d(\omega t)\right] =$$

$\int_0^{2\pi}\frac{1}{2}\cos(2\omega t)\,d(\omega t) = 0$; $\sin 0 = 0$; $\sin 2\pi = 0$

$$= \frac{I_0^2}{2\pi}\,\frac{1}{2}\cdot 2\pi = \frac{I_0^2}{2\pi} = \frac{I_0}{\sqrt{2}}$$

- 

$$U_{sr} = \frac{1}{T}\int_0^T u(t)\,dt$$

$$U_{sr} = \frac{1}{1ms}\int_0^{1ms}\frac{5V}{1ms}\cdot t\;dt =$$

$T = 1ms$

$u(t) = k \cdot t$

$u(1ms) = 5V$

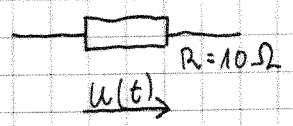
$\Big\} \; k = \frac{5V}{1ms}$

$$= \frac{1}{1ms}\,k\cdot\frac{t^2}{2}\Big|_0^{1ms} = \frac{1}{1ms}\cdot\frac{5V}{1ms}\cdot\frac{(1ms)^2}{2} =$$

$$= \boxed{2{,}5\,V}$$

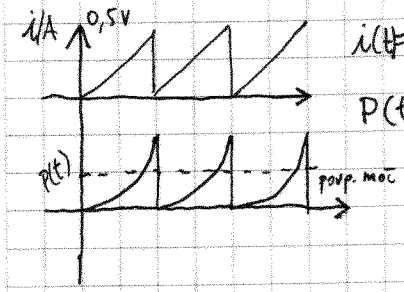
$$U_{ef}^2 = \frac{1}{1ms}\int_0^{1ms}(kt)^2\,dt = \frac{1}{1ms}\cdot k^2\cdot\frac{t^3}{3}\Big|_0^{1ms} = \frac{1}{1ms}\,\frac{25V^2}{1ms^2}\cdot\frac{(1ms)^3}{3} =$$

$$= \frac{25}{3}\,V^2 \rightarrow U_{ef} = \boxed{\frac{5}{\sqrt{3}}\,V}$$



iščemo povprečno moč za periodični signal

$u(t) = R \cdot i(t)$



$i(t) = u(t)/R$

$P(t) = i(t) \cdot u(t) = i^2(t) \cdot R = u^2/R$

$$P = \frac{1}{T}\int_0^T P(t)\,dt = \frac{1}{T}\int_0^T i^2 R\,dt = R \cdot \frac{1}{T}\int_0^T i^2\,dt =$$

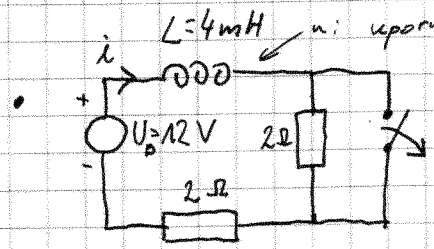
$$= R\,I_{ef}^2 = U_{ef}^2/R = \frac{25}{3 \cdot 10}\,W = \boxed{\frac{2,5}{3}\,W}$$

Prehodni pojavi

UPOR	$u(t) = R\,i(t)$	$i(t) = u(t)/R$
TULJAVA	$u(t) = L\,di/dt$	$i(t) = \frac{1}{L}\int_{-\infty}^{t} u(t)\,dt$
KONDENZATOR	$u(t) = \frac{1}{C}\int_{-\infty}^{t} i(t)\,dt$	$i(t) = dQ/dt = C\frac{du}{dt}$

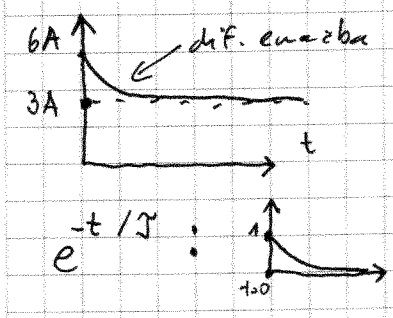
1. Kirchoffov zakon $\sum_{k=1}^{n} i_k(t) = 0$ v spojišču

2. Kirchoffov zakon $\sum_{j=1}^{m} u_j(t) = \emptyset$ v zanki



$i(t=0-) = 12V/2\Omega = 6A$

$i(t \to \infty) = 12V/4\Omega = 3A$



$$i(t) = 3A + (6A - 3A)\,e^{-t/\tau} =$$

$$= 3\,(1 + e^{-t/\tau})\,A$$

Začetni pogoji

$i_L(0^-) = i_L(0^+)$ tok skozi tuljavo ostane enak tik po vklopu

$u_c(0^-) = u_c(0^+)$

$i(t=0^+) = i(t=0^-) = 6A$ ← začetni pogoj

Zapišemo dif. enačbo za stanje po preklopu

$U_B = U_L + 2U_R = L\frac{di}{dt} + 2Ri$

Potrebno je rešiti dif. enačbo z upoštevanjem začetnega pogoja.

Rešitev iz homogenega dela diferencialne enačbe

$0 = L\frac{di}{dt} + 2Ri$

$\frac{di}{i} = -\frac{2R}{L}dt$ / $\int$ $\qquad \frac{2R}{L} = \frac{1}{\tau} \rightarrow \tau = \frac{L}{2R}$

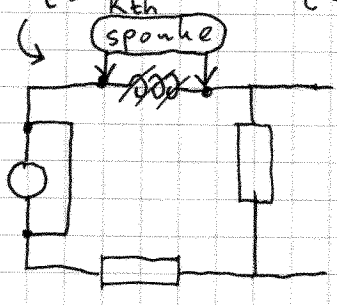
$\int_{i(0)}^{i(t)} \frac{di}{i} = -\int_0^t \frac{2R}{L}dt$

$\log i = -t/\tau + K$

$i = i(0)\, e^{-t/\tau}$

$i(t) = A \cdot e^{-t/\tau} + B$ vse rešitve so te oblike

$\tau = \frac{L}{R_{th}}$ $\qquad \tau = R_{th} \cdot C$



Zp: $i(t>0^+) = 6A$

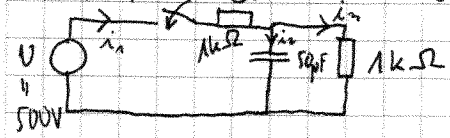
$i(t\rightarrow\infty) = 3A$

$6A = A \cdot e^0 + B = A + B \rightarrow A = 6 - B = 3$

$3A = A \cdot e^{-\infty} + B = B \rightarrow B = 3$

$i(t) = 3A \cdot e^{-t/\tau} + 3A$

Določite časovno odvisnost tokov



$U = i_1 \cdot R + \frac{1}{C}\int i_2\,dt$ $\quad u_c$

$U = i_1 R_1 + i_3 R_2$

$i_1 = i_2 + i_3$

$u_c = U_R$

$\frac{1}{C}\int i_2\,dt = i_3 \cdot R$

$\frac{1}{C}\int i_2\,dt = i_1 R - i_2 R$

$\frac{1}{C}\int i_1\,dt = U - \frac{1}{C}\int i_2\,dt - i_2 R$

$\frac{2}{C}\int i_2\,dt = u - i_2 R \quad / d/dt$

$\frac{2}{C} i_2 = 0 - \frac{di_2}{dt} R$

$\frac{di_2}{i_2} = -\frac{2}{RC}\,dt$

$\tau = RC/2$

$i_2(t) = A \cdot e^{-t/\tau} + B$

← ob vklopu toka samo tu skozi

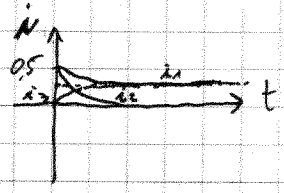
$i_2(t=0)$ $500V/1k\Omega = 0{,}5A$

$i_2(t=\infty) = 0A$

$0{,}5 = A + B \rightarrow A = +0{,}5$

$0 = 0 + B \rightarrow B = 0$

$\boxed{i_2(t) = 0{,}5\,(e^{-t/\tau})\,A}$



$i_1(t) = Ae^{-t/\tau} + B$

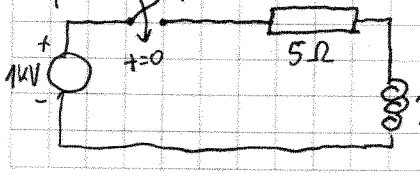
$i_1(t=0^+) = 0{,}5A$ ← samo skozi kondenzator $= A + B \rightarrow A = 0{,}25A$

$i_1(t=\infty) = 0{,}25A$ ← oba upora $= B \uparrow = 0{,}25A$

$\boxed{i_1(t) = 0{,}25\,(1 + e^{-t/\tau})\,A}$

$i_3 = i_1 - i_2 = \boxed{0{,}25A\,(1 - e^{-t/\tau})}$

Koliko % končne magnetne energije se akumulira v magnetnem polju tuljave v času dveh časovnih konstant po vklopu stikala?



$W_m = \frac{1}{2}Li^2$

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14.5.08

$$U = iR + L\frac{di}{dt}$$

ZP: $i_L(0^+) = i_L(0^-) = 0\,A$

$$(U - iR)\frac{1}{L}\,dt = di$$

$$\int_0^t \frac{1}{L}\,dt = \int_{i(0^+)}^{i(t)} \frac{di}{U - iR} \qquad \frac{1}{L}\,t = -\frac{1}{R}\log(U - iR)\Big|_{i(0^+)}^{i(t)} \qquad \begin{matrix} U - iR = z \\ -R\,di = dz \end{matrix}$$

$$-\frac{R}{L}\,t = \log\frac{U - iR}{U}$$

$$e^{-\frac{R}{L}t} = U - iR$$

$$i = \frac{U}{R}\left(1 - e^{-\frac{R}{L}t}\right) = \frac{U}{R}\left(1 - e^{-t/\tau}\right)$$

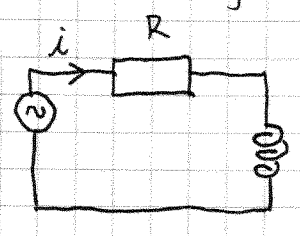
$$\tau = L/R$$

$$i(t = 2\tau) = \frac{U}{R}\left(1 - e^{-2}\right)$$

$$i(t \to \infty) = U/R$$

$$\frac{W(t = 2\tau)}{W(t \to \infty)} = \frac{(1/2)L\,i^2(2\tau)}{(1/2)L\,i^2(\infty)} = \boxed{\left(1 - e^{-2}\right)^2} = 0{,}748 \to 75\%$$

Analiza vezij z izmeničnimi signali



$u_g(t) = 10\sin(\omega t)$

$\omega = 50\,Hz$

$u_g(t) = u_R(t) + u_L(t) =$

$10\cdot\sin(\omega t) = i(t)R + L\frac{di}{dt}$

$R = 2\,\Omega \qquad L = 10\,mH$

ponavadi z nastavkom: $i = A\sin(\omega t) + B\cos(\omega t)$

$$10\sin(\omega t) = 2(A\sin(\omega t) + B\cos(\omega t)) + 0{,}01(A\omega\cdot\cos(\omega t) - B\omega\sin(\omega t))$$

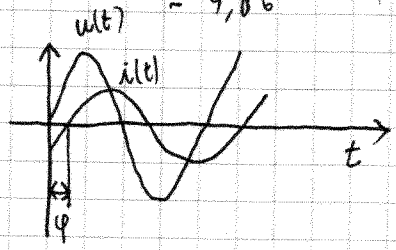
$$\begin{matrix} 10 = 2A - 0{,}5B \\ 0 = 2B + 0{,}5A \end{matrix} \Big\} \to \begin{matrix} 10 = -8B - 0{,}5B \to B = -10/8{,}5 \simeq -1{,}1765 \\ A = -2B\cdot 2 = -4B \qquad A \simeq 4{,}7 \end{matrix}$$

$i(t) = (4{,}7 \sin(\omega t) - 1{,}18 \cos(\omega t))\,A$ ← amperov

$= K \cdot \sin(\omega t + \varphi) = 4{,}86 \sin(\omega t - 14°)\,A$

$K = \sqrt{4{,}7^2 + (-1{,}18)^2} = 4{,}86$

$\varphi = \operatorname{arctg} \frac{-1{,}18}{4{,}7} = -0{,}25\ rad = -14°$



napetost "prehiteva" tok
induktivni karakter vezja

$u_R(t) = 2 \cdot i(t)$

$u_L(t) = L \cdot di/dt = L \frac{d}{dt}(4{,}86 \sin(\omega t + \varphi)) =$

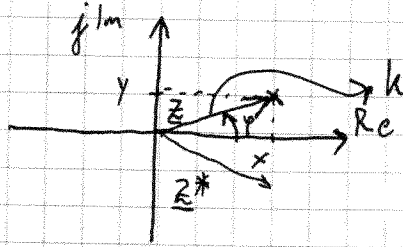
$= 10\,mH \cdot 4{,}86\,A \cdot 50\,s^{-1} \cos(\omega t + \varphi)$

Kompleksni račun

$\underline{Z} = X + jY$

$x = Re\{\underline{Z}\}$ $\quad y = Im\{\underline{Z}\}$

$j = \sqrt{-1} \rightarrow j^2 = -1$ imaginarno število



kompleksor

$|\underline{Z}| = Z = \sqrt{X^2 + Y^2}$ $\quad \varphi = atg(Y/X)$

$\underline{Z} = Z \angle \varphi$

Eulerjev obratec: $e^{j\alpha} = \cos\alpha + j \sin\alpha$ ↙ polarni zapis

$\underline{Z} = Z \cdot \cos\alpha + jZ \sin\alpha = Z(\cos\alpha + j \sin\alpha) = Z e^{j\alpha}$

Konjugacija $\quad \underline{Z} = X + jY \quad \underline{Z} = Z \cdot e^{j\alpha}$

$\underline{Z}^* = X - jY \quad \underline{Z}^* = Z \cdot e^{-j\alpha}$

$\underline{Z}_1 \underline{Z}_2 = Z_1 e^{j\alpha_1} \cdot Z_2 e^{j\alpha_2} = \underbrace{Z_1 Z_2}_{\text{amplituda}} e^{j(\alpha_1 + \alpha_2)}$

• vse operacije: $\underline{Z}_1 = 2 + j3$, $\underline{Z}_2 = 4 - j5$

$\underline{Z}_1 + \underline{Z}_2 = 6 - j2$

$\underline{Z}_1 - \underline{Z}_2 = -2 + j8$

$\underline{Z}_1 \cdot \underline{Z}_2 = (2 + j3)(4 - j5) = 8 - 10j + 12j + 15 = 23 + j2$

$\underline{Z}_1 / \underline{Z}_2 = \frac{(2 + j3)}{(4 - j5)} \cdot \frac{(4 + j5)}{(4 + j5)} = \frac{-7 + j22}{16 + 25} = \frac{-7 + j22}{41}$

→ $i(t) = 2\cos(\omega t)\ A \rightarrow \underline{i}(t) = 2A \cdot (\cos(\omega t) + j\sin(\omega t)) = 2e^{j\omega t}\ A$

$i(t) = Re\{\underline{i}(t)\}$

$i(t) = I_{max}\cos(\omega t + \varphi) \rightarrow \underline{i}(t) = I_m e^{j(\omega t + \varphi)} = \underline{I_m e^{j\varphi}} \cdot e^{j\omega t} = \underline{I}\, e^{j\omega t}$

$\underline{I}$ ↑ kompleksor toka

• $u = 10\sin(\omega t)\ V = Ri + L\frac{di}{dt} = 10 \cdot \cos(\omega t - \pi/2)$

$\underline{i}(t) = I_{max} e^{j(\omega t + \varphi)}$

$Re\{10 \cdot e^{j(\omega t - \pi/2)}\} = R \cdot Re\{I_m e^{j\omega t} \cdot e^{j\varphi}\} \cdot L\, Re\{I_m \cdot j\omega \cdot e^{j(\omega t + \varphi)}\}$ / $: e^{j(\omega t)}$

$\underbrace{10 \cdot e^{-j\pi/2}}_{U_g} = R \cdot \underbrace{I_{max} e^{j\varphi}}_{\text{kompleksor toka}} + L\, I_m\, j\omega e^{j\varphi}$ ni več dif. enačbe

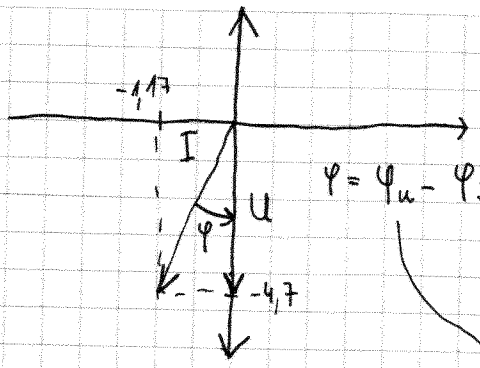
U_g ↑ ↘ kompleksor napetostnega signala

$\underline{U_g} = R \cdot \underline{I} + j\omega L \underline{I} \rightarrow \underline{U} = 10e^{-j\pi/2}\ V = -j10\ V$ ($\cos(-\pi/2) + j\sin(-\pi/2)) = -j$

$\underline{I} = \frac{\underline{U}}{R + \underbrace{j\omega L}_{\text{kompleksna upornost}}} = \frac{-j10\ V}{2\Omega + j0{,}5\Omega} =$

$= \frac{-j10\ (2 - j0{,}5)}{(2 + j0{,}5)(2 - j0{,}5)} = \frac{-j10(2 - j0{,}5)}{4{,}25} = -j4{,}7 - 1{,}17 = \boxed{-1{,}17 - j4{,}7}$

kompleksor toka



$= (-1,17 - j\,4,7)A = \underbrace{\sqrt{1,17^2 + 4,7^2}}_{\text{amplituda}}\, e^{j\,\mathrm{Atg}\left(\frac{-4,7}{-1,17} + \pi\right)}$

!!! Prištejes π, ker je v tretjem kvadrantu.

$\varphi = \varphi_u - \varphi_I$ ... 360-103 ... 270 - 257 = 13°

4,86A

$i(t) = \mathrm{Re}\{I_e\, e^{j\omega t}\} = \mathrm{Re}\{4,86 \cdot e^{j(\omega t + 257°)}\ A\} =$

$= 4,86 \cos(\omega t + 257°)A = 4,86 \sin(\omega t - 13°)A$

$\underline{U}_R = R\,\underline{I}_R$

$\underline{U}_L = j\omega L\,\underline{I}_L$

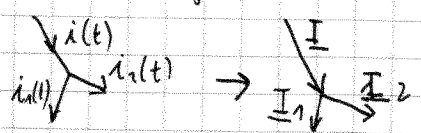
$\underline{U}_C = \dfrac{\underline{I}_C}{j\omega C}$

2. Kirchof. zakona

$\sum_{i=1}^{n} \underline{I}_i = 0$ v spojišču

$\sum_{i=1}^{n} U_i = 0$ v zanki

Tok $i(t) = 3\cos(\omega t + 30°)A$ se razdeli v dve veji. Kolikšen je $i_2(t)$, če je $i_1(t) = 2\cos(\omega t - 45°)A$?



$\underline{I} = \underline{I}_1 + \underline{I}_2$

$\underline{I} = ?$

$i(t) = 3e^{j(\omega t + 30°)} \Rightarrow \underline{I} = 3e^{j30°}\ A$

$i_1(t) \rightarrow \underline{I}_1 = 2e^{-j45°}$

$\underline{I}_2 = \underline{I} - \underline{I}_1 = 3e^{j30°}\ A - 2e^{-j45°}\ A = 3(\cos 30° + j\sin 30°) - 2(\cos 45° + j\sin(-45°))$

↑ eulerjeva

$= 3\cdot(\sqrt{3}/2) + 3j(1/2) + -2(\sqrt{2}/2) + 2j(\sqrt{2}/2) =$

$= \dfrac{3\sqrt{3} - 2\sqrt{2}}{2} + j\,\dfrac{3 + 2\sqrt{2}}{2} = \boxed{(1.18 + j\,2.9)\ A}$

kompleksor toka 2

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Akta

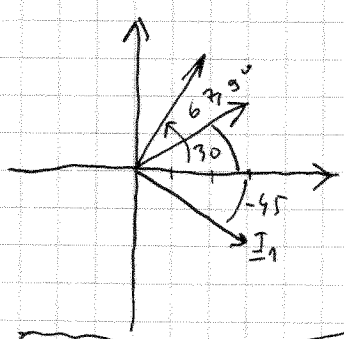
$I = \sqrt{1.18^2 + 2.9^2}$ $\varphi = \text{Arctg}\,(2,9/1.18) = 67,9°$

$= 3.14\ A$

$\underline{I}_2 = 3,14 \cdot e^{j67,9°}\ A$

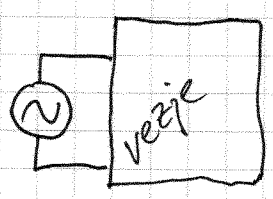
$i_n(t) = \text{Re}\{3,14 \cdot e^{j67,9°} \cdot e^{j\omega t}\}$

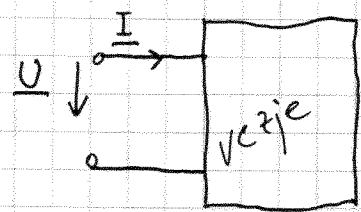
$= 3,14\,(\cos(\omega t + 67,9°))$ ← časovni signal



sinusni signal se pretvori v cosinusnega

Impedanca (admitanca)





$\underline{Z} = \dfrac{\underline{U}}{\underline{I}}$

$\underline{U} = \underline{Z} \cdot \underline{I}$ Ω

$\underline{Y} = 1/\underline{Z}$ S

	$\underline{Z}$	$\underline{Y}$
upor	R	$1/R = G$
kond	$1/j\omega C$	$j\omega C$
tulj	$j\omega L$	$1/j\omega L$

$\underline{Z} = R + j(\otimes)$ (⊗: reaktanca)

$X_c = -1/\omega C$

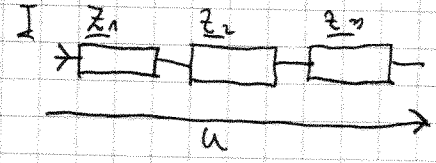
$X_L = \omega L$

$\underline{Y} = G + jB$ (B: susceptanca)

$B_c = \omega C$

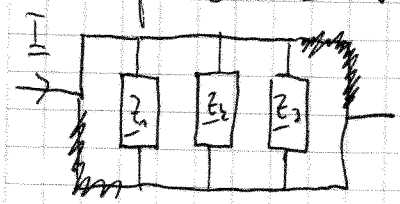
$B_L = -1/\omega L$

Zaporedna vezava impedanc



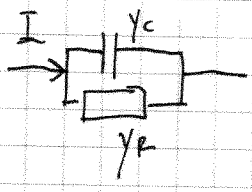
$$\underline{U} = \underline{I}(\underline{Z}_1 + \underline{Z}_2 + \underline{Z}_3 + \dots) = \underline{I} \cdot \underline{Z}$$

Vzporedna vezava admitanc



$$I = \underline{U}(\underline{Y}_1 + \underline{Y}_2 + \underline{Y}_3 + \dots) = \underline{U} \cdot \underline{Y}$$

- Določite admitance vzporedne vezave $C = 2\,\mu F$ in $R = 100\,\Omega$, $\omega = 1000\,s^{-1}$



$$\underline{Y}_R = 1/R = 0{,}01\ S$$

$$\underline{Y}_C = j\omega C = j\,0{,}002\ S$$

$$\underline{Y} = \underline{Y}_R + \underline{Y}_C = 0{,}01\ S + j\,0{,}002\ S$$

- Tok v vezje $i(t) = 20\cos(\omega t)^{+0°}\ mA$, koliko je $u(t)$. Na prejšnjem vezju.

$$\underline{U} = \underline{Z} \cdot \underline{I}$$

$$\underline{I} = 20\,mA\, e^{j\cdot 0} = \underline{20\,mA} \quad \text{kompleksor toka}$$

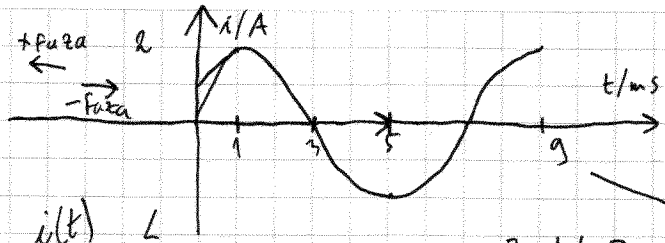
$$\underline{Z} = 1/\underline{Y} = \frac{1}{0{,}01\ S + j\,0{,}002\ S} = 10^3 \frac{1 \quad (10 + j2)}{(10 + j2)(10 - j2)} =$$

$$= 10^3 \cdot \frac{10 + j2}{100 + 4} = \frac{10^3(10 + j2)}{104}\ \Omega$$

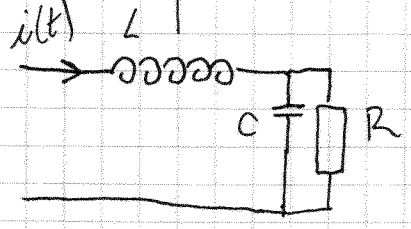
$$\underline{U} = \underline{I} \cdot \underline{Z} = 20\,mA \cdot \frac{10^3(10 + j2)}{104 \sim 100}\ \Omega \simeq (2 + 0{,}4j)\ V$$

Časovni signal $u(t)$

$$u(t) = \sqrt{4 + 0{,}16} \cdot \cos\left(1000t + \arctg\frac{-0{,}4}{2}\right) \simeq 2 \cdot \cos(1000t - 11{,}3°)$$



Določite časovni potek toka s katerim kondenzator, če je tok v vezje podan na sliki.



$R = 4\,k\Omega$
$C = 1\,\mu F$
$L = 2\,mH$

$\rightarrow T = 8\,ms \rightarrow \omega = 2\pi/T = \pi/4\ ms^{-1}$

iz grafa: $i(t) = I_{max} \cdot \sin(\omega t + \varphi)$
$= 2A \cdot \sin\left(\frac{\pi}{4} ms^{-1} \cdot t + \varphi\right)$

$t = 1ms$, $I = 2A$ $\rightarrow$ $2A = 2A \cdot \sin\left(\frac{\pi}{4\,ms} 1ms + \varphi\right)$

$\arcsin 1 = \pi/4 + \varphi$
$\pi/2 - \pi/4 = \varphi \rightarrow \varphi = \pi/4$

$i(t) = 2A \cdot \sin(\pi/4\ ms^{-1} + \pi/4)$

$\underline{I} = 2 e^{j\pi/4 - j\pi/2} = 2e^{-j\pi/4}\ A$ ← ker je sinusni

$\underline{I}_1 = \underline{I} \frac{\underline{Z}_2 \underline{Z}_1}{\underline{Z}_1 + \underline{Z}_2} \quad / : \underline{Z}_1$

$Z_2 = R$

$Z_1 = 1/j\omega C = (4 \cdot 10^3 / j\omega)\ \Omega$

$I_1 = 2 \cdot e^{-\pi/4}\ A \cdot \frac{4000\ \Omega}{\frac{4 \cdot 10^3}{j\omega} + 4 \cdot 10^3\ \Omega} = 2 \cdot e^{-j\pi/4} \frac{\pi}{-j + \pi} =$

v polarno: $\frac{\pi}{\sqrt{\pi^2 + 1} e^{j \arctg(-1/\pi)}} = \frac{\pi}{3,3} \cdot e^{-j17,6°}$

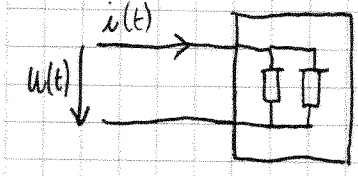
$= 2e^{-j\pi/4} \cdot \frac{\pi}{3,3} \cdot e^{j17,6} = \frac{2\pi}{3,3} \cdot e^{-j27,4°}\ A$

$i_1 = 1,9 (\cos(\omega t - 27,4°))\ A$

21.5.08

$i(t) = 50\cos(\omega t)\ A$
$u(t) = 100\cos(\omega t - 30°)\ V$
$\omega = 5\cdot 10^4\ s^{-1}$

Realizirajte vezje z vzporedno vezavo dveh elementov

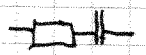


računamo admitance

$$\underline{Y} = \frac{1}{\underline{U}} = \frac{50A}{100 e^{j\cdot 30°} V} = \frac{1}{2} e^{j30°} S =$$

$$= \left(\frac{1}{2}\cos 30° + j\frac{1}{2}\sin 30°\right) S = \frac{\sqrt{3}}{4} S + j\frac{1}{4} S$$

$G = \frac{\sqrt{3}}{4} S \qquad \omega C = \frac{1}{4} S \qquad C = \frac{1}{5\cdot 4\cdot 10^4 s^{-1}} S = 5\,pF$

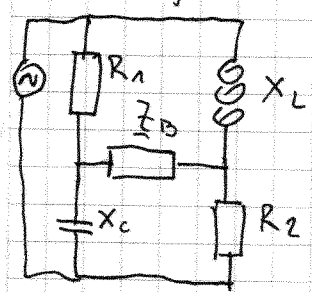
Zaporedna vezava: 

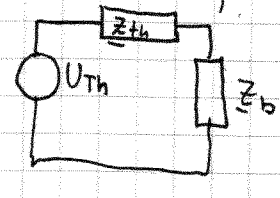
$\underline{Z} = 2\cos(-30°) + j2\sin(-30°)\,\Omega$

$\to (\sqrt{3} + j(-1))\,\Omega \to R = \sqrt{3}$

$$\frac{1}{\omega C} = 1 \to C = \frac{1}{5\cdot 10^4} F$$

Določite ~~upor~~ z bremena tako, da se bo na njem trošila največja moč.





$\underline{Z}_{Th} = R_{Th} + jX_{Th}$

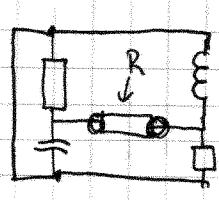
$\underline{Z}_B = R_b + jX_b$

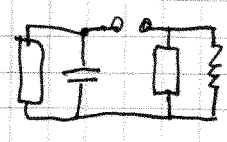
$$P_b = \frac{1}{2} I^2 \cdot Re\{\underline{Z}_b\} = \frac{1}{2}\left|\frac{U_{Th}}{(R_{Th} + j\cancel{X_{Th}}) + (R_b + j\cancel{X_b})}\right|^2 \cdot R_b$$

$X_{Th} = -X_b$ } Največja moč

$R_{Th} = R_b$ } Sledi $\underline{Z}_{Th} = \underline{Z}_{Th}^*$

$\omega = 5\,kHz$
$C = 10\,nF$
$L = 2\,mH$
$R_1 = R_2 = 10\,\Omega$





$$\underline{Z}_{Th} = R_1 \parallel \underline{Z}_C + R_2 \parallel \underline{Z}_L$$

$$\underline{Z}_C = -1/j\omega C = -j\frac{1}{5\cdot 10^3\,s^{-1}\cdot 10^{-5}\,F} = -j\,20\,\Omega$$

$$\underline{Z}_L = j\omega L = j\,5\cdot 10^3\,s^{-1}\cdot 2\cdot 10^{-3}\,H = j10\,\Omega$$

$$\underline{Z}_{th} = \frac{10\cdot(-j20)}{10-j20}\,\Omega + \frac{10\cdot j10}{10+j10}\,\Omega = \frac{-j20(1+j2)}{(1-j2)(1+j2)}\,\Omega +$$

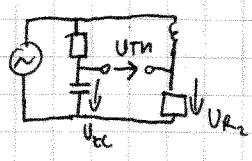
$$+\frac{j10(1-j)}{(1+j)(1-j)}\,\Omega = \frac{-j20(1+j2)}{5} + j5(1-j)\,\Omega =$$

$$= -j4 + 8 + j5 + 5\,\Omega = (13 + j)\,\Omega$$

$\underline{Z}_b = (13 - j)\,\Omega$ $\qquad$ R_b

$$\boxed{P_{bmax} = \frac{U_{Th}^2}{8\,R_b}}$$

$$\underline{U}_{Th} = \underline{U}\,\frac{\underline{Z}_C}{R_1 + \underline{Z}_C} - \underline{U}\,\frac{R^2}{\underline{Z}_L + R_2} =$$



$$= \underline{U}\left[\frac{-j40}{10-j40} - \frac{10}{j10+10}\right] =$$

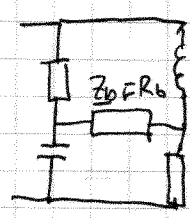
napaka 40 → 20!

$$= \underline{U}\left[\frac{-4j(1+j4)}{9} - \frac{1\,(1-j)}{2}\right] = \underline{U}\,\frac{1}{18}\left(-j8(1+j4) - 9 + j9\right) =$$

$$= \frac{\underline{U}}{18}(23 + j) \qquad |U_{Th}| \approx \left(\frac{100\cdot 23}{18}\right) V = 127{,}8\,V$$

$$P_{bmax} = \frac{127{,}8^2\,V^2}{8\cdot 13\,\Omega} = \underline{157\,W}$$

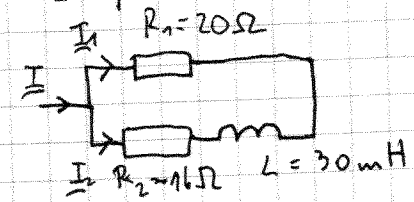
Poseben primer



$R_b = |\underline{Z}_{Th}|$

$$P_{bmax} = \frac{U_{Th}^2}{4\left(Re(\underline{Z}_{th}) + |\underline{Z}_{Th}|\right)}$$

Določite delovno moč P_{R1} in P_{R2}, kjer Q_{L2}, če vzbujamo $i(t)/A = 20 \sin(400\,s^{-1}\,t)$



$\underline{S} = \frac{1}{2}\underline{U}\,\underline{I}^*$ $\quad S = \frac{1}{2}UI$ navidezna

$P = S\cos\varphi$ $\quad Q = S\sin\varphi$

$P = \mathrm{Re}\{\underline{S}\}$ delovna $\quad Q = \mathrm{Im}\{\underline{S}\}$ jalova

$S = \frac{1}{2}\underline{I}\,\underline{Z}\,\underline{I}^* = \frac{1}{2}I^2\underline{Z} = \frac{1}{2}U^2\underline{Y}^*$

$\cos\varphi$ .... faktor moči

$\underline{Z}_{R2} = 16\,\Omega \quad \underline{Z}_L = j\,12\,\Omega \quad Z_{R2L} = (16 + j\,12)\,\Omega$

$\underline{I} = 20\,A\,e^{-j\pi/2} = -j\,20\,A$

Tokovni delilnik $\underline{I}_1 = \underline{I}\,\dfrac{\underline{Z}_{R_2} + \underline{Z}_L}{\underline{Z}_{R_1} + \underline{Z}_{R_2} + \underline{Z}_L}$

$\underline{I}_2 = \underline{I}\,\dfrac{\underline{Z}_{R_1}}{\underline{Z}_{R_1} + \underline{Z}_{R_2} + \underline{Z}_L} = -j\,20\,A\,\dfrac{20\,\Omega}{(36 + j\,12)\,\Omega} =$

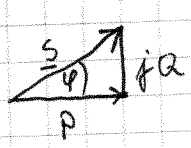
$= \dfrac{-j\,400}{12(3+j)} = \dfrac{1}{3}(-10 - j\,30) = -\dfrac{10}{3} - j\,10\,A$

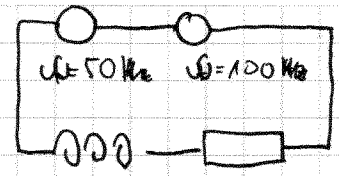
$\underline{I}_1 = \underline{I} - \underline{I}_2 = (3{,}33 - j\,10)\,A$

$S_{R1} = \frac{1}{2} \cdot 20\,\Omega \left| 3{,}33 - j\,10 \right|^2 = 1110\,W$

$S_{R_2L} = \frac{1}{2}(16 + j\,10)\,|-3{,}33 - j\,10|^2 = (888 + j\,666)$

$S = P + jQ_P$

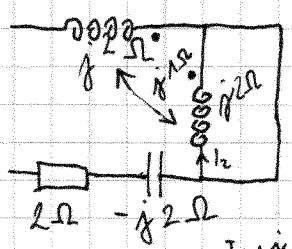




$U_1(t) = 5\cos(50\,s^{-1}t)\,V$

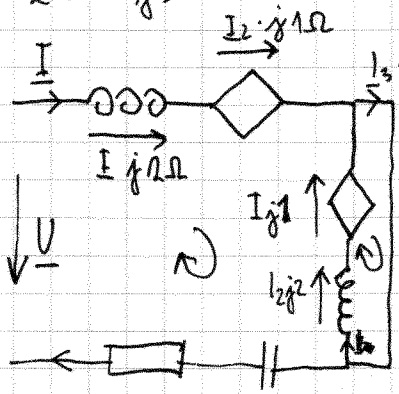
$U_2(t) = 10\cos(100\,s^{-1}t)\,V$

superpozicija časovnih signalov



brez sklopljenja je

$Z_{vh} = j2\Omega - j2\Omega + 2\Omega = 2\Omega$



Z sklopljenimi elementi

$Z_{vh} = \underline{U} / \underline{I} = \frac{1V}{\underline{I}}$

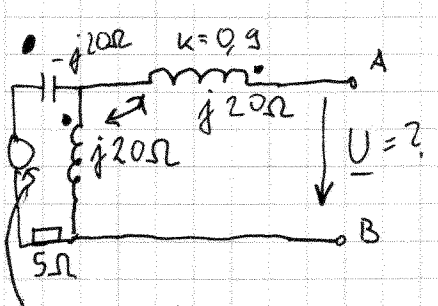
$\underline{U} = \underline{I}\, j2 + j\underline{I}_2 - j\underline{I} - j2\underline{I}_2 + \underline{I}(-j2) + \underline{I}_2 2$

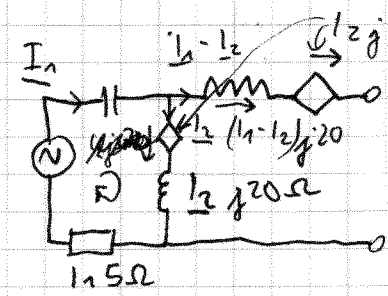
$0 = \underline{I}_2 j2 + \underline{I}\, j$

$\underline{I}_2 = -\frac{1}{2}\underline{I}$

$U = \underline{I}\left(j2 - \frac{1}{2}j - j + j - j2 + 2\right) = \underline{I}\left(2 - \frac{1}{2}j\right)$

$Z_{vh} = \frac{\underline{U}}{\underline{I}} = \left(2 - \frac{1}{2}j\right)\Omega$





$M = k\sqrt{L_1 L_2}$

$j\omega M = j\,k\sqrt{\omega L_1 \omega L_2} =$

$= j\,k\,20\Omega = j\cdot 18\Omega$

$10\angle 0°\,V = 10\cos(\omega t)\,V$

$\left.\begin{aligned} \underline{U} &= -I_1 j20 - (I_1 - I_2) j18 + I_1 5 + I_2 j20 \\ 0 &= U_{AB} + I_2 j18 - (I_1 - I_2) j20 - (I_1 - I_2) j18 + I_2 j20 \end{aligned}\right\} I_1 = I_2$

$10\,V = 5\,I_1 \quad I_1 = 2\,A$

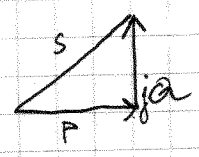
$U_{AB} = 2\,j18 + 2\cdot j20 = j76\,V$

28.5.08

Omsko induktivno breme navidezne moči 100 kVA in $\cos\varphi = 0{,}8$ je priključeno na napetost 500 V / 50 Hz. Da bi izboljšali $\cos\varphi$, priključimo vzporedno bremenu kondenzator jalove moči 50 kVAr. Za koliko se zmanjša tok v dovodnih žicah.

$S = 100$ kVA

kondenzacija delovne moči

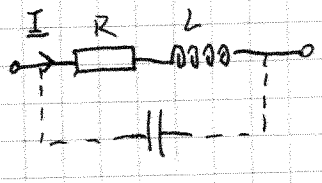


$U_{ef} = 500$ V

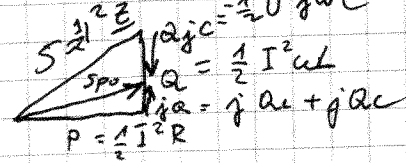
$Q_c = 50$ kVAr

$$I_{pred} = \frac{S_{pred}}{U_{ef}} = \frac{100\text{ kVA}}{500\text{ V}} = 200\text{ A}$$

$S = \frac{1}{2} U I = U_{ef} I_{ef}$



$S = \frac{1}{2} I^2 \underline{Z}$



$$\underline{S}_c = \frac{1}{2} U^2 \underline{Y}_c^* = \frac{1}{2} U^2 (j\omega C)^* = -\frac{1}{2} U^2 j\omega C$$

$$I_{po,ef} = \frac{S_{po}}{U_{ef}}$$

$P = S\cos\varphi = 80$ kVA

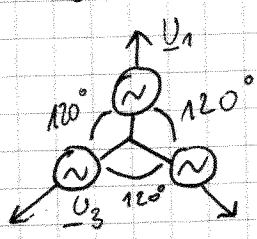
$Q_L = \sqrt{S^2 - P^2} = 60$ kVAr

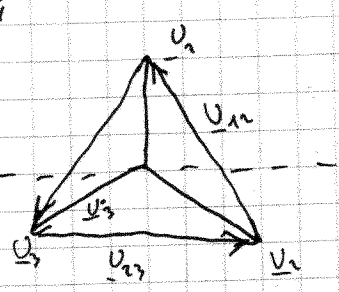
$Q = 60\text{ kVAr} - 50\text{ kVAr} = 10$ kVAr

$$S_{po} = \sqrt{P^2 + Q^2} = \sqrt{80^2 + 10^2}\text{ kVA} = 80{,}6\text{ kVA}$$

$$\frac{S_{po}}{500\text{V}} = \frac{80{,}6\text{ kVA}}{500\text{ V}} \simeq 161\text{ A}$$

Trifazni sistemi





$\underline{U}_1 = j\,U_f$

$\underline{U}_2 = \underline{U}_f\, e^{-j120°}$

$\underline{U}_3 = \underline{U}_F\, e^{-j240°}$

$\underline{U}_2 = U_f\left(\frac{\sqrt{3}}{2} - j\frac{1}{2}\right)$

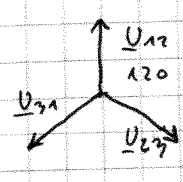
$\underline{U}_3 = \underline{U}_f\left(-\frac{\sqrt{3}}{2} - j\frac{1}{2}\right)$

$\underline{U}_{12} = \underline{U}_1 - \underline{U}_2 = \underline{U}_{mf} \cdot e^{j30°}$

$\underline{U}_{31} = \underline{U}_{mf}\, e^{-j120°}$

$\underline{U}_{mf} = \sqrt{3}\, U_f$

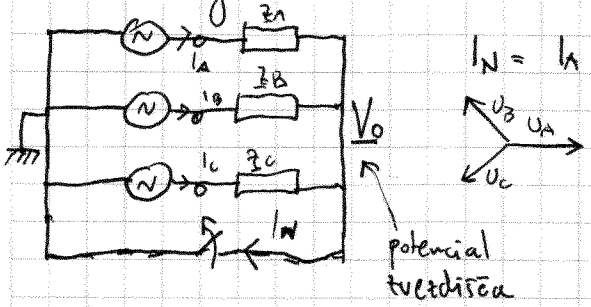
$\underline{S} = \underline{U} \cdot \underline{I}^*$ za efektivne vrednosti



Akta

Vezava zvezda s ali brez ničelnega vodnika

Na simetrični trofazni generator z $\underline{U}_A = 230\,V$ je priključeno nesimetrično breme z $\underline{Z}_A = j100\,\Omega$, $\underline{Z}_B = j100\,\Omega$ $Z_C = 100\,\Omega$. Določite tok v nevtralnem vodniku in potencial zvezdišča ob prekinitvi ničelnega vodnika



$$I_N = I_A + I_B + I_C = \frac{\underline{U}_A}{\underline{Z}_A} + \frac{\underline{U}_B}{\underline{Z}_B} + \frac{\underline{U}_C}{\underline{Z}_C} =$$

$$= \frac{230\,V}{j100\,\Omega} + \frac{230\,V\,e^{-j120°}}{j100\,\Omega} + \frac{230\,V\,e^{j120°}}{100\,\Omega} =$$

$$= (3{,}15 + j2)\,A$$

brez nevtralnega vodnika

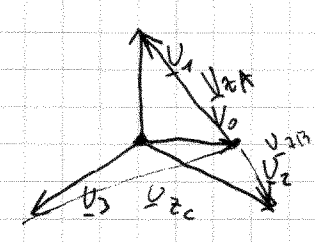
$$I_A + I_B + I_C = 0$$

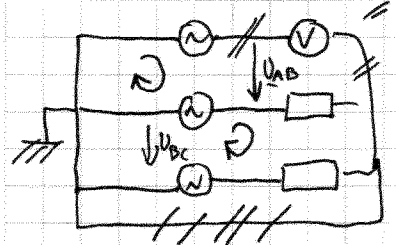
$$(\underline{V}_0 - \underline{V}_A)\underline{Y}_A + (\underline{V}_0 - \underline{V}_B)\underline{Y}_B + (\underline{V}_0 - \underline{V}_C)\underline{Y}_C = 0$$

$$V_0 = \frac{V_A Y_A + V_B Y_B + V_C Y_C}{Y_A + Y_B + Y_C} = \ldots = (189{,}23 - j83{,}84)\,V$$

moč $S_A = ?$; $S_A = |\underline{V}_0 - \underline{V}_A|^2 \cdot \underline{Y}_A^*$

$$\underline{S} = \underline{U}\,\underline{I}^* = \underline{U}^2 \cdot \underline{Y}^* = \underline{I}^2 \cdot \underline{Z}$$





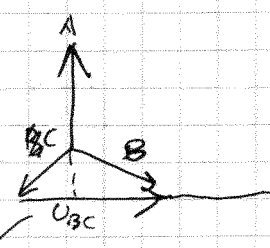
$U_{VOLTM.}$ ⇒ $I_A = 0\,A$

U_{mf} / U_f

$3 \times 21\,V$ / $12\,V$

$Z_B = Z_C = 100\,\Omega$

Koliko kaže voltmeter ?



$$U_V - I_B Z_B + \underline{U}_B - \underline{U}_A = 0 \rightarrow U_{VM} = \underline{I}_B \underline{Z}_B + \underline{U}_{AB}$$

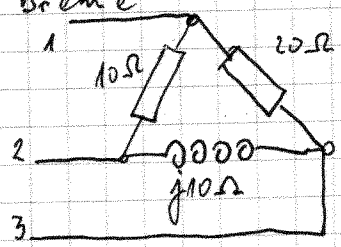
$$U_{BC} = \underline{I}_B (\underline{Z}_B + \underline{Z}_C)$$

$$\underline{U}_V = \frac{U_{BC}}{\underline{Z}_B + \underline{Z}_C} + U_{AB} \qquad \underline{U}_V = \frac{21\,V}{200\,\Omega} + 21\,V\,e^{j120}$$

$= 0,105 + 21\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = 0,105 - 10,5 + j\,18,19 = (-10,4 + j\,18,19)\,V$

$|\underline{U}_V| = \sqrt{10,4^2 + 18,19^2}\,V = 21,6\,V$

Vezje v trikot priključeno na simetrični trofazni sistem $3\times 400/231\,V$. Kolikšno navidezno moč prejema breme



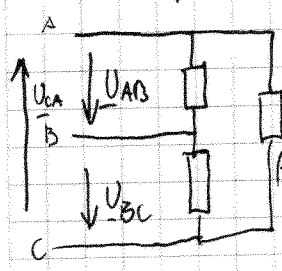
$\underline{S} = \underline{S}_{12} + \underline{S}_{23} + \underline{S}_{31}$

$S = U^2 \underline{Y}^* = \frac{(400\,V)^2}{10\Omega} + \frac{(400\,V)^2}{20\Omega} + \frac{(400\,V)^2}{-j\,10\Omega} =$

$= \frac{400^2}{10}\left(1 + \frac{1}{2} + j\right) = 16\cdot 10^3\left(\frac{3}{2} + j\right)\,VA$

(delovna, jalova)

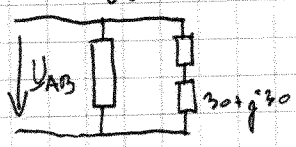
Simetrično breme je priključeno na $3\times 380/220\,V$. Pri obratovanju vodnik fazne C včasih odklopimo



Δ kolikšno je razmerje delovne moči v enem in drugem režimu obratovanja.

C priključen: $\underline{S}_C = U_{CA}^2\,\underline{Y}_{CA}^* = (380V)^2 \cdot \frac{1}{(30 - j30)\Omega} = \frac{380^2}{30}\,\frac{(1+j)}{2}\,VA$

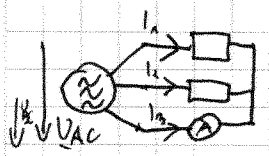
$P_C = \frac{380^2}{60}\,W = 2406,7\,W$



$\downarrow U_{AB}/2$ $\qquad \underline{S}' = \left|\frac{U_{AB}}{2}\right|^2 Y_{AC}^* = \frac{1}{4}\,\underline{S}_C$

$P_C' = \frac{1}{4}\,P_C$

Določite efektivni tok ampermetra. $3\times400\,V$, $R = 50\,\Omega$



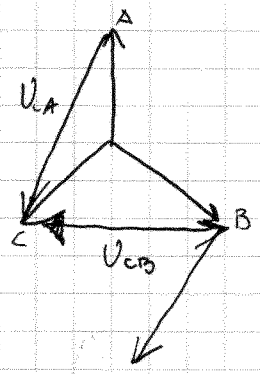
$\underline{I}_A = ?$

$\underline{I}_C = -(\underline{I}_A + \underline{I}_B)$

$\underline{I}_C = -\frac{\underline{U}_{AC}}{R} - \frac{\underline{U}_{BC}}{R}$

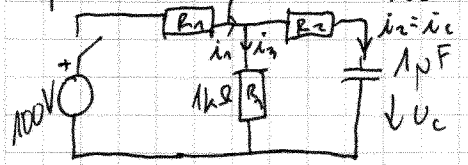
$= \frac{\underline{U}_{CA}}{R} + \frac{\underline{U}_{CB}}{R} = \frac{1}{50}\,\Omega^{-1}\left(400\angle-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) - j400)V =$

$= \frac{400}{50}\left(-\frac{1}{2} - \left(1+\frac{\sqrt{3}}{2}\right)\right)A$



I_C = ... absolutno $\underline{I}_C$

Določite časovni potek napetosti na kondenzatorju po vklopu stikala.



$R_1 = R_2 = R_3 = 1\,k\Omega$

$i = C\frac{du}{dt}$ $\quad y = Ae^{-t/\tau} + B$

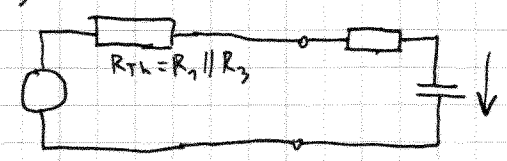
$i_1 - i_2 - i_3 = 0$

$i_2 R_2 + u_C - i_3 R_3 = 0$) +

$i_1 R_1 + i_3 R_3 - 100V = 0$

$i_2 = i_C = C\frac{du_C}{dt}$

$i_1 R_1 + i_2 R_2 + u_C - 100\,V = 0$



$U_{Th} = 100V\,\frac{R_3}{R_1+R_3} = 50V$

$R_{TH} = 0{,}5\,k\Omega$

$U_{Th} = i_C(R_{TH} + R_2) + U_C$

$U_{Th} = C\frac{du_C}{dt}\underbrace{(R_1 \| R_3 + R_2)}_{1{,}5\,k\Omega} + U_C$

5g

$$50\,V = 1{,}5\,k\Omega \cdot 1\,nF \frac{du_c}{dt} + u_c$$

RC člen

$$\frac{du_c}{dt} = -\frac{1}{\tau} A \cdot e^{-t/\tau}$$

$$50\,V = 1{,}5\,ms \left(-\frac{1}{\tau} A e^{-t/\tau}\right) + B + A e^{-t/\tau} + B$$

$$50\,V = B$$

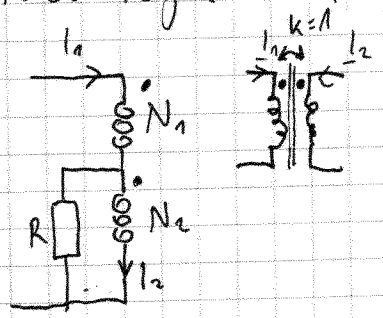
$$u_c(t) = A e^{-t/\tau} + B$$

$$0\,V = A + B$$

$$A = -B$$

$$u_c(t) = 50\left(1 - e^{-t/\tau}\right) V$$

Izrazite impedanco dvopola, ki vključuje tuljavi idealnega transformatorja



$\frac{U_1}{U_2} = \frac{N_1}{N_2} = n$ napetostna prestava

$\frac{I_1}{I_2} = -\frac{1}{n}$ tokovna prestava

$Z_{VHODNA} = n^2 Z_{BREMENA}$

$\underline{S}_1 \simeq \underline{S}_2$

$$\underline{Z}_{Vh} = \frac{U_1 + U_2}{I_1}$$

$$\underline{I}_1 = \frac{U_2}{R} + I_2$$

$$\underline{I}_2 = -n\underline{I}_1$$

$$\underline{I}_1 = \frac{U_2}{R} - n\underline{I}_1$$

$$U_2 = R(\underline{I}_1 + n\underline{I}_1)$$

$$\underline{U}_1 = n\underline{U}_2$$

$$Z_{vh} = \frac{n U_2 + U_2}{I_1} = \frac{(n+1) R I_1 (n+1)}{I_1} = R(n+1)^2$$

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Aleta