Fourierova transformacija

$$X(f)=\int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt$$

$$x(t)=\int_{-\infty}^{\infty} X(f)e^{j2\pi ft}df$$

Zadrževalnik ničtega reda

$$G_0(s)=\frac{1-e^{-sT}}{s}$$

Z-transformacija

$$\mathcal{Z}\left\{x(k)\right\}=X(z)=\sum_{k=0}^{\infty}x(k)z^{-k}$$

$$\mathcal{Z}^{-1}\left\{X(z)\right\}=x(k)=\frac{1}{2\pi j}\oint_C X(z)z^{k-1}dz=\sum \text{Res}\left[X(z)z^{k-1}\right]; \quad \text{Res}_{z=a} f(z)=\frac{1}{(q-1)!}\lim_{z\to a}\frac{d^{q-1}}{dz^{q-1}}\left[(z-a)^q f(z)\right]$$

	$x(k)$	$X(z)$
Teorem linearnosti	$ax_1(k)+bx_2(k)$	$aX_1(z)+bX_2(z)$
Teorem časovnega premika v desno	$x(k-m)$	$z^{-m}X(z)$
Teorem časovnega premika v levo	$x(k+m)$	$z^m\left[X(z)-\sum_{k=0}^{m-1}x(k)z^{-k}\right]$
Teorem eksponencialnega dušenja	$a^{-k}x(k)$	$X(az)$
Teorem začetne vrednosti	$\lim_{k\to 0}x(k)$	$\lim_{z\to\infty}X(z)$
Teorem končne vrednosti	$\lim_{k\to\infty}x(k)$	$\lim_{z\to 1}(z-1)X(z)$
Teorem množenja s k^r	$k^r x(k)$	$(-z\frac{d}{dz})^r X(z)$
Teorem diferenciranja funkcije po parametru	$\frac{\partial}{\partial a}x(k,a)$	$\frac{\partial}{\partial a}X(z,a)$
Teorem konvolucije	$\sum_{m=0}^{k}x(m)h(k-m)$	$X(z)H(z)$

Povezava med ravninama z in s

$$z=e^{sT}$$

Diskretna konvolucija

$$y(k)=\sum_{m=0}^{k}u(m)h(k-m)=\sum_{m=0}^{k}h(m)u(k-m)\xleftrightarrow{Z-trans.}Y(z)=H(z)U(z)$$

Diskretna Fourierova transformacija

$$X(mF)=\sum_{k=0}^{N-1}x(kT)e^{-\frac{j2\pi mk}{N}}$$

$$x(kT)=\frac{1}{N}\sum_{m=0}^{N-1}X(mF)e^{\frac{j2\pi mk}{N}}; \quad T=\frac{1}{f_s}, \ F=\frac{1}{t_p}$$

Prostor stanj

$$\left.\begin{aligned}\mathbf{x}(k+1)&=\mathbf{A}\mathbf{x}(k)+\mathbf{b}u(k)\\ y(k)&=\mathbf{c}^T\mathbf{x}(k)+du(k)\end{aligned}\right\}\leftrightarrow\frac{Y(z)}{U(z)}=\mathbf{c}^T(z\mathbf{I}-\mathbf{A})^{-1}\mathbf{b}+d$$

Odziv diskretnega sistema

$$\mathbf{x}(k)=\mathbf{A}^k\mathbf{x}(0)+\sum_{i=1}^{k}\mathbf{A}^{(k-i)}\mathbf{b}u(i-1); \quad \mathbf{A}^k=\mathcal{Z}^{-1}\left[(z\mathbf{I}-\mathbf{A})^{-1}z\right]=\mathbf{\Theta}\mathbf{\Lambda}^k\mathbf{\Theta}^{-1}$$

Frekvenčni odziv diskretnih sistemov

$$H(z)\Big|_{z=e^{j\omega T}}=H(e^{j\omega T})=A(\omega)e^{j\beta(\omega)}$$

Diskretni ekvivalenti zveznih sistemov:

- Metoda prvih diferenc: $s\sim\frac{z-1}{T}$
- Metoda zadnjih diferenc: $s\sim\frac{z-1}{Tz}$
- Trapezna formula: $s\sim\frac{2}{T}\frac{z-1}{z+1}$
- Načrtovanje diskretnih filtrov: $s\sim C\dfrac{1-z^{-1}}{1+z^{-1}}$, a) $C=\dfrac{2}{T}$, b) $C=\omega_r \operatorname{ctg}\dfrac{\omega_r T}{2}$

- Metoda stopnične invariance (prenosne funkcije): $H(z) = (1 - z^{-1})\mathcal{Z}\left\{\mathcal{L}^{-1}\left[\frac{G(s)}{s}\right]\Big|_{t=kT}\right\}$
- Metoda stopn. invariance (prostor stanj): $\mathbf{A}_{dis} = e^{\mathbf{A}_{zv}T} \doteq \mathbf{I} + \mathbf{A}_{zv}T$, $\mathbf{b}_{dis} = \int_0^T e^{\mathbf{A}_{zv}\tau}\mathbf{b}_{zv}d\tau = \mathbf{A}_{zv}^{-1}(\mathbf{A}_{dis} - \mathbf{I})\mathbf{b}_{zv} \doteq \mathbf{b}_{zv}T$

Pretvorbe med zapisi v prostoru stanj

$$\mathbf{A}_t = \mathbf{T}^{-1}\mathbf{A}\mathbf{T} \quad \mathbf{b}_t = \mathbf{T}^{-1}\mathbf{b} \quad \mathbf{c}_t^T = \mathbf{c}^T\mathbf{T} \quad d_t = d$$

Diagonalna kanonična oblika

$$\mathbf{T} = \mathbf{\Theta}$$

Vodljivost, spoznavnost

$$\mathbf{Q}_v = \begin{bmatrix}\mathbf{b} & \mathbf{A}\mathbf{b} & \cdots & \mathbf{A}^{n-1}\mathbf{b}\end{bmatrix} \quad \mathbf{T}_v = \mathbf{Q}_v\mathbf{W}$$

$$\mathbf{Q}_s = \begin{bmatrix}\mathbf{c}^T \\ \mathbf{c}^T\mathbf{A} \\ \vdots \\ \mathbf{c}^T\mathbf{A}^{n-1}\end{bmatrix} \quad \mathbf{T}_s = \left(\mathbf{W}\mathbf{Q}_s\right)^{-1} \quad \mathbf{W} = \begin{bmatrix} a_{n-1} & a_{n-2} & \cdots & 1 \\ a_{n-2} & a_{n-3} & & 0 \\ \vdots & & \ddots & \\ 1 & 0 & & 0 \end{bmatrix}$$

Modificiran Routhov kriterij

$$w = \frac{z-1}{z+1}$$

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$$b_k = \begin{vmatrix} a_0 & a_{n-k} \\ a_n & a_k \end{vmatrix} \qquad c_k = \begin{vmatrix} b_0 & b_{n-1-k} \\ b_{n-1} & b_k \end{vmatrix}$$

Splošni linearni regulator

$$\mathrm{A}_{zz} = PA + QBz^{-d} \quad l = \max\{m+\mu, m+d+\nu\}$$

P-regulator: $\mu = m + d - 1, \nu = m - 1$

I-regulator: $\mu = m + d, \nu = m$

Regulator stanj

$$\det\left(z\mathbf{I} - \mathbf{A} + \mathbf{b}\mathbf{k}^T\right) = 0 \quad \mathbf{k}_v^T = \begin{bmatrix}\alpha_n - a_n & \cdots & \alpha_1 - a_1\end{bmatrix}$$

Opazovalnik stanj

$$\det\left(z\mathbf{I} - \mathbf{A} + \mathbf{h}\mathbf{c}^T\right) = 0 \quad \mathbf{h}_s^T = \begin{bmatrix}\beta_n - a_n & \cdots & \beta_1 - a_1\end{bmatrix}$$

Tabela Laplaceove in z-transformacije

$x_z(t)$	$x(k) = x_z(kT)$	$\mathcal{L}\{x_z(t)\}$	$\mathcal{Z}\{x(k)\}$
	$\delta(kT)$		1
1	1	$\frac{1}{s}$	$\frac{z}{z-1}$
t	kT	$\frac{1}{s^2}$	$\frac{Tz}{(z-1)^2}$
t^2	k^2T^2	$\frac{2}{s^3}$	$\frac{T^2 z(z+1)}{(z-1)^3}$
t^3	k^3T^3	$\frac{6}{s^4}$	$\frac{T^3 z(z^2+4z+1)}{(z-1)^4}$
t^n	k^nT^n	$\frac{n!}{s^{n+1}}$	$\lim_{a\to 0} \frac{\partial^n}{\partial a^n} \frac{z}{z-e^{aT}}$
e^{-at}	e^{-akT}	$\frac{1}{s+a}$	$\frac{z}{z-e^{-aT}}$
te^{-at}	kTe^{-akT}	$\frac{1}{(s+a)^2}$	$\frac{Tze^{-aT}}{(z-e^{-aT})^2}$
t^2e^{-at}	$k^2T^2e^{-akT}$	$\frac{2}{(s+a)^3}$	$\frac{T^2ze^{-aT}(z+e^{-aT})}{(z-e^{-aT})^3}$
t^ne^{at}	$k^nT^ne^{akT}$	$\frac{n!}{(s-a)^{n+1}}$	$\frac{\partial^n}{\partial a^n} \frac{z}{z-e^{aT}}$
$1-e^{-at}$	$1-e^{-akT}$	$\frac{a}{s(s+a)}$	$\frac{(1-e^{-aT})z}{(z-1)(z-e^{-aT})}$
$at-1+e^{-at}$	$akT-1+e^{-akT}$	$\frac{a^2}{s^2(s+a)}$	$\frac{(aT-1+e^{-aT})z^2+(1-aTe^{-aT}-e^{-aT})z}{(z-1)^2(z-e^{-aT})}$
$e^{-at}-e^{-bt}$	$e^{-akT}-e^{-bkT}$	$\frac{b-a}{(s+a)(s+b)}$	$\frac{z(e^{-aT}-e^{-bT})}{(z-e^{-aT})(z-e^{-bT})}$
$\sin\omega_0 t$	$\sin\omega_0 kT$	$\frac{\omega_0}{s^2+\omega_0^2}$	$\frac{z\sin\omega_0 T}{z^2-2z\cos\omega_0 T+1}$
$\cos\omega_0 t$	$\cos\omega_0 kT$	$\frac{s}{s^2+\omega_0^2}$	$\frac{z(z-\cos\omega_0 T)}{z^2-2z\cos\omega_0 T+1}$
$e^{-at}\sin\omega_0 t$	$e^{-akT}\sin\omega_0 kT$	$\frac{\omega_0}{(s+a)^2+\omega_0^2}$	$\frac{ze^{-aT}\sin\omega_0 T}{z^2-2ze^{-aT}\cos\omega_0 T+e^{-2aT}}$
$e^{-at}\cos\omega_0 t$	$e^{-akT}\cos\omega_0 kT$	$\frac{s+a}{(s+a)^2+\omega_0^2}$	$\frac{z^2-ze^{-aT}\cos\omega_0 T}{z^2-2ze^{-aT}\cos\omega_0 T+e^{-2aT}}$