



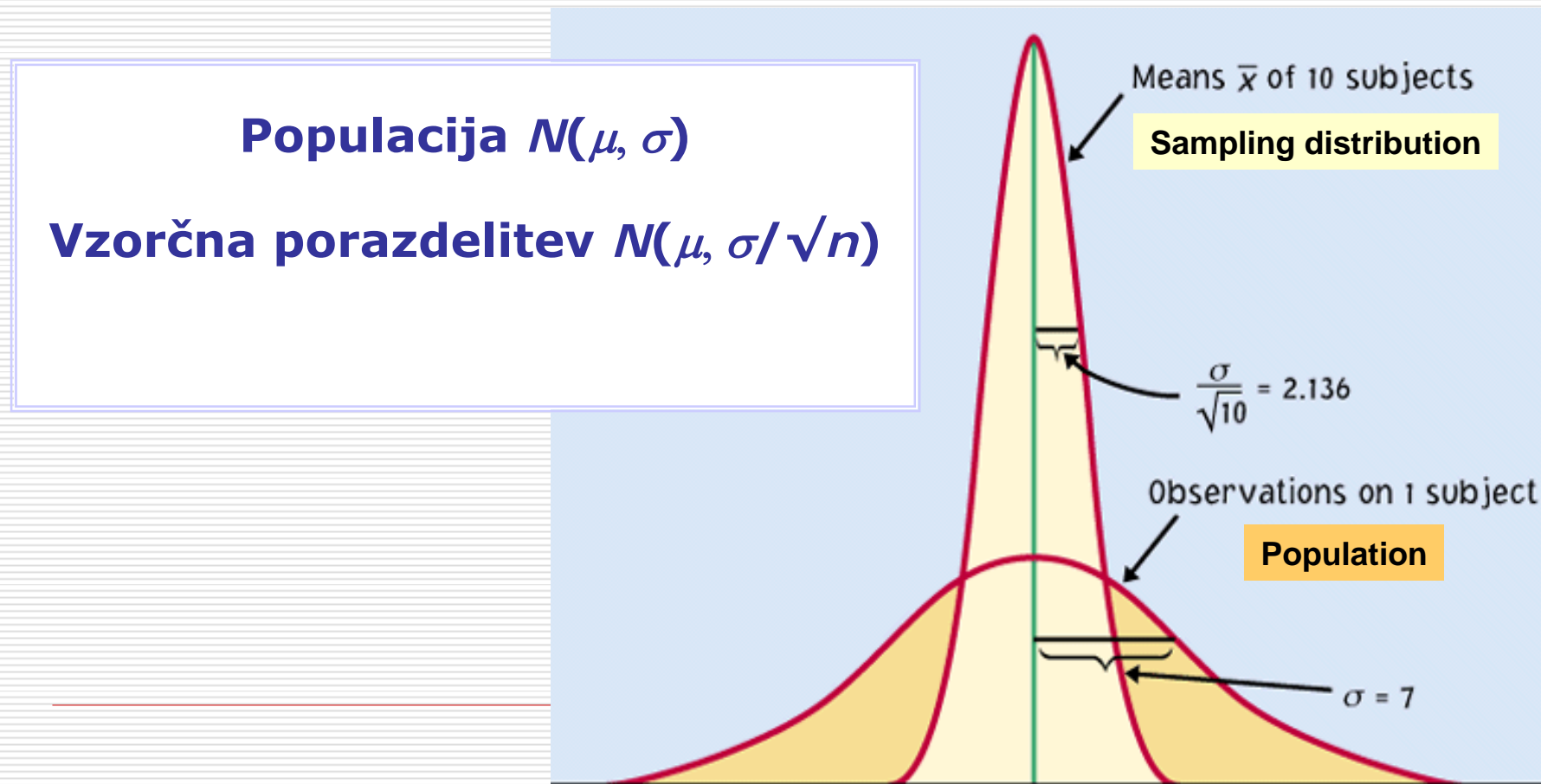
Testiranje hipotez o razliki med dvema vzorcema

Iztok Grabnar

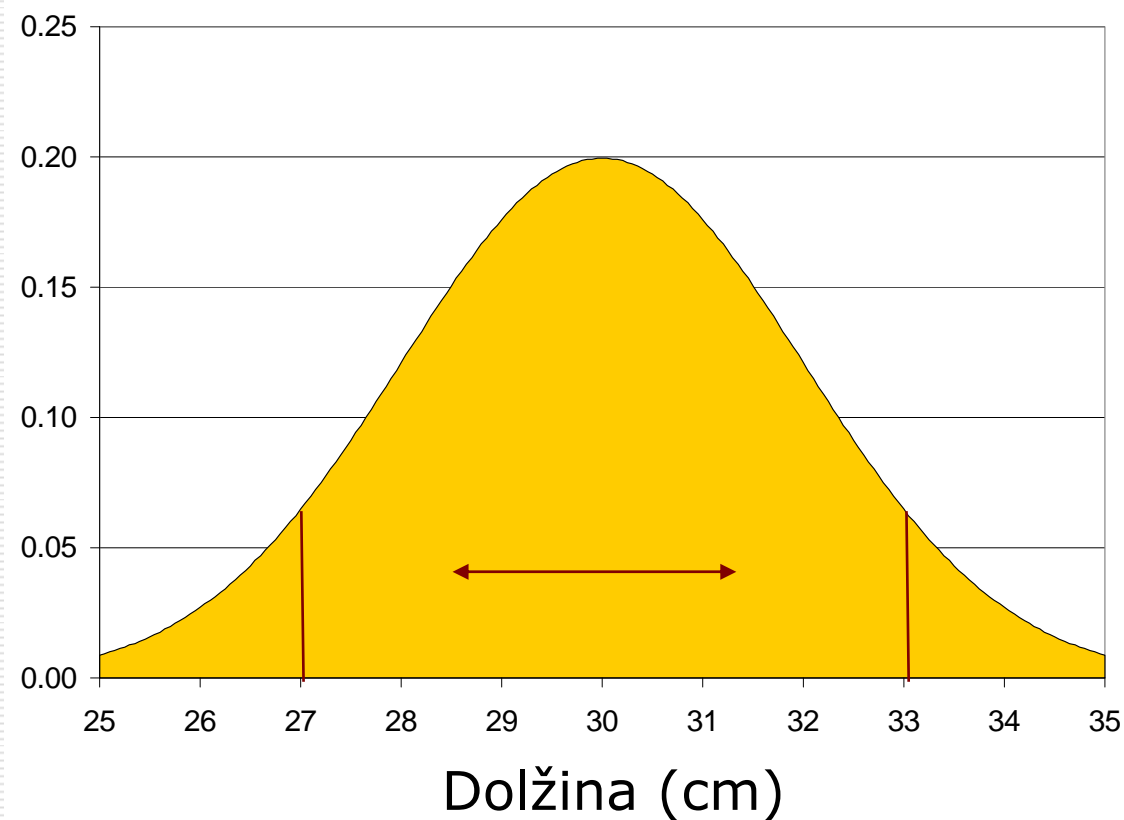
Univerza v Ljubljani, Fakulteta za farmacijo

November, 2010

Centralni limitni izrek



Intervalna ocena za aritmetično sredino



$$\mu = 30 \text{ cm}$$
$$\sigma = 1.5 \text{ cm}$$

Poznana varianca

$$\bar{X} - z_{\alpha/2} \frac{\sigma}{\sqrt{N}} \leq \mu \leq \bar{X} + z_{\alpha/2} \frac{\sigma}{\sqrt{N}}$$

Stopnja tveganja (α)	Enostransko tveganje	Dvostransko tveganje
0.1	1.282	1.645
0.05	1.645	1.960
0.025	1.960	2.241
0.01	2.326	2.576
0.005	2.576	2.807

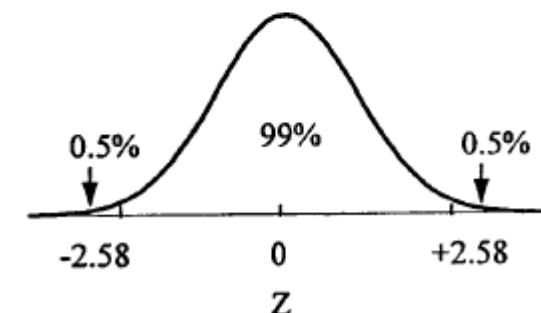
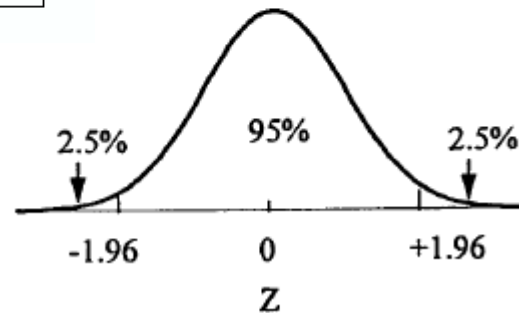
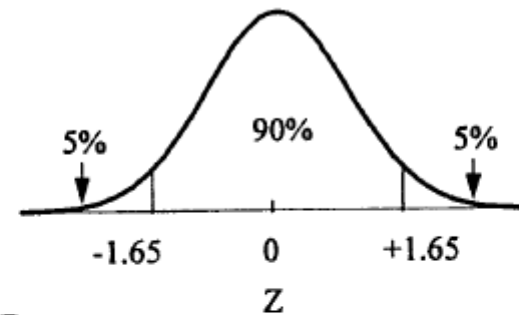
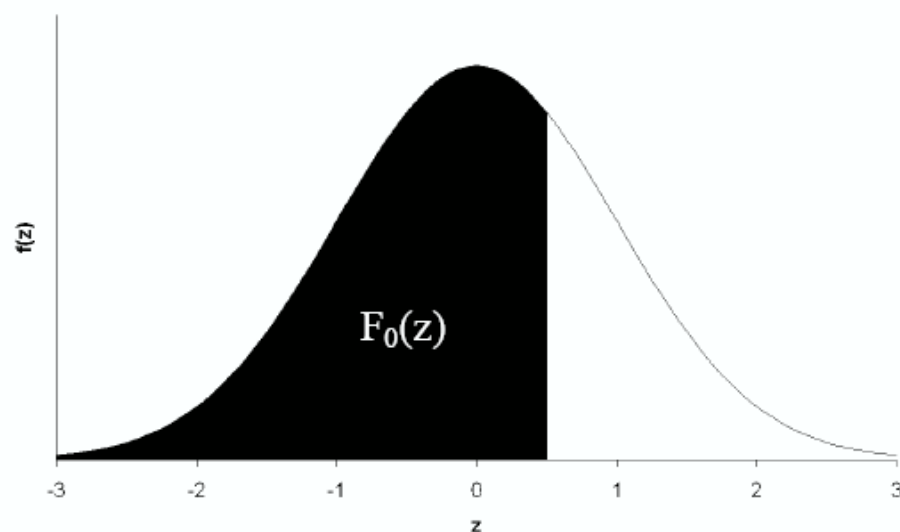


Tabela standardizirane normalne distribucije.



Stopnja tveganja (α)	Enostransko tveganje	Dvostransko tveganje
0.1	1.282	1.645
0.05	1.645	1.960
0.025	1.960	2.241
0.01	2.326	2.576
0.005	2.576	2.807

$F_0(z)$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879

Prava varianca ni poznana Ocenjena iz vzorca

$$\bar{X} - t_{\alpha/2} \frac{s}{\sqrt{N}} \leq \mu \leq \bar{X} + t_{\alpha/2} \frac{s}{\sqrt{N}}$$

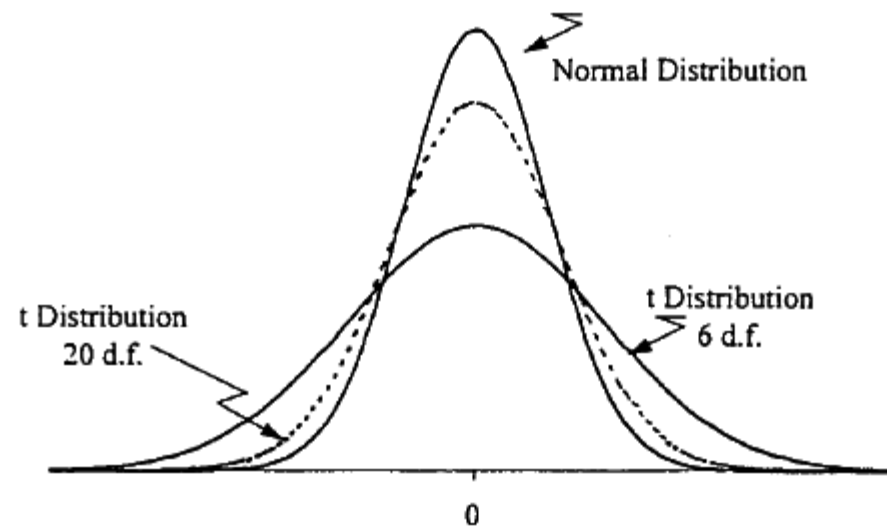
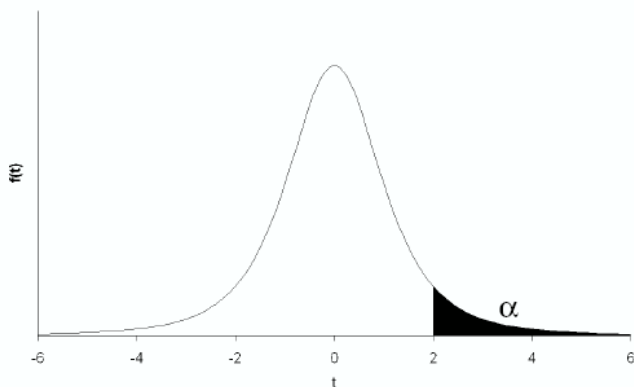


Figure 5.9 *t* distribution compared to the standard normal distribution.

Studentova porazdelitev

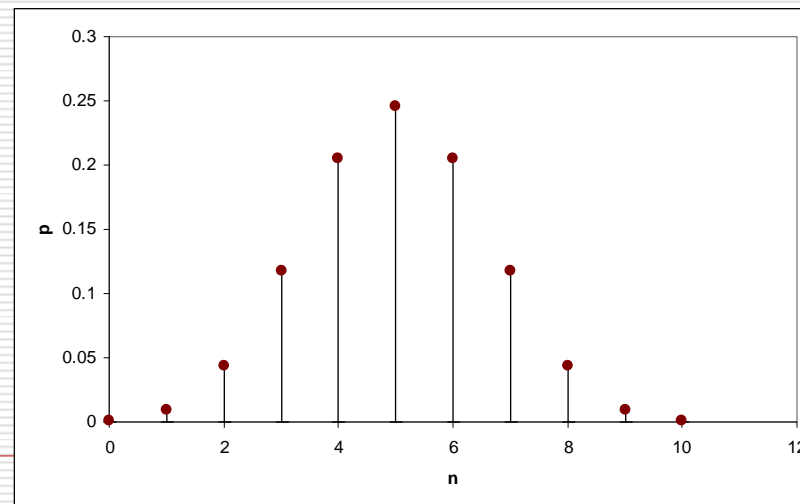


Število prostostnih stopenj	Stopnja tveganja (α)				
	0.4	0.2	0.1	0.05	0.01
Dvostransko tveganje	0.4	0.2	0.1	0.05	0.01
Enostransko tveganje	0.2	0.1	0.05	0.025	0.005
1	1.376	3.078	6.314	12.706	63.656
2	1.061	1.886	2.920	4.303	9.925
3	0.978	1.638	2.353	3.182	5.841
4	0.941	1.533	2.132	2.776	4.604
5	0.920	1.476	2.015	2.571	4.032
6	0.906	1.440	1.943	2.447	3.707
7	0.896	1.415	1.895	2.365	3.499
8	0.889	1.397	1.860	2.306	3.355
9	0.883	1.383	1.833	2.262	3.250

Intervalna ocena za delež

Aproksimacija z normalno porazdelitvijo

$$\hat{p} - Z_{\alpha} \sqrt{\frac{\hat{p}\hat{q}}{N}} \leq p \leq \hat{p} + Z_{\alpha} \sqrt{\frac{\hat{p}\hat{q}}{N}}$$

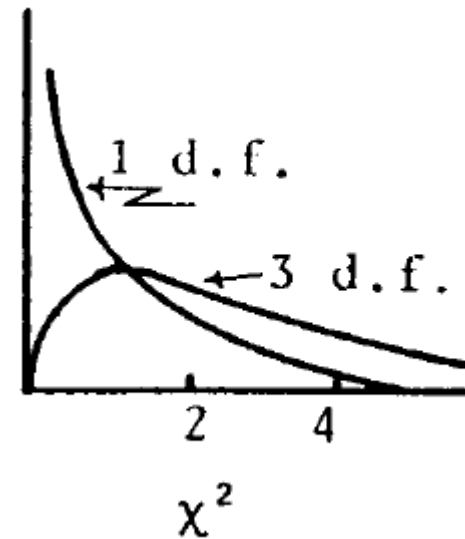


Interválna ocena za delež

$$s^2(n-1)/\chi_{\alpha/2}^2 \leq \sigma^2 \leq s^2(n-1)/\chi_{\alpha/2}^2$$

Table 5.19 Short Table of Lower and Upper Cutoff Points for Chi-Square Distribution

Degrees of Freedom	Lower 2.5%	Lower 5%	Upper 95%	Upper 97.5%
2	0.0506	0.1026	5.99	7.38
3	0.216	0.352	7.81	9.35
4	0.484	0.711	9.49	11.14
5	0.831	1.15	11.07	12.83
6	1.24	1.64	12.59	14.45
7	1.69	2.17	14.07	16.01
8	2.18	2.73	15.51	17.53
9	2.70	3.33	16.92	19.02
10	3.25	3.94	18.31	20.48
15	6.26	7.26	25.00	27.49
20	9.59	10.85	31.41	34.17
30	16.79	18.49	43.77	46.98
60	40.48	43.19	79.08	83.30
120	91.58	95.76	146.57	152.21



Preizkušanje statističnih hipotez

- H_0 : ničelna hipoteza
- H_a : alternativna hipoteza

Table 5.6 Alpha and Beta Probabilities in Hypothesis Testing (Errors When Accepting or Rejecting H_0)

	H_0 is true	H_a (a specific alternative) is true
H_0 is rejected	Alpha (α)	1 – beta
H_0 is accepted	1 – alpha	Beta (β)

Absence of evidence of the effect is not evidence of absence of the effect

Preizkušanje statističnih hipotez za en vzorec

Table 5.5 Results of 20 Single-Tablet Assays from a Modification of a Process with a Historical Mean of 5.01 mg

5.13	5.04	5.09	5.00
4.98	5.03	5.01	4.99
5.20	5.08	4.96	5.18
5.08	5.06	5.02	5.24
4.99	5.17	5.06	5.00
$\bar{X} = 5.0655$ mg		$S = 0.0806$	
σ (historical) = 0.11			

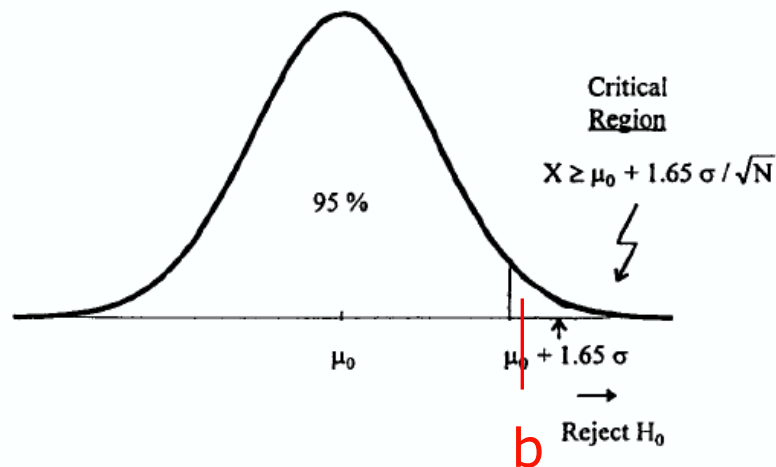
En vzorec, prava varianca je poznana

Enostransko tveganje

Enostranski test

$$H_0: \mu \geq b$$

$$H_a: \mu < b$$

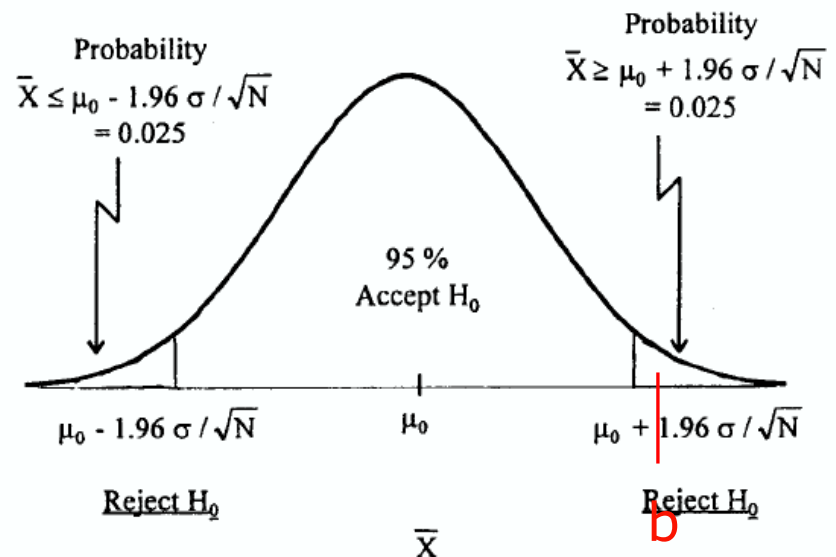


Dvostransko tveganje

Dvostranski test

$$H_0: \mu = b$$

$$H_a: \mu \neq b$$



En vzorec, prava varianca je poznana

Enostransko tveganje

$$H_0: \mu - b \geq 0$$

$$H_a: \mu - b < b$$

$$Z_{\text{exp}} = \frac{\mu - b}{\frac{\sigma}{\sqrt{N}}}$$

$$Z_{\text{tab}} = -1.645$$

Dvostransko tveganje

Dvostranski test

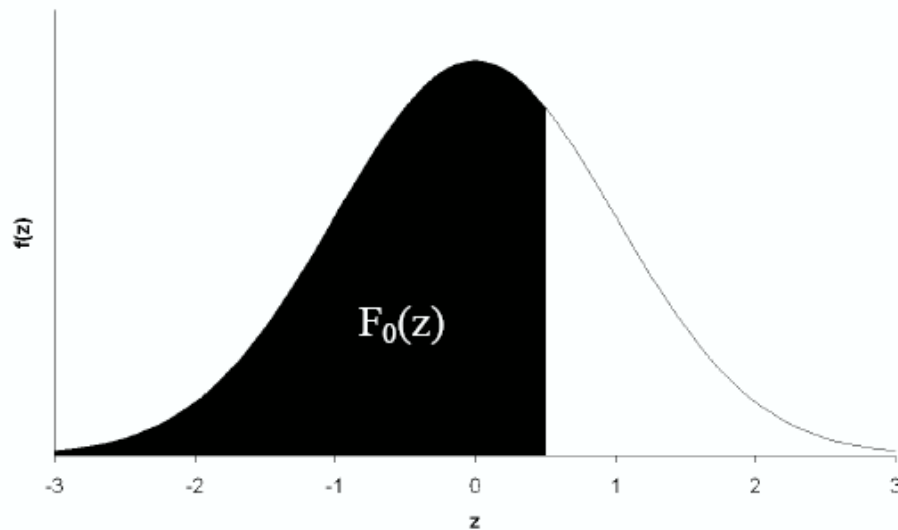
$$H_0: \mu = b$$

$$H_a: \mu \neq b$$

$$Z_{\text{exp}} = \frac{|\mu - b|}{\frac{\sigma}{\sqrt{N}}}$$

$$Z_{\text{tab}} = 1.96$$

$$\alpha = 0.05$$



$$Z_{\text{exp}} > Z_{\text{tab}} \Leftrightarrow 1 - F_0(Z_{\text{exp}}) < \alpha$$

H_0 zavrnemo in sprejmemo H_a
Razlika je statistično značilna!

Enostransko tveganje

~~$$H_0: \mu - b \geq 0$$

$$H_a: \mu - b < b$$~~

Dvostransko tveganje

Dvostranski test

~~$$H_0: \mu - b = 0$$

$$H_a: \mu - b \neq 0$$~~

En vzorec, prava varianca ni poznana t-test za en vzorec

Enostransko tveganje

$$H_0: \mu - b \geq 0$$

$$H_a: \mu - b < b$$

Dvostransko tveganje

Dvostranski test

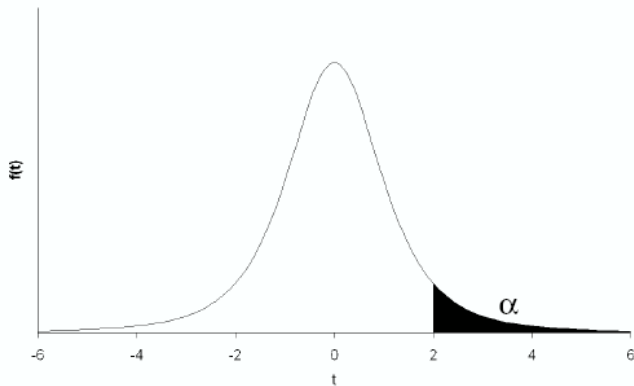
$$H_0: \mu = b$$

$$H_a: \mu \neq b$$

$$t_{\text{exp}} = \frac{\mu - b}{\frac{s}{\sqrt{N}}}$$

$$t_{\text{exp}} = \frac{|\mu - b|}{\frac{s}{\sqrt{N}}}$$

Studentova porazdelitev



t

Število prostostnih stopenj	Stopnja tveganja (α)				
	0.4	0.2	0.1	0.05	0.01
Dvostransko tveganje	0.4	0.2	0.1	0.05	0.01
Enostransko tveganje	0.2	0.1	0.05	0.025	0.005
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2	1.061	1.886	2.920	4.303	9.925
3	0.978	1.638	2.353	3.182	5.841
4	0.941	1.533	2.132	2.776	4.604
5	0.920	1.476	2.015	2.571	4.032
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7	0.896	1.415	1.895	2.365	3.499
8	0.889	1.397	1.860	2.306	3.355
9	0.883	1.383	1.833	2.262	3.250

Dva neodvisna vzorca t-test za dva neodvisna vzorca

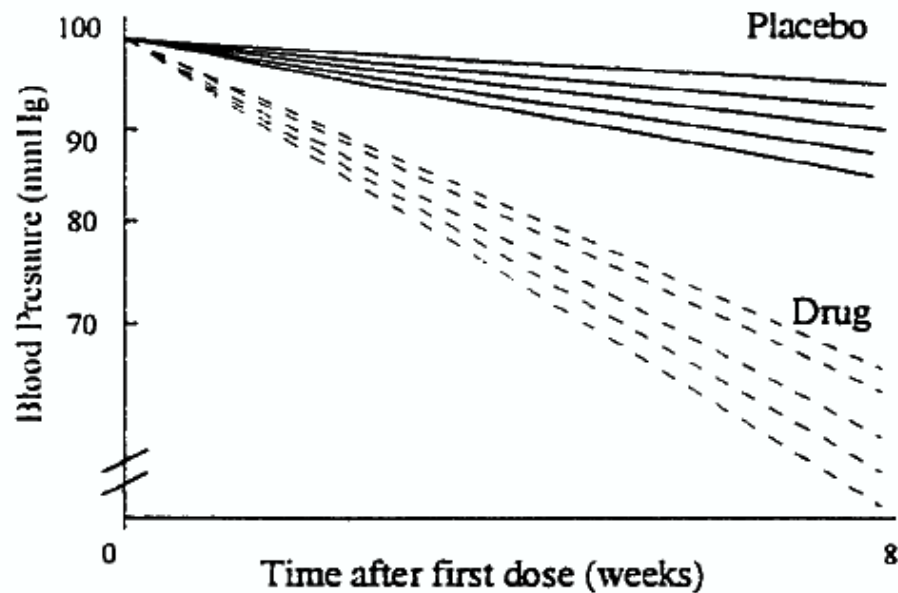
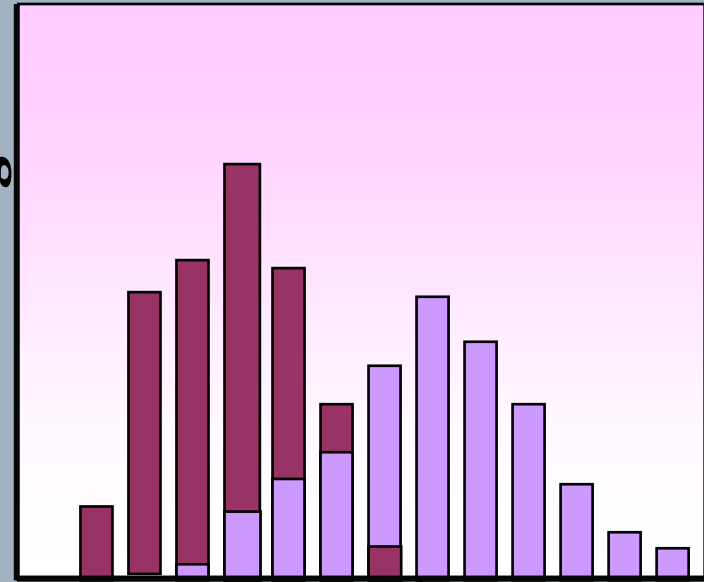


Figure 5.6 Mark of a real drug effect: A large difference between drug and placebo with small variation.



**Blood pressure
mm of Hg**



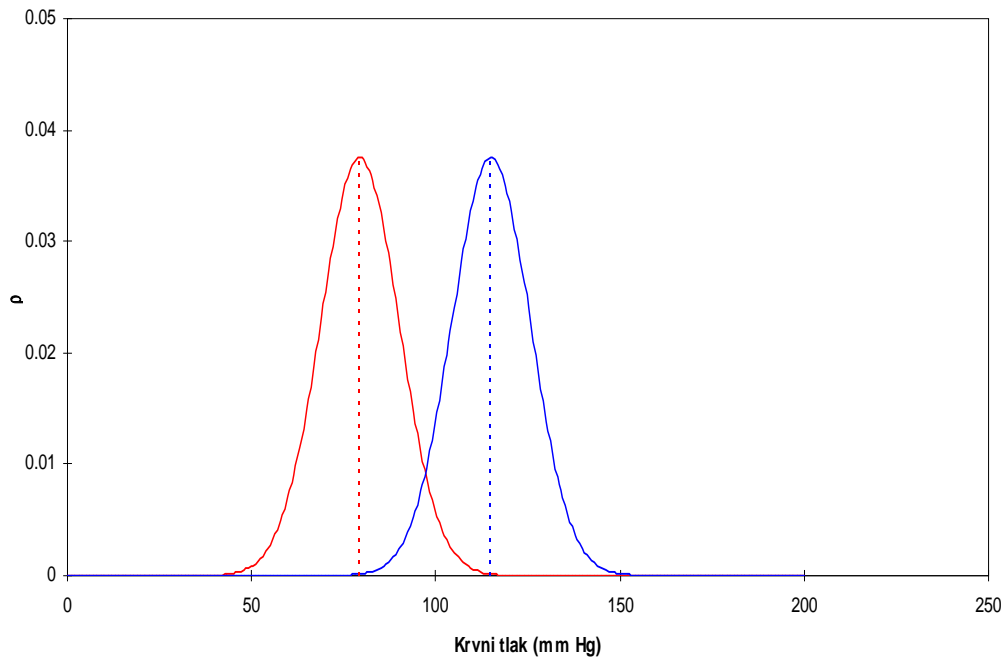
μ_c

$\mu_{t/t}$

Groups

Control

Treatment



Dva vzorca s poznanimi variancama

$$Z_{\text{exp}} = \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{N_1} + \frac{\sigma_2^2}{N_2}}}$$

Pravi varianci nista poznani, a sta enaki Vzorčna ocena skupne variance

$$s_p^2 = \frac{\sum (x_{1,i} - \bar{X}_1)^2 + \sum (x_{2,i} - \bar{X}_2)^2}{N_1 - 1 + N_2 - 1}$$

$$s_p^2 = \frac{(N_1 - 1)s_1^2 + (N_2 - 1)s_2^2}{N_1 + N_2 - 2}$$

$$10 < N < 20 \quad s_1^2 / s_2^2 < 4$$

$$t_{\text{exp}} = \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{s_p^2}{N_1} + \frac{s_p^2}{N_2}}}$$

$$t_{\text{tab}}(\alpha, df = N_1 + N_2 - 2)$$

Pravi varianci nista poznani, a sta različni Behrens-Fisher test

$$t'_{\text{exp}} = \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{N_1} + \frac{s_2^2}{N_2}}}$$

t_{tab}

$$N_1 = N_2 = N$$

$$df = N - 1$$

$$N_1 \neq N_2$$

$$t_{\text{tab}} = (w_1 t_1 + w_2 t_2) / (w_1 + w_2)$$

$$w_1 = s_1^2 / N_1, w_2 = s_2^2 / N_2$$

Test enakosti varianc

F-test

$$H_0: \sigma_1^2 = \sigma_2^2$$

$$H_a: \sigma_1^2 \neq \sigma_2^2$$

$$F_{\text{exp}} = \frac{s_1^2}{s_2^2}$$

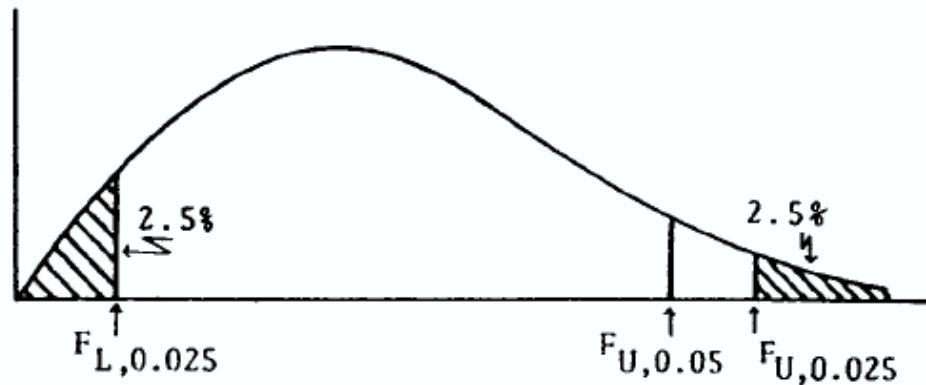


Figure 5.12 Example of two-sided cutoff points in an F distribution.

Dva odvisna vzorca parni t-test

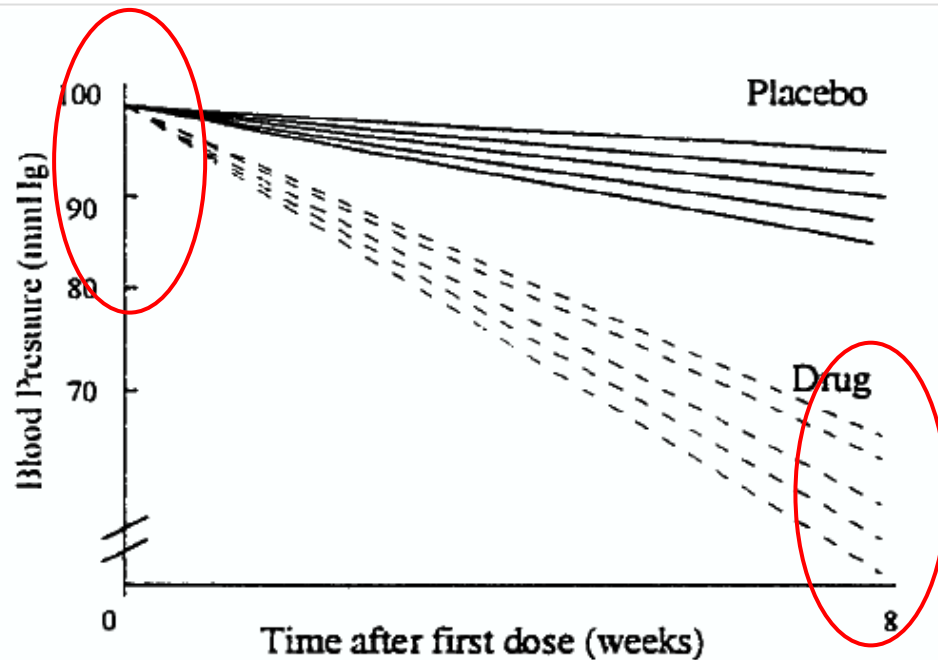


Figure 5.6 Mark of a real drug effect: A large difference between drug and placebo with small variation.

Deleži

En vzorec

$$Z_{\text{exp}} = \frac{|\hat{p} - p_0| - \frac{1}{2N}}{\sqrt{\frac{p_0 q_0}{N}}}$$

Yatesova korektura

Dva neodvisna vzorca

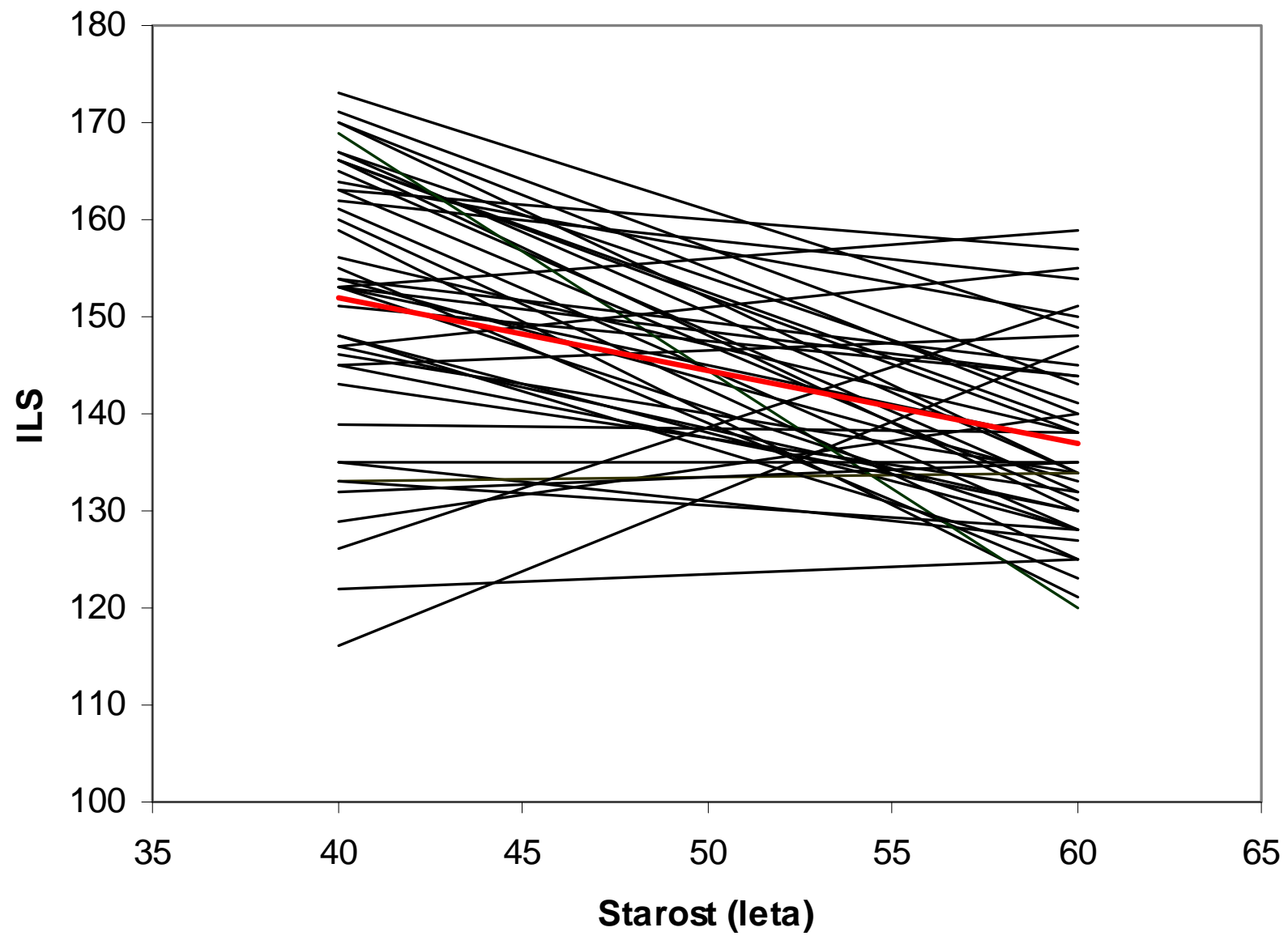
$$Z_{\text{exp}} = \frac{|\hat{p}_a - \hat{p}_b|}{\sqrt{\frac{p_0 q_0}{N_a} + \frac{p_0 q_0}{N_b}}}$$

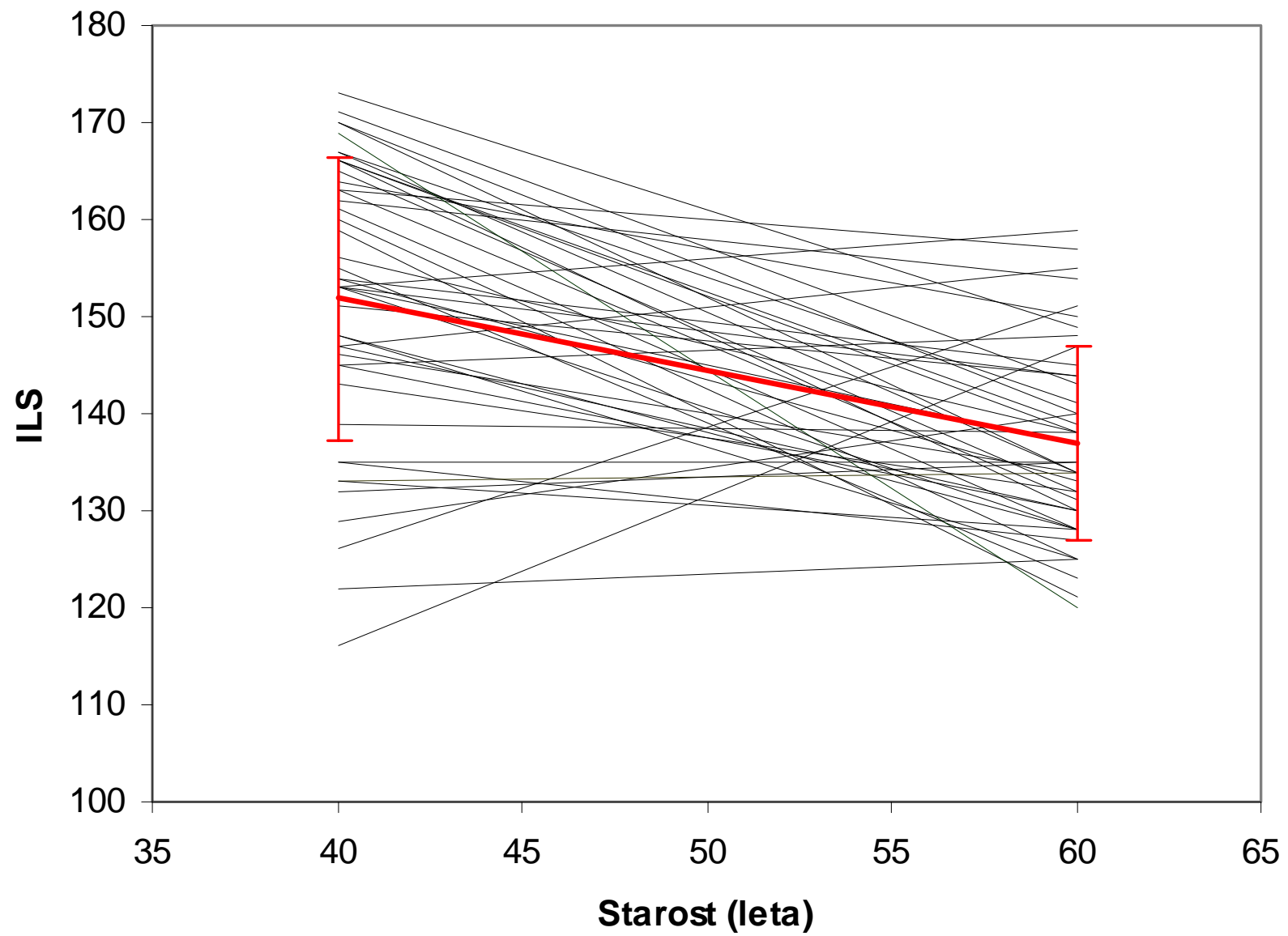
$$p_0 = \frac{\hat{p}_a N_a + \hat{p}_b N_b}{N_a + N_b}$$

Primer 1

Ali se stres s starostjo spremeni?

ILS (Index of Life Stress) pri skupini 100 žensk starosti 40 let. Drugi vzorec pridobljen pri podskupini 45 žensk, ko so stare 60 let.





T-test za dva odvisna vzorca

Paired Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	ILS 40	151.84	45	14.483	2.159
	ILS 60	136.87	45	9.949	1.483

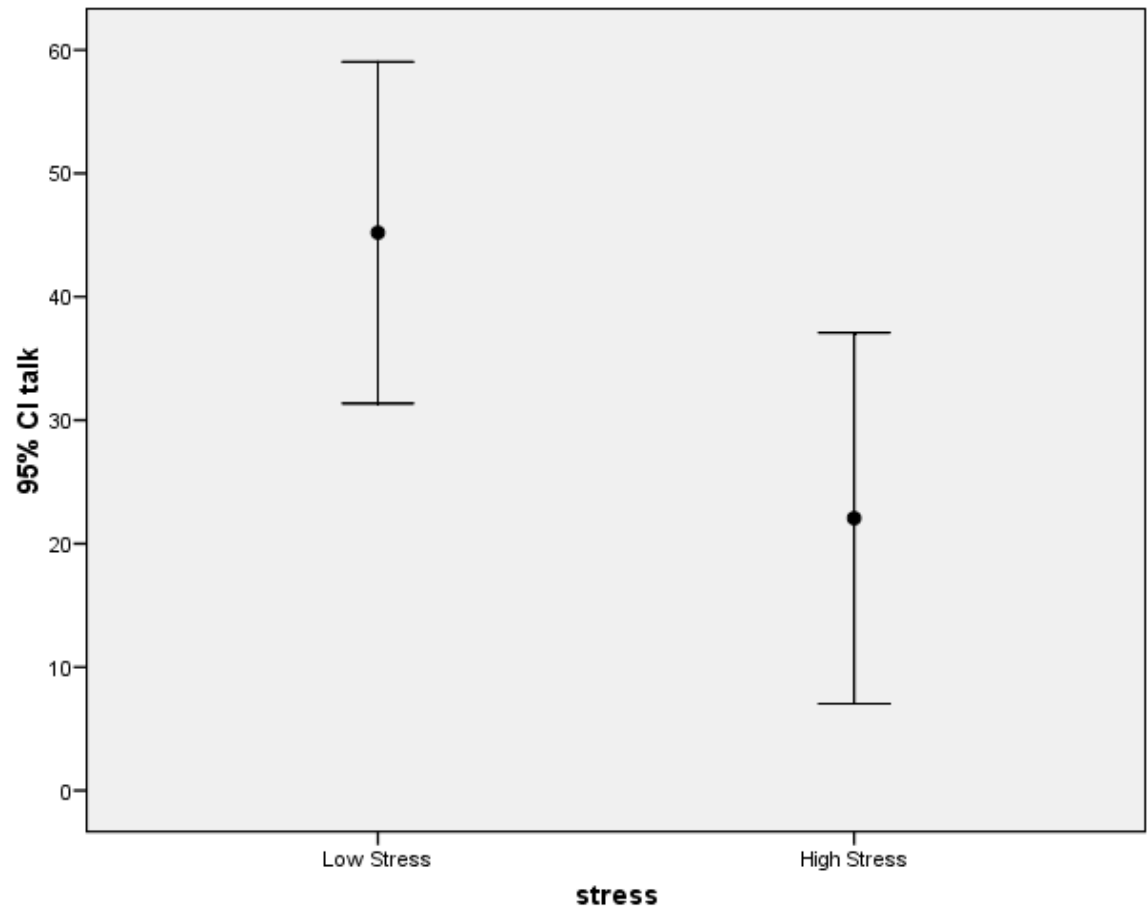
Paired Samples Test

		Paired Differences							
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference		t	df	Sig. (2-tailed)
					Lower	Upper			
Pair 1	ILS 40 - ILS 60	14.978	17.266	2.574	9.791	20.165	5.819	44	.000

Primer 2

Low Stress	High Stress
31	41
5	93
2	12
78	0
32	3
17	29
60	0
30	2
62	1
79	4
62	42
65	18
37	60
58	24
60	2

Ali ljudje govorimo več, ko smo nervozni?



T-test za dva neodvisna vzorca

Group Statistics

stress		N	Mean	Std. Deviation	Std. Error Mean
talk	Low Stress	15	45.20	24.969	6.447
	High Stress	15	22.07	27.136	7.006

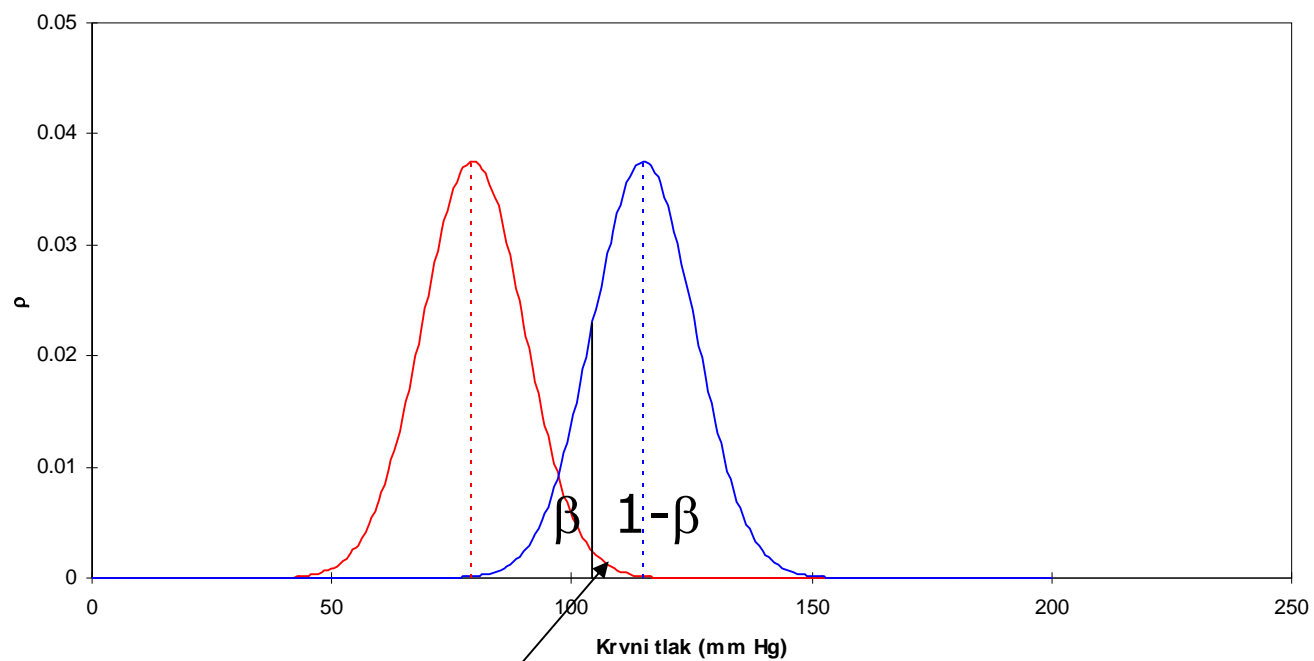
Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
talk	Equal variances assumed	.023	.881	2.430	28	.022	23.133	9.521	3.630	42.637
	Equal variances not assumed			2.430	27.808	.022	23.133	9.521	3.624	42.643

Varianci se ne razlikujeta značilno

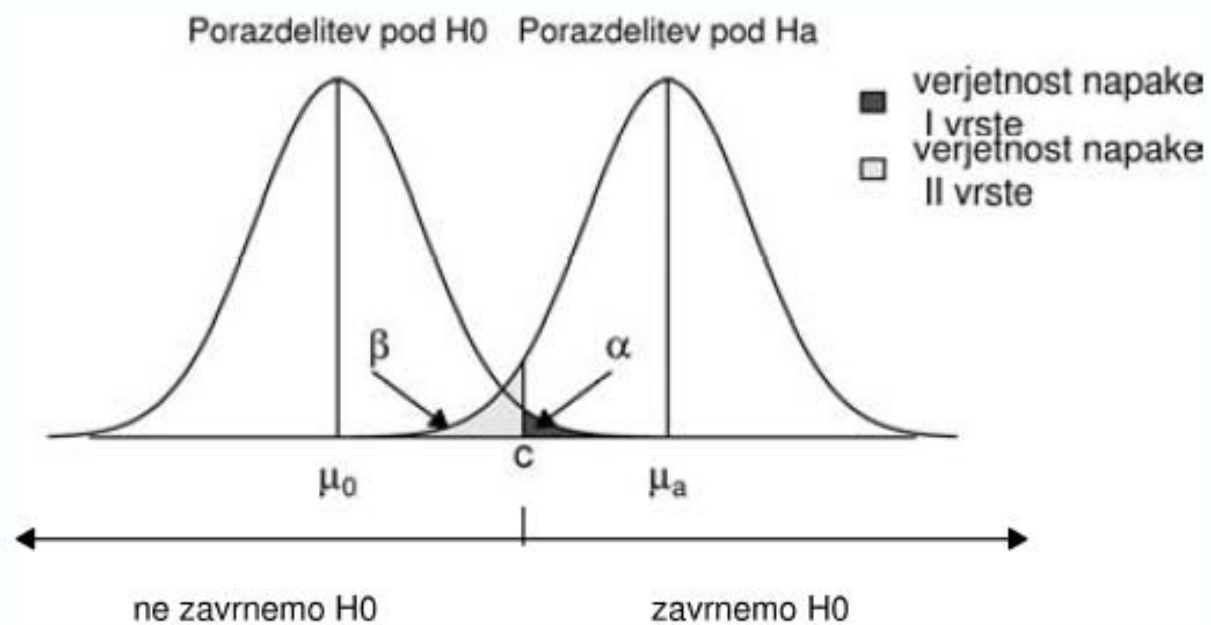
Povprečji sta značilno različni

Moč testa



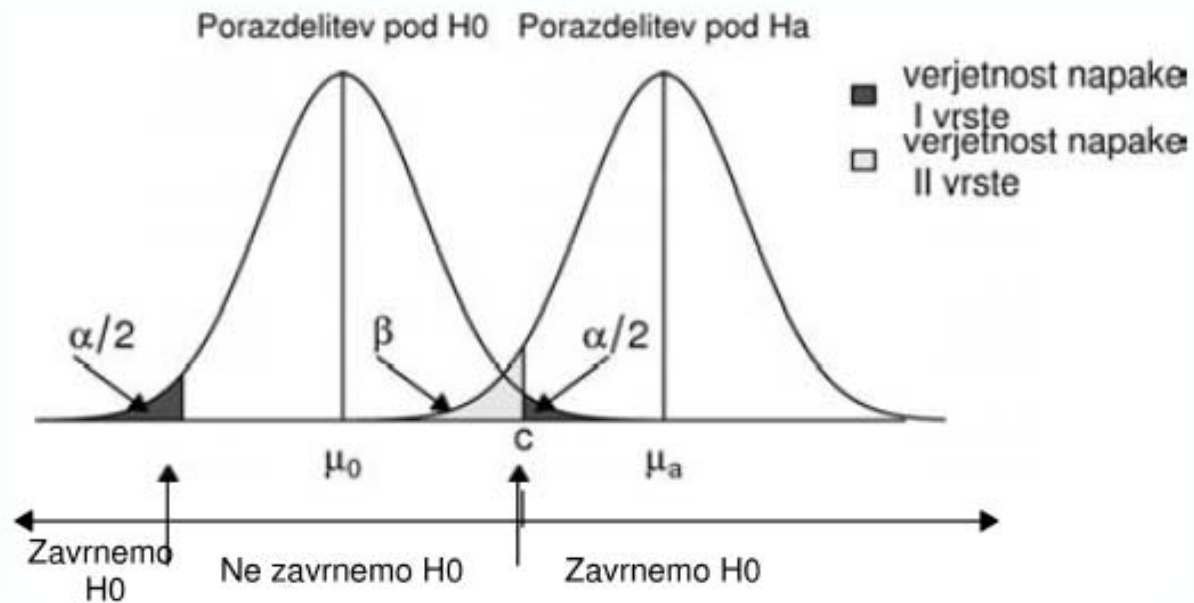
α

Moč testa



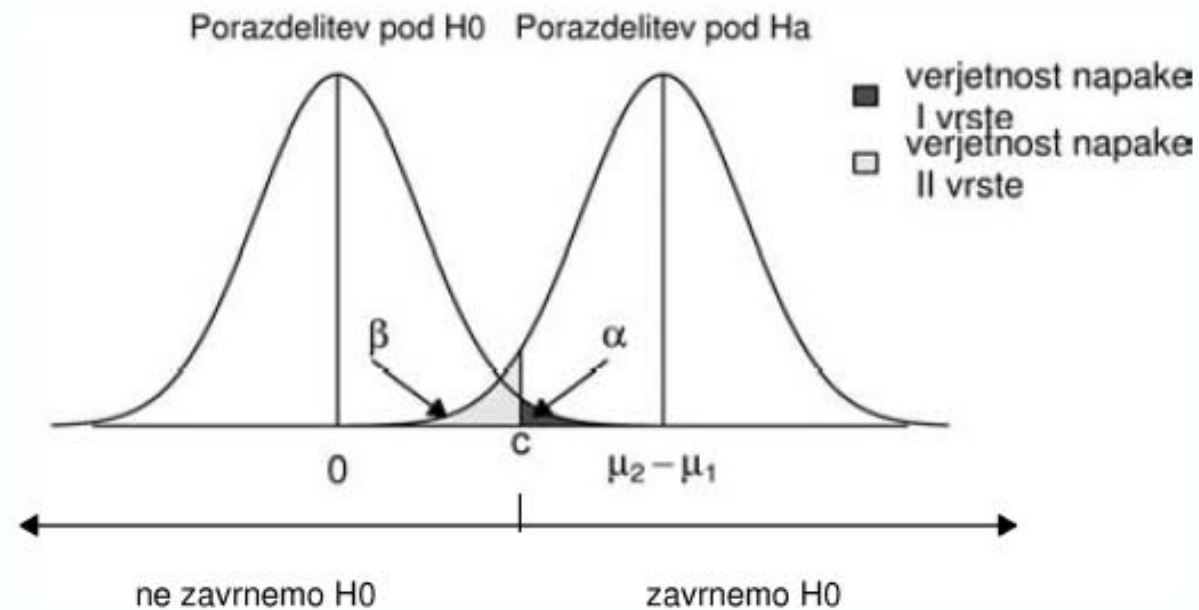
Slika 1: Testiranje hipoteze o povprečju populacije z enostranskim testom.

Dvostranski test



Slika 2: Testiranje hipoteze o povprečju populacije z dvostranskim testom.

Enostranski test



Slika 3: Dva vzorca, enostranski test za $H_0: \mu_1 = \mu_2$ proti alternativni hipotezi, da je $H_0: \mu_2 > \mu_1$.

Moč testa – poznana varianca

En vzorec

$$N = \left(\frac{\sigma}{\Delta} \right)^2 (Z_{\alpha} + Z_{\beta})^2$$

Dva neodvisna vzorca

$$N = 2 \left(\frac{\sigma}{\Delta} \right)^2 (Z_{\alpha} + Z_{\beta})^2$$

Moč testa – varianca ni poznana

En vzorec

$$N = \left(\frac{s}{\Delta} \right)^2 (Z_{\alpha} + Z_{\beta})^2 + 0.5Z_{\alpha}^2$$

Dva neodvisna vzorca

$$N = 2 \left(\frac{s}{\Delta} \right)^2 (Z_{\alpha} + Z_{\beta})^2 + 0.25Z_{\alpha}^2$$

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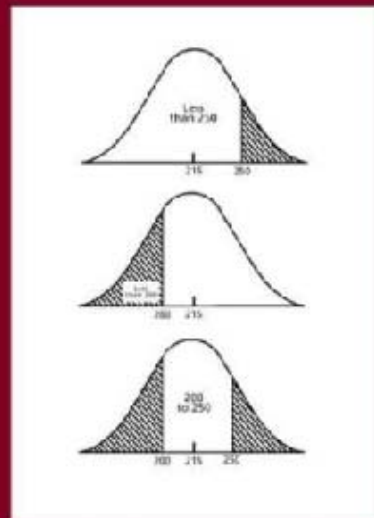
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