Univerza
v Ljubljani
Fakulteta
za gradbeništvo
in geodezijo

Katedra za metalne konstrukcije

JEKLENE KONSTRUKCIJE I

7.0 UVOD V STABILNOST JEKLENIH KONSTRUKCIJ

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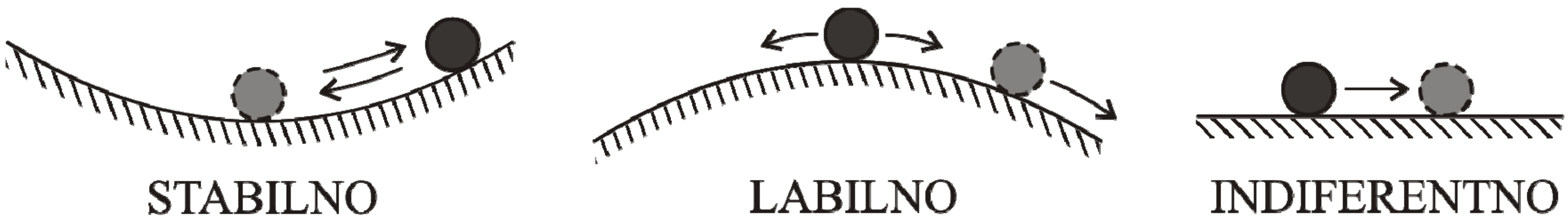
Sodelavec: Blaž Čermelj

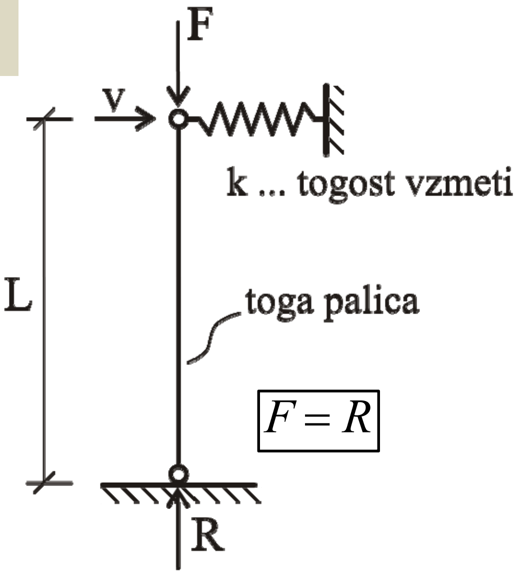
7.1 Osnovni pojmi

RAVNOTEŽJE – vrsta ravnotežja, izmikanje iz ravnotežne lege

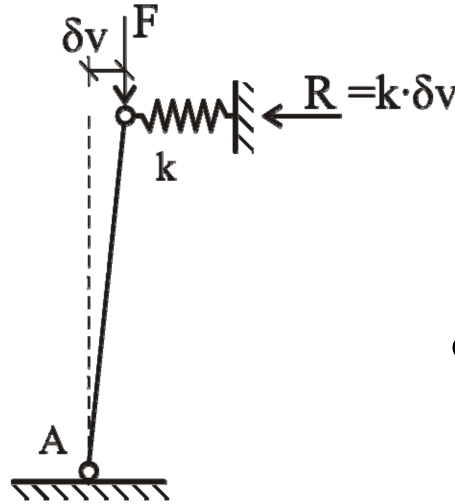


Za ugotavljanje ravnotežja **konstrukcijo izmaknemo iz ravnotežne lege**, **ravnotežne enačbe** zapišemo v **deformirani legi konstrukcije.**

PRIMER



ZAČETNA LEGA



DEFORMIRANA LEGA

δv ... virtualni pomik

- ❑ poljuben
- ❑ možen

$$\sum M^{A}:$$

$$F\,\delta v - k\,\delta v\,L = 0$$

$$\left(F - k\,L\right)\delta v = 0$$

Zanima nas **netrivialna rešitev**:

$$\delta v \neq 0 \quad \rightarrow \quad F - k\,L = 0$$

$$\boxed{F_{cr} = k\,L}$$

Ravnotežje:

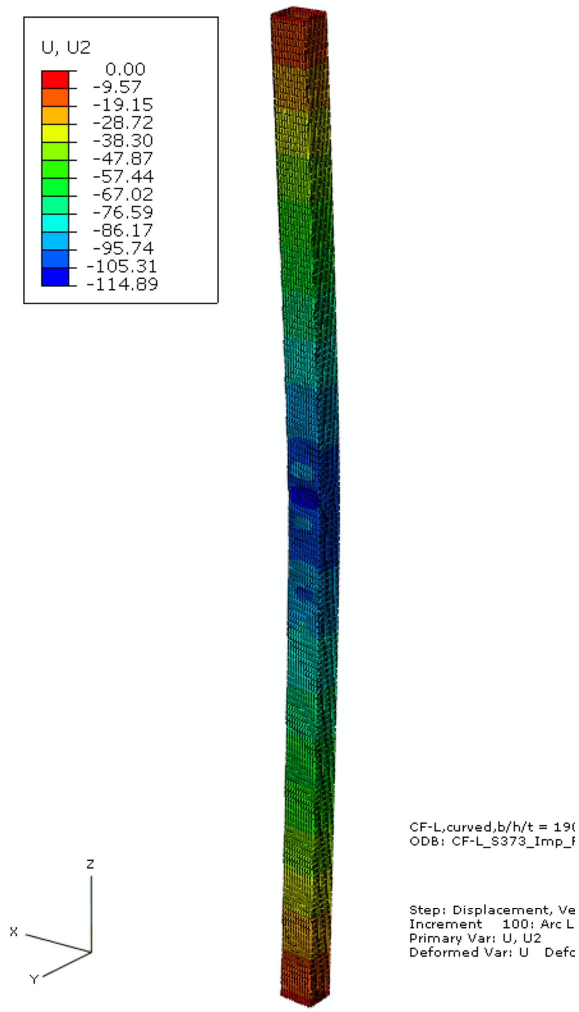
$F < F_{cr}$ **stabilno**

$F \geq F_{cr}$ nestabilno

Primer uklona tlačenega stebra

Preizkušanec v laboratoriju

Računalniška simulacija uklona

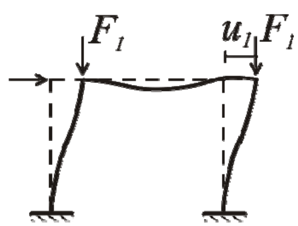


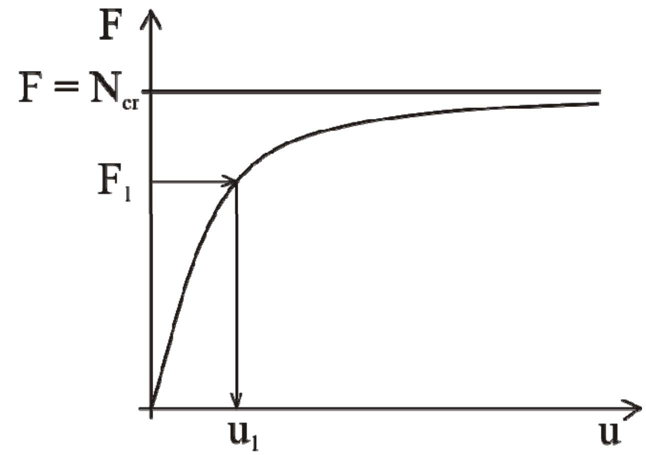
Analiza konstrukcij

$[K] = [K(N)]$ ... togost sistema

- NAPETOSTNI PROBLEM:

  nehomogen sistem enačb: $[K]\{u\} = \{F\}$

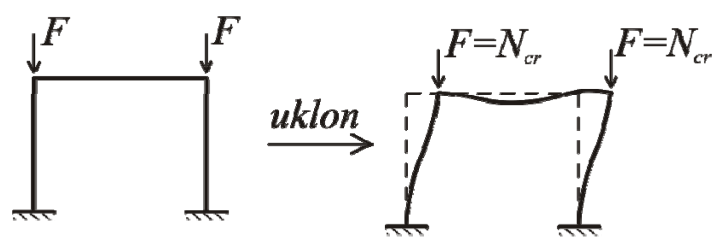




- RAČUN N_{cr} (idealni sistemi):

  homogen sistem: $[K(N)]\{u\} = \{0\}$

  netrivialna rešitev: $\boxed{\det[K(N)] = 0 \rightarrow N_{cr}}$



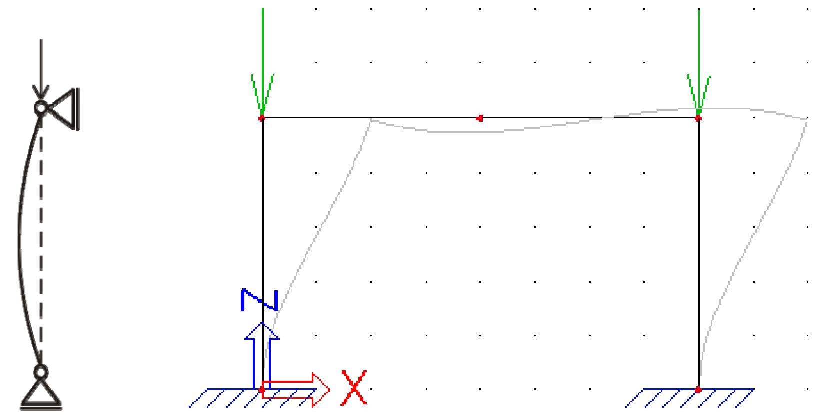
$\{u\}$ pri $N = N_{cr}$

- lastna vrednost
- lastni vektor – uklonska oblika

Značilni primeri nestabilnosti

Nestabilnost v konstrukcijah povzroča **tlačna napetost**.

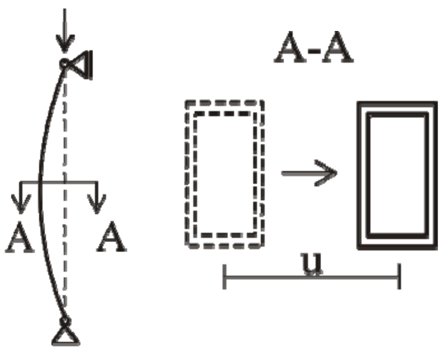
a) <u>UKLON TLAČENIH PALIC</u>



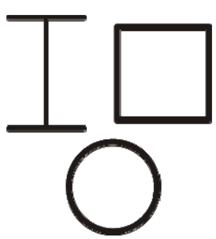
Mejna nosilnost je povezana z **uklonom oz. izgubo stabilnosti palic**.

Trije načini uklona tlačenih palic

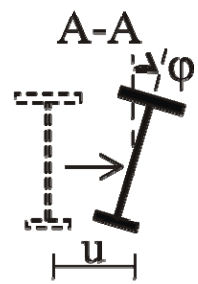
UPOGIBNI UKLON



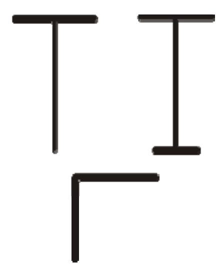
Cevni in škatlasti prerezi, navadni odprti profili.



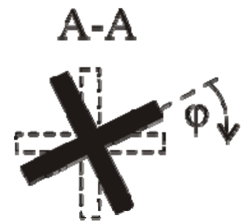
UPOGIBNO – TORZIJSKI UKLON



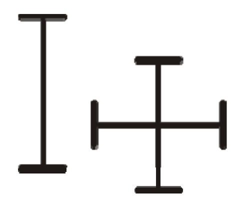
Nesimetrični odprti prerezi z majhno torzijsko togostjo.



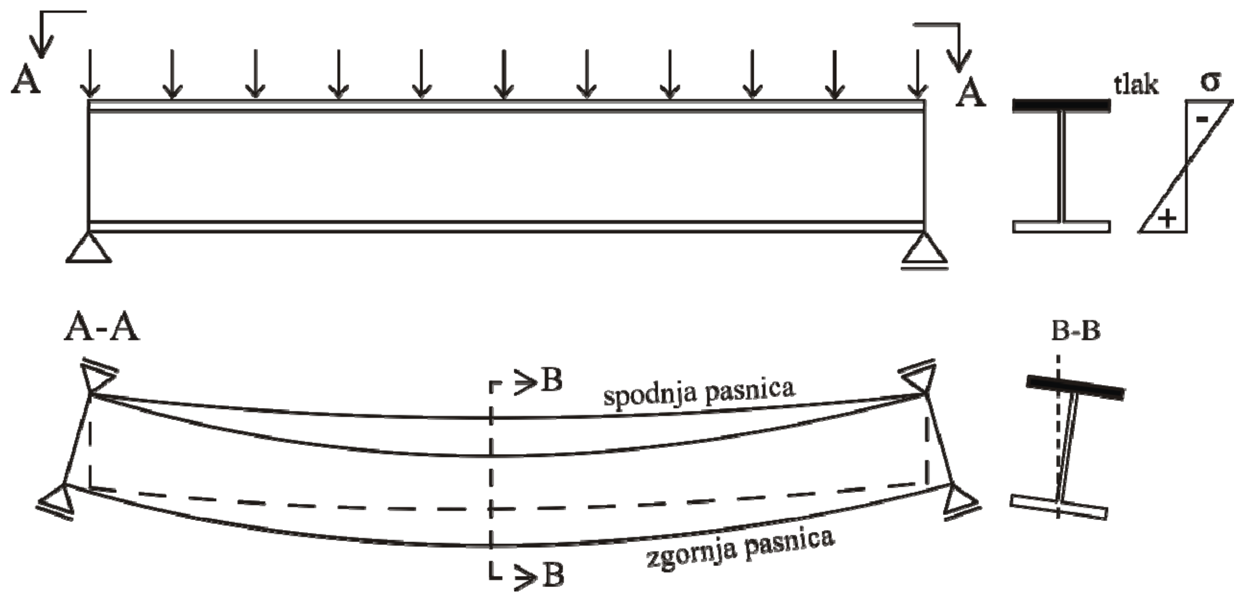
TORZIJSKI UKLON



Dvojnosimetrični odprti prerezi z majhno torzijsko togostjo.



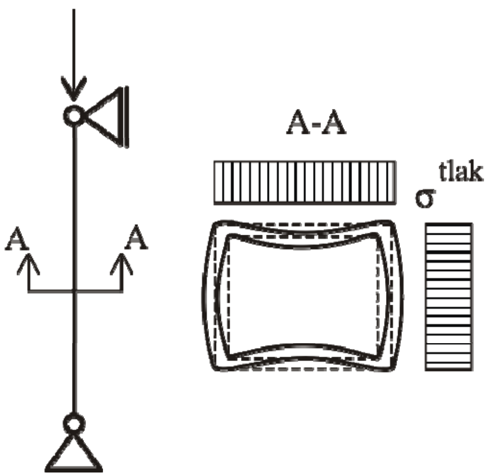
b) BOČNA ZVRNITEV UPOGIBNIH NOSILCEV



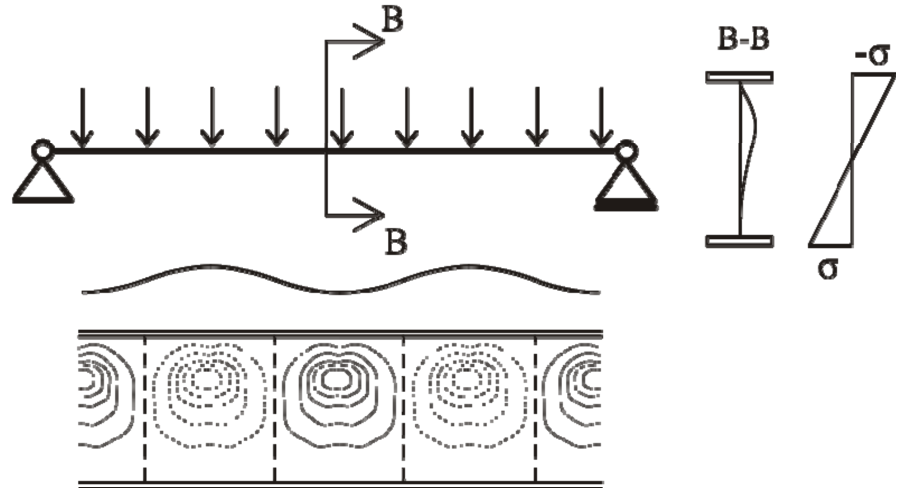
Uklon tlačne pasnice izven ravnine nosilca zaradi upogibnih momentov okoli močne osi → tlačna sila v zgornji pasnici.

c) <u>LOKALNO IZBOČENJE PLOČEVIN</u>

Vitka stojina: izbočenje v tlačeni coni



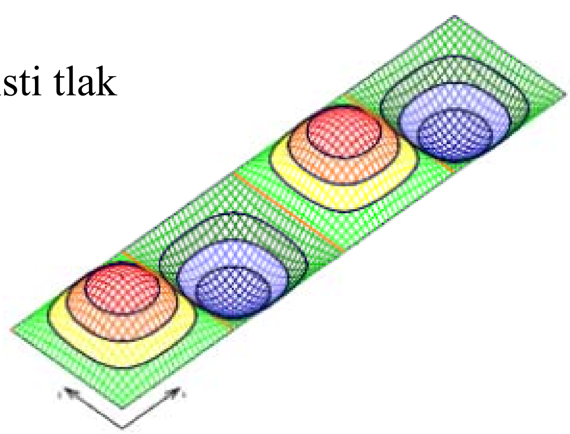
ČISTI TLAK



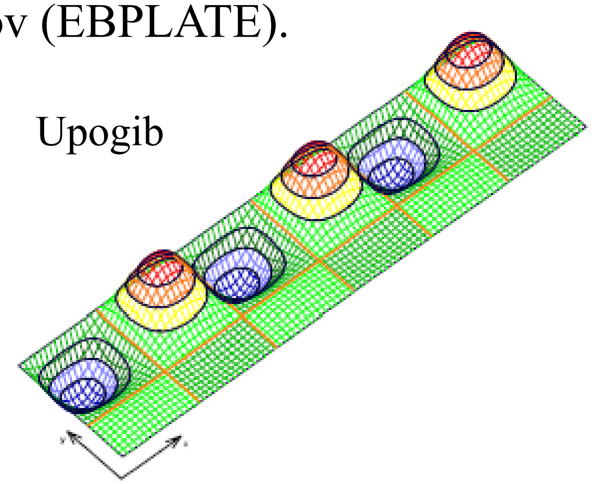
UPOGIB

- Primer simulacije izbočenja pločevine brez ojačitev obremenjene s konstantnim potekom tlačnih napetosti vzdolž krajših robov (EBPLATE).

Čisti tlak



Upogib

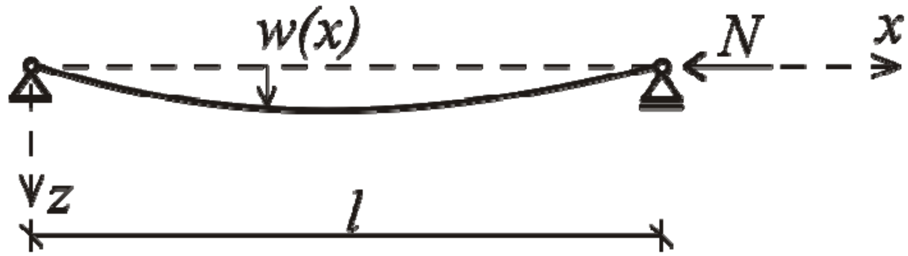


- Skupno pri vseh treh pojavih nestabilnosti:
  - vitek element;
  - tlačna napetost (N – uklon, M – bočna zvrnitev, σ - lokalno izbočenje).

- Možne so kombinacije zgoraj naštetih nestabilnosti.

7.2 Upogibni uklon tlačenih palic

Deformirana lega:



$$A, I, E, l$$

Ravnotežje momentov: $M(x) = N \cdot w(x)$

Konstitucijska zveza: $\dfrac{1}{R} \approx w'' = -\dfrac{M}{EI}$

$$M = -w'' EI$$

$$\boxed{EI\, w'' + N\, w = 0} \qquad \boxed{w'' + \omega^2\, w = 0} \qquad \omega^2 = \frac{N}{EI}$$

Nastavek za pomike, ki reši zgornjo homogeno dif. enačbo:

$$\boxed{w = A\sin(\omega x) + B\cos(\omega x)}$$

Robni pogoji:

$$x = 0 \qquad w = 0: \quad A\sin(\omega \cdot 0) + B\cos(\omega \cdot 0) = 0 \rightarrow B = 0$$

$$x = l \qquad w = 0: \quad A\sin(\omega \cdot l) = 0$$

netrivialna rešitev:

$$A \neq 0 \quad \rightarrow \quad \sin(\omega l) = 0 \rightarrow N_{cr}$$

$$\omega l = \{0, \pm\pi, \pm 2\pi, ...\} \ \text{ oz. } \ \omega l = n\pi, \quad n \in \mathbb{Z}$$

$$\omega l = n\pi \qquad\qquad n \in \{1,2,3...\}$$

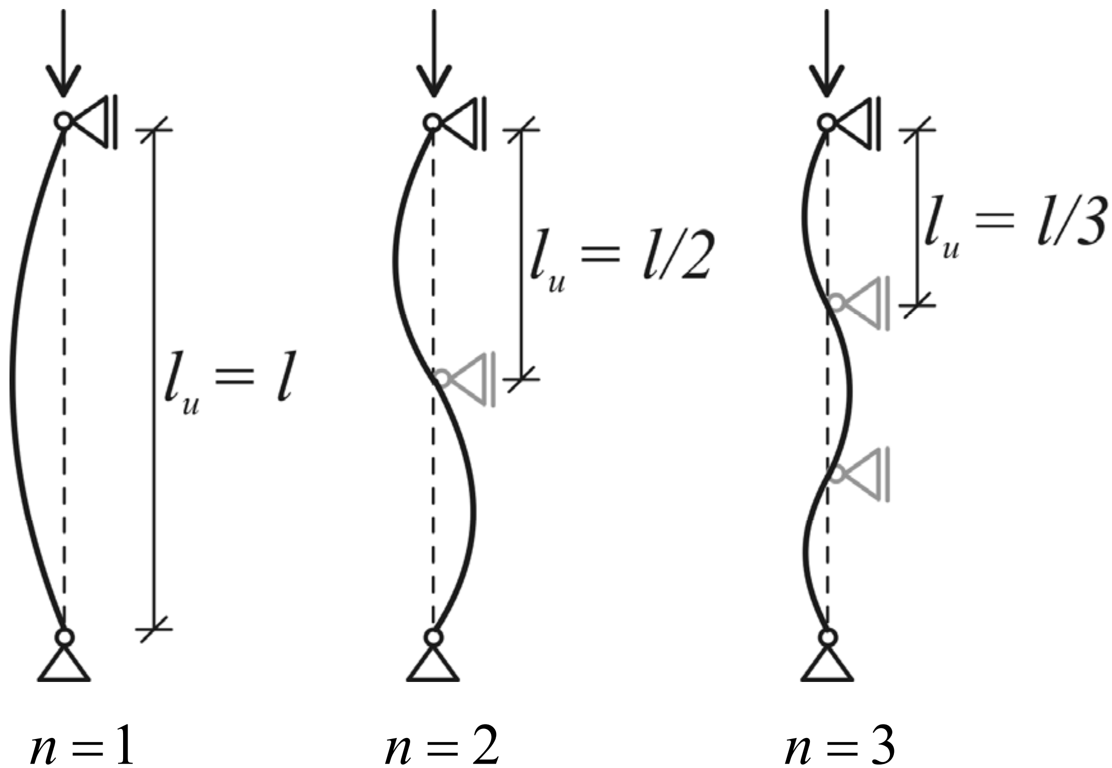
$$\omega^2 = \frac{n^2 \pi^2}{l^2} = \frac{N_{cr}}{EI} \quad \Rightarrow \quad \boxed{N_{cr} = n^2 \frac{\pi^2 EI}{l^2}}$$

Euler-jeva kritična sila

Uklonska dolžina l_u – razdalja med prevojnimi točkami (uklonske oblike)

$$l_u = \frac{l}{n}$$

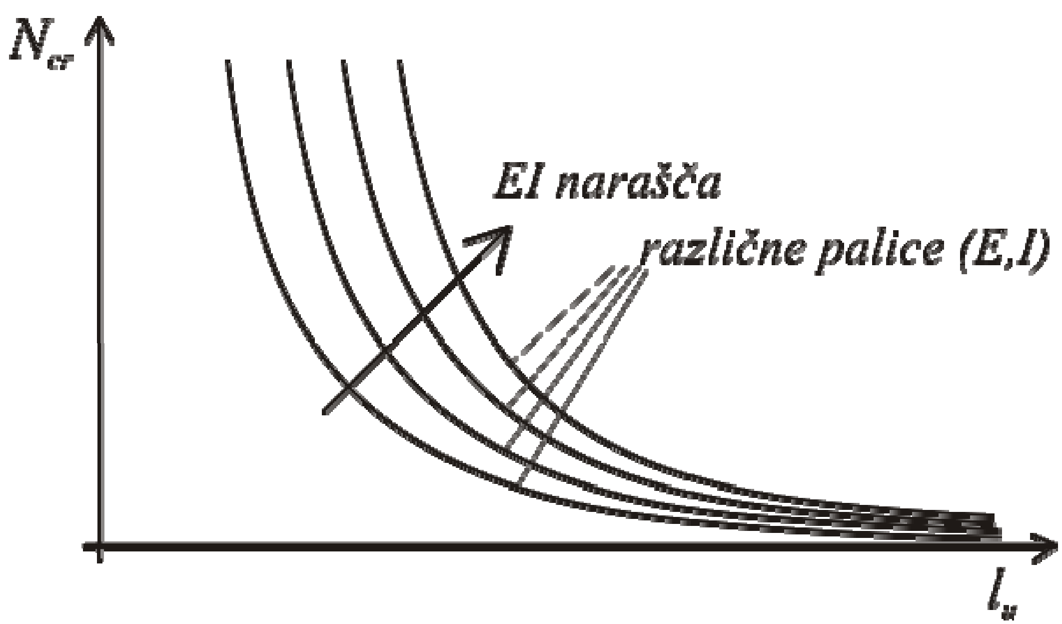
$$N_{cr} = \frac{\pi^2 EI}{l_u^2}$$



$$N_{cr} = n^2 \frac{\pi^2 EI}{l^2}$$

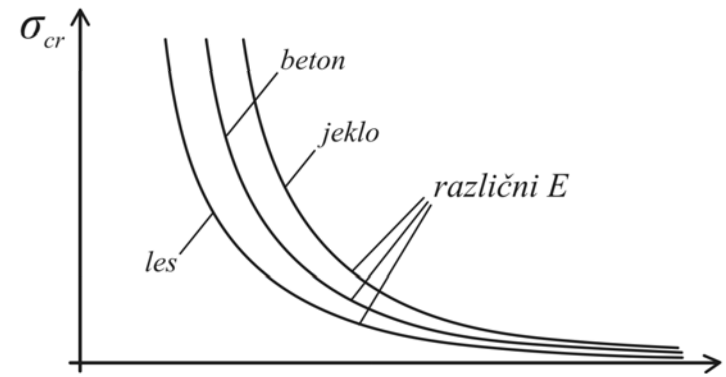
$$N_{cr,MIN} \Rightarrow n = 1, \quad l_u = l$$

$$N_{cr,MIN} = \frac{\pi^2 EI}{{l_u}^2}$$



$$\frac{N_{cr}}{A} = \sigma_{cr} = \frac{\pi^2 EI}{A l_u^2} = \frac{\pi^2 E}{\lambda^2} \Rightarrow \boxed{\sigma_{cr} = \frac{\pi^2 E}{\lambda^2}}$$

$$\frac{I}{A} = i^2 \qquad \lambda = \frac{l_u}{i}$$ **vitkost**



$$\frac{\sigma_{cr}}{f_y} = \frac{\pi^2 E}{\lambda^2 f_y}$$

$$\frac{N_{cr}}{A f_y} = \frac{\sigma_{cr}}{f_y} = \frac{\lambda_1^2}{\lambda^2} = \frac{1}{\bar{\lambda}^2} \Rightarrow$$

Povsem brezdimenzionalni zapis:

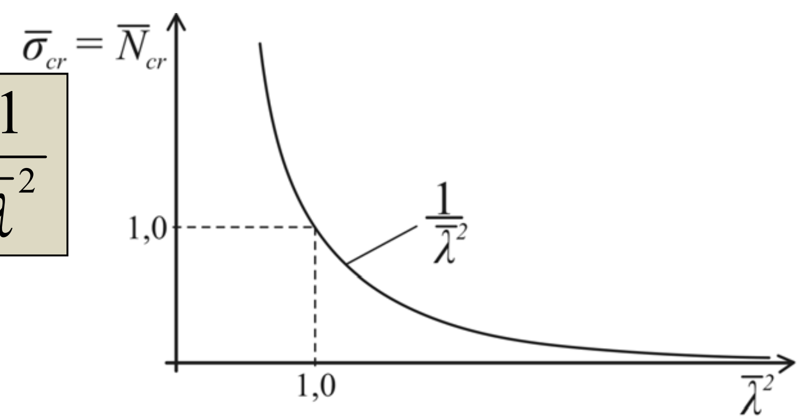
$$\boxed{\overline{N}_{cr} = \frac{N_{cr}}{N_{pl}} = \frac{1}{\bar{\lambda}^2}}$$

$$\bar{\lambda} = \sqrt{\frac{N_{pl}}{N_{cr}}}$$

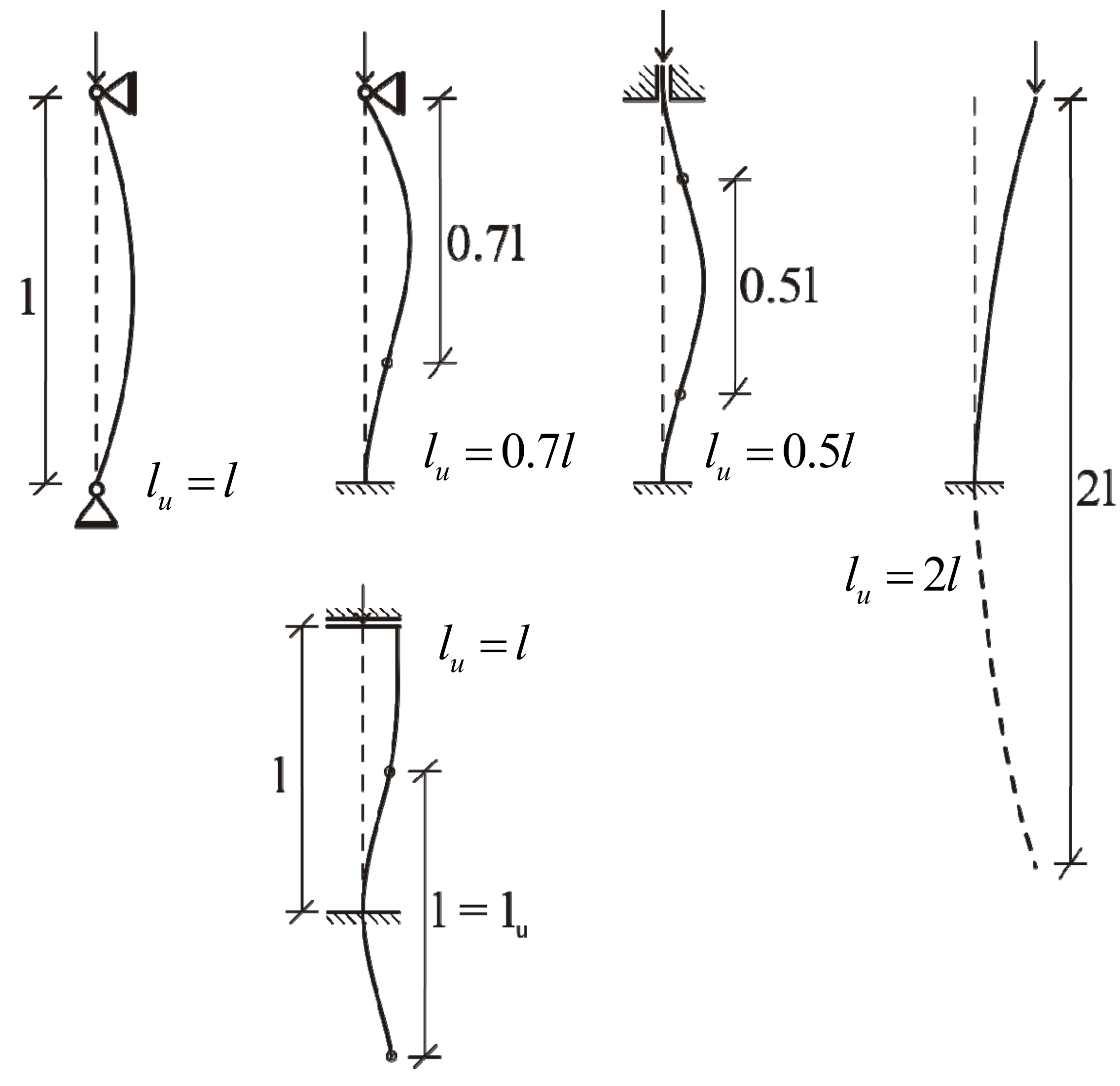
$$\lambda_1 = \pi \sqrt{\frac{E}{f_y}}$$

$$\bar{\lambda} = \frac{\lambda}{\lambda_1}$$

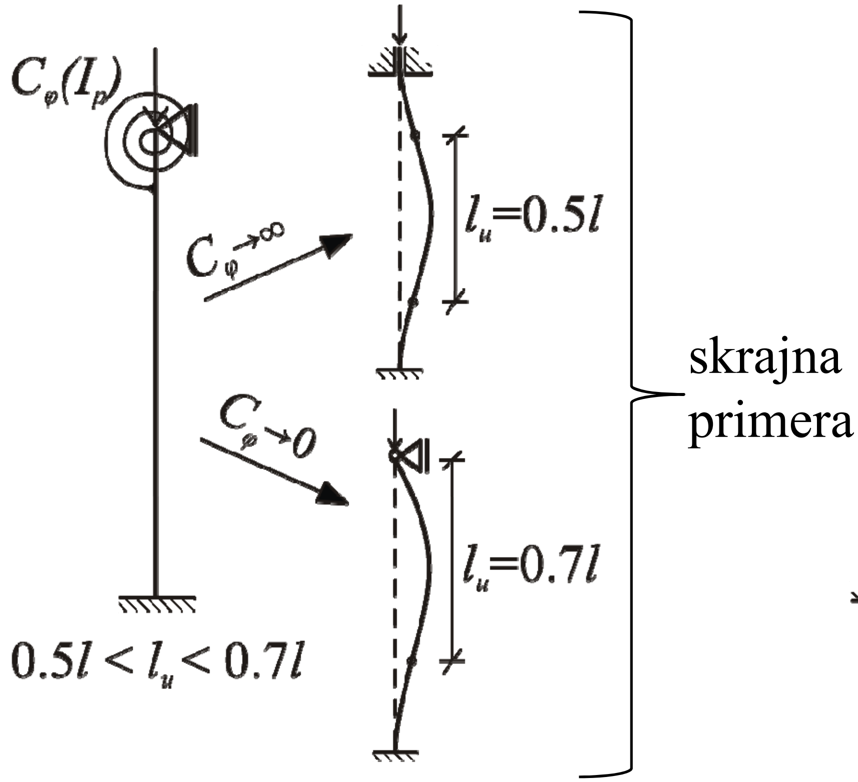
$$N_{pl} = A f_y$$

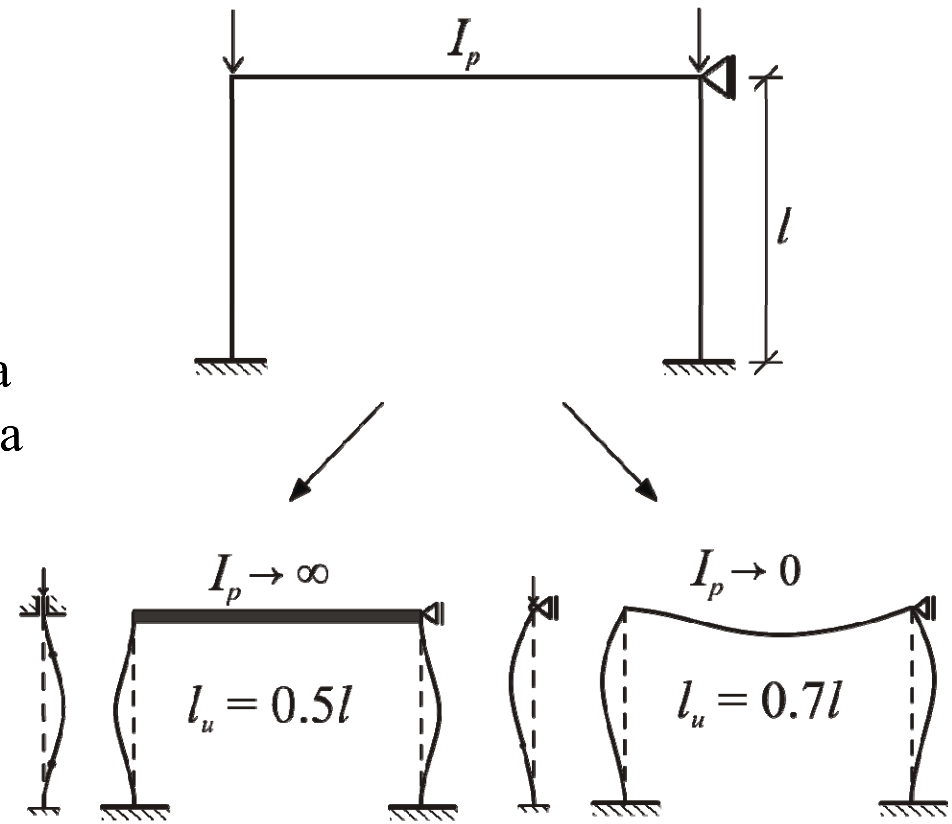


Uklonske dolžine



Uklonske dolžine

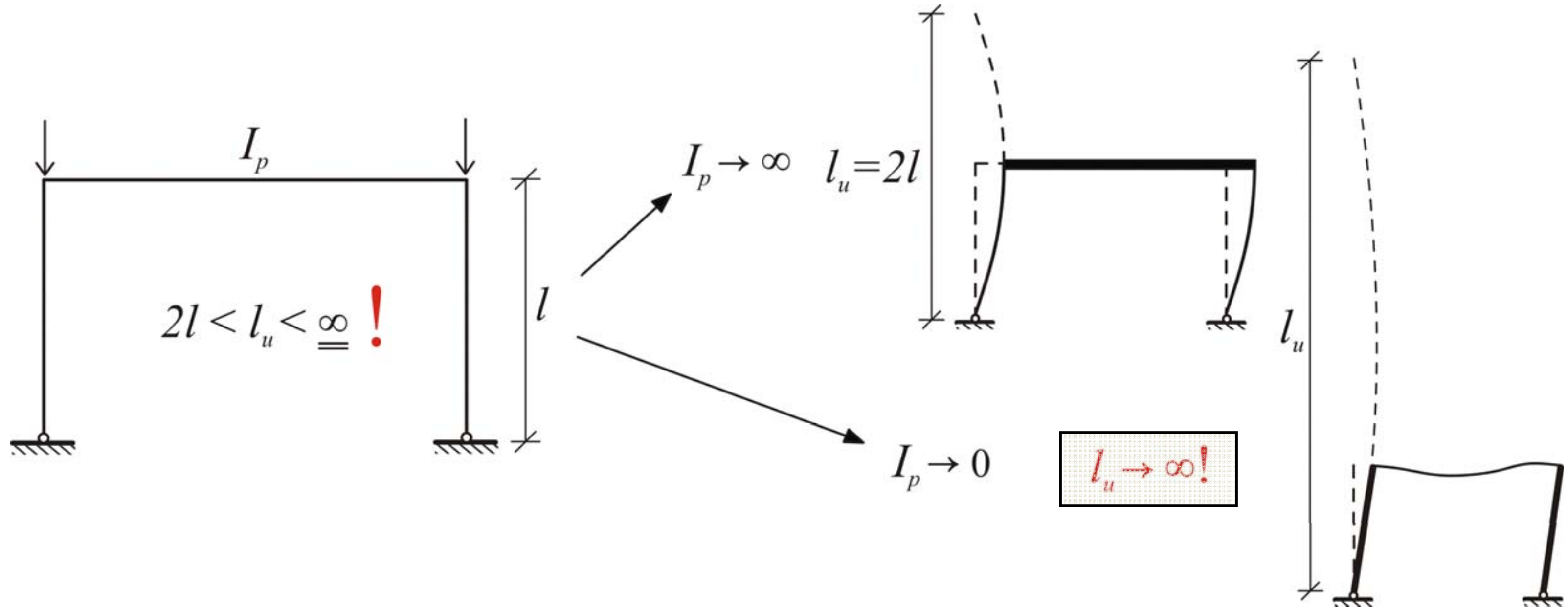




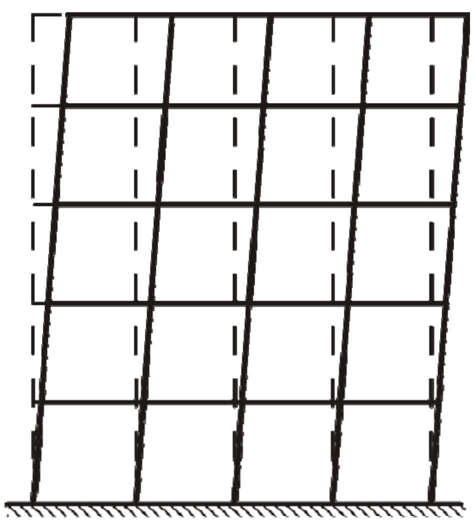
$$\frac{2\pi^2 EI}{l^2} \leq N_{cr} \leq \frac{4\pi^2 EI}{l^2}$$

$$N_{cr} = \frac{4\pi^2 EI}{l^2}$$

$$N_{cr} = \frac{2\pi^2 EI}{l^2}$$



Dimenzioniranje pomičnih okvirjev:

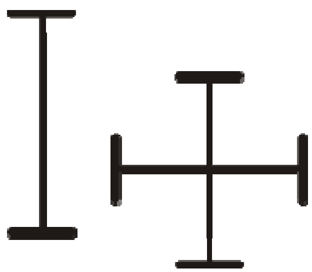


Priporočeno:

teorija drugega reda
+
nepopolnosti

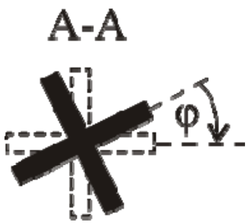
Torzijski uklon

Dvojnosimetrični odprti prerezi z majhno torzijsko togostjo:

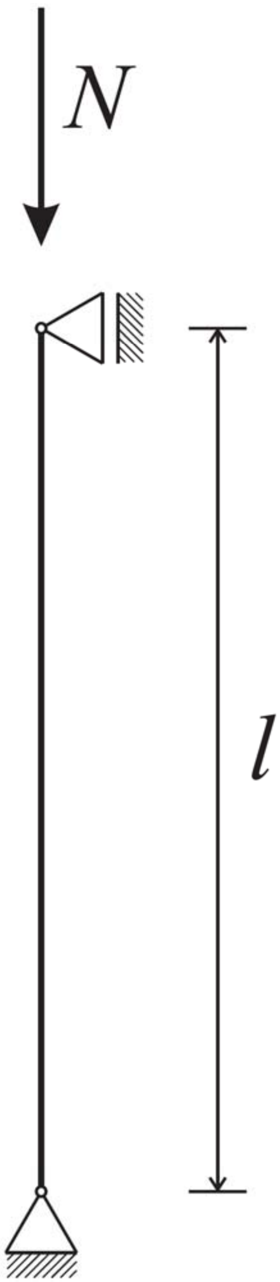


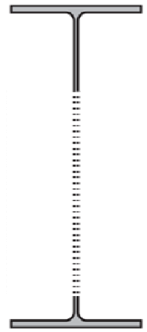
$$N_{cr} = \frac{A}{I_p}\left(\frac{\pi^2 EI_w}{l^2} + GI_t\right)$$

Enakomerna (Saint-Vernantova) torzija:

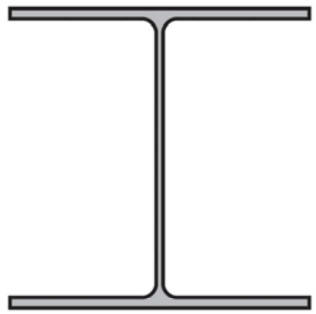


$$N_{cr} = \frac{A}{I_p} GI_t$$

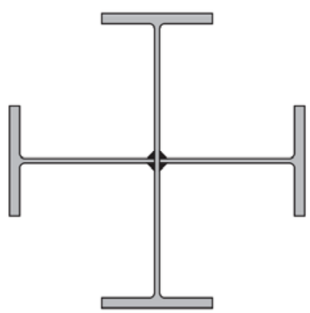




IPE 500			
l [m]	$N_{cr,y}$ [kN]	$N_{cr,z}$ [kN]	$N_{cr,T}$ [kN]
10	9990	444	2259
18	3083	137	1846
28	1274	57	1738



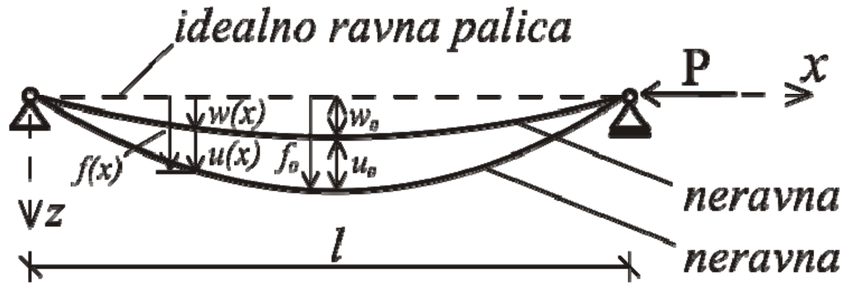
HEB 300			
l [m]	$N_{cr,y}$ [kN]	$N_{cr,z}$ [kN]	$N_{cr,T}$ [kN]
10	5217	1774	8146
18	1610	548	7078
28	665	226	6798



2 HEA 800			
l [m]	$N_{cr,y}$ [kN]	$N_{cr,z}$ [kN]	$N_{cr,T}$ [kN]
10	65503	65503	15588
18	20217	20217	10845
28	8355	8355	9602

Upoštevanje začetne geometrijske nepopolnosti

NERAVNA PALICA



$w(x)$

$f(x) = w(x) + u(x)$

V začetni legi neravne palice - brez napetosti

$$f(x) = w(x) + u(x) \quad \text{odmik od osi } x$$

$$x = l/2: \quad f_0 = w_0 + u_0$$

Znano:

$w(x)$: začetna neravna geometrija palice

$$w(x) = w_0 \sin\frac{\pi x}{l}$$

P: tlačna osna sila

Iščemo:

$u(x)$ oziroma $f(x)$

pri znani $w(x)$ in P

Ravnotežna enačba za neravno palico

$$\boxed{EI\,u_{,xx} + P\cdot(u+w) = 0}$$ momentni pogoj

Notranji moment (w ne sodeluje pri ukrivljenosti, ker v začetni legi palica ni obremenjena).

Zunanja obtežba

$$u_{,xx} + \omega^2 (u+w) = 0 \qquad ;\omega^2 = \frac{P}{EI}$$

$$u_{,xx} + \omega^2 u = -\omega^2 w_0 \sin\frac{\pi x}{l}$$

Za *u(x)* predpostavimo rešitev (za nehomogeno dif. enačbo, ki avtomatično izpolni tudi robne pogoje):

$$u(x) = A\sin\frac{\pi x}{l}$$

$$u_{,xx} = -\frac{\pi^2}{l^2} A\sin\frac{\pi x}{l}$$

$$-\frac{\pi^2}{l^2} A\sin\frac{\pi x}{l} + \omega^2 A\sin\frac{\pi x}{l} = -\omega^2 w_0 \sin\frac{\pi x}{l}$$

$$\sin\frac{\pi x}{l}\left(A\left(\omega^2 - \frac{\pi^2}{l^2}\right) + \omega^2 w_0 \right) = 0$$

$$A = -\frac{\omega^2 w_0}{\omega^2 - \frac{\pi^2}{l^2}} = -\frac{w_0}{1 - \frac{\pi^2}{l^2 \omega^2}} = \frac{w_0}{\frac{\pi^2}{l^2 \omega^2} - 1} = \frac{w_0}{\frac{P_{cr}}{P} - 1}$$

$$A = \frac{w_0}{\frac{P_{cr}}{P} - 1} \quad , P_{cr} = \frac{\pi^2 EI}{l^2}$$

$$u(l/2) = u_0 = A \sin\frac{\pi}{l}\frac{l}{2} = A \sin\frac{\pi}{2} = A \cdot 1 = A \;\Rightarrow\; u_0 = A$$

$$f_0 = u_0 + w_0 = w_0 \left(\frac{1}{\frac{P_{cr}}{P} - 1} + 1 \right) = \frac{w_0}{1 - \frac{P}{P_{cr}}}$$

$$\boxed{k_\delta = \frac{1}{1 - \frac{P}{P_{cr}}}}$$

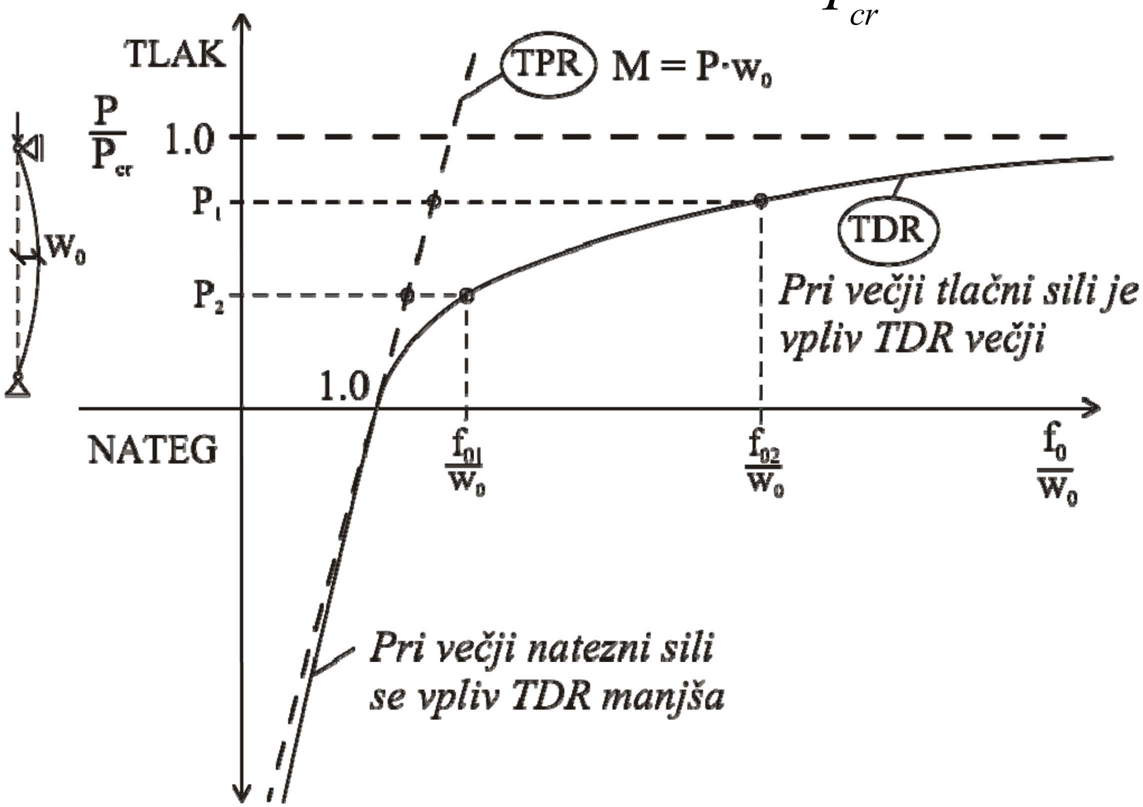
Amplifakcijski
koeficient TDR

$$\Rightarrow \quad \boxed{f_0 = w_0 \cdot k_\delta}$$

$$\boxed{\begin{aligned} w^{II} &\approx w^{I} k_\delta \\ M^{II} &\approx M^{I} k_\delta \end{aligned}}$$

Približni račun količin po TDR

$$k_\delta = \frac{f_0}{w_0} = \frac{1}{1 - \dfrac{P}{P_{cr}}} \quad \Rightarrow \quad \boxed{\frac{P}{P_{cr}} = 1 - \frac{1}{\dfrac{f_0}{w_0}}}$$

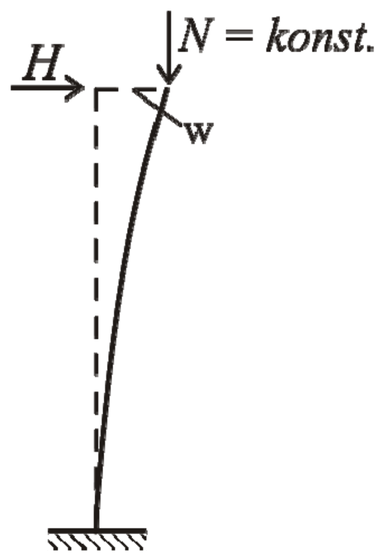


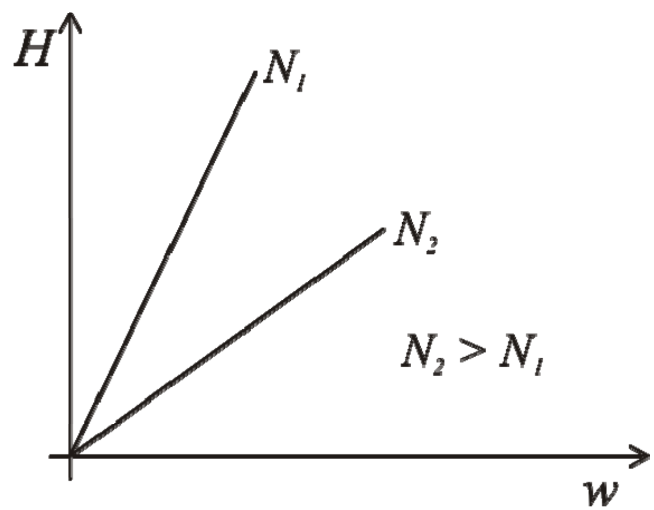
$$\boxed{\frac{f_{02}}{f_{01}} > \frac{P_2}{P_1}}$$

Pomiki naraščajo hitreje kot sile.

Konstantna osna sila

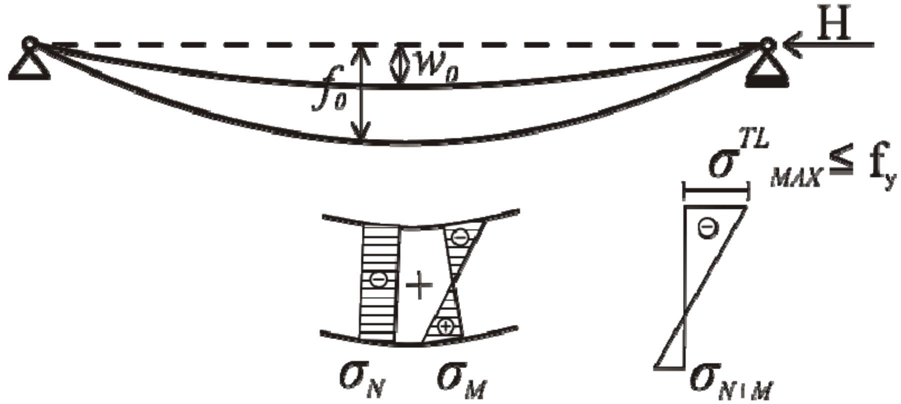
Vrednost k_δ **konstantna**: $$k_\delta = \frac{1}{1 - \frac{N}{N_{cr}}}$$





Vpliv TDR se kaže samo v **zmanjšani togosti**.

Vpliv nepopolnosti na nosilnost



$$f_0 = w_0\, k_\delta$$

$$M^{II} = N \cdot f_0 = N\, w_0\, k_\delta$$

$$\sigma^{TL}_{MAX} = \frac{N}{A} + \frac{M^{II}}{W_{el}} \leq f_y \quad / : f_y$$

$$\frac{N}{N_{pl}} + \frac{N\, w_0\, k_\delta}{M_{el}} \leq 1.0 \qquad N_{pl} = A\, f_y$$

$$M_e = W_{el}\, f_y$$

$$\frac{N}{N_{pl}}+\frac{N\,w_0\,k_\delta\,A}{W_{el}\,f_y\,A}\le 1.0 \quad \Longleftarrow \quad \eta=\frac{w_o\,A}{W_{el}}$$

$$\frac{N}{N_{pl}}+\frac{N\,\eta}{N_{pl}\left(1-\dfrac{N}{N_{cr}}\right)}\le 1.0 \quad \Longleftarrow \quad \bar{N}=\frac{N}{N_{pl}}$$

$$\bar{N}+\bar{N}\,\eta\frac{1}{\left(1-\bar{N}\bar{\lambda}^2\right)}\le 1.0 \quad \Longleftarrow \quad \bar{\lambda}^2=\frac{N_{pl}}{N_{cr}}$$

$$\eta\frac{1}{\left(1-\bar{N}\bar{\lambda}^2\right)}\le\left(\frac{1}{\bar{N}}-1\right)$$

$$\boxed{\eta\le\left(\frac{1}{\bar{N}}-1\right)\left(1-\bar{N}\bar{\lambda}^2\right)}$$

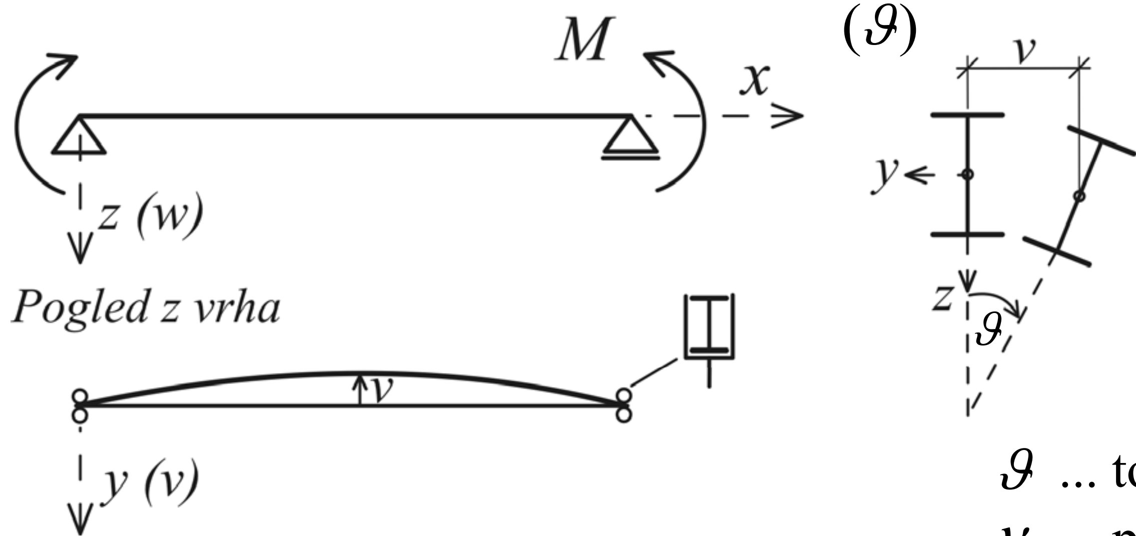
pri “=” začetek plastifikacije

pri “<” začetna nepopolnost η **ne povzroči začetka plastifikacije**

$$w_0=f\left(N\right)\ \text{ali}\ N=f\left(w_0\right)$$

7.3 Bočna zvrnitev

Elastičen in **idealno raven nosilec**.



ϑ ... torzijski zasuk

v ... premik težišča v y smeri

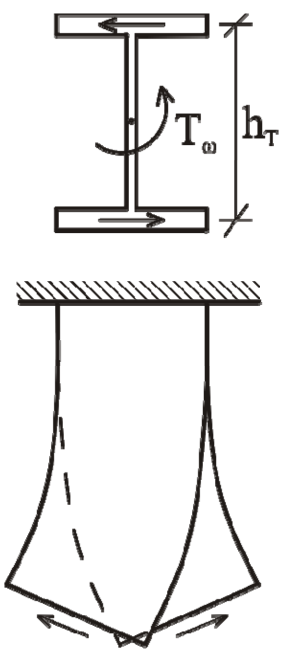
Robni pogoji (VILIČASTA PODPORA):

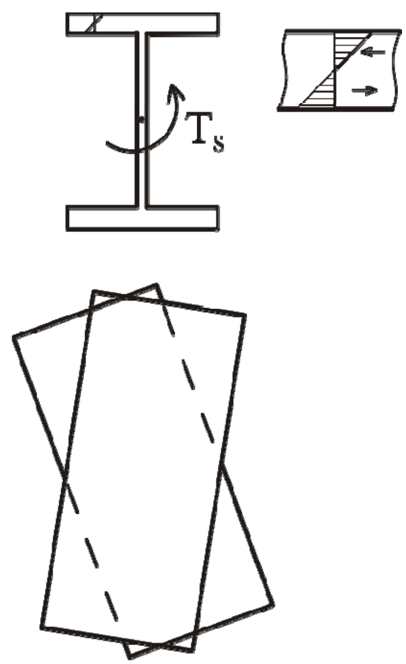
$$\vartheta(0)=0 \qquad \vartheta(L)=0$$
$$\vartheta_{,xx}(0)=0 \qquad \vartheta_{,xx}(L)=0$$

$$v(0)=0 \qquad v(L)=0$$
$$v_{,xx}(0)=0 \qquad v_{,xx}(L)=0$$

Ravnotežna enačba za bočno zvrnitev nosilca s konstantnim potekom momentov

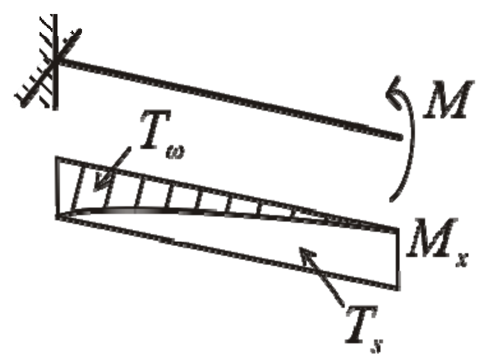
$$\underbrace{EI_\omega \vartheta_{,xxxx}}_{\text{delež ovirane torzije}} - \underbrace{GI_t \vartheta_{,xx}}_{\text{delež enakomerne torzije (čista, Saint-Venantova) - samo strigi.}} - \underbrace{\frac{M^2}{EI_z}\vartheta}_{\text{obtežba}} = 0$$





Torzijski moment

$$M_x = T_\omega + T_s$$

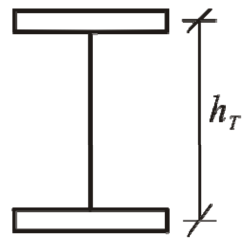


$$G = \frac{E}{2(1+\nu)}$$ ... strižni modul $\left(G_{JEKLO} = 8100\, kN/cm^2, \nu = 0.3\right)$

I_ω = vzbočitveni vztrajnostni moment (tabele c_M)

"vztrajnostni moment ovirane torzije"

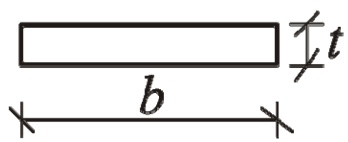
$$I_\omega = I_z \frac{h_T^2}{4}$$ za dvojnosimetrične I - profile



$I_z = I_{MIN}$ (šibka os)

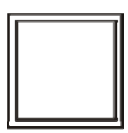
I_t = Saint–Venantov torzijski vztrajnostni moment

$$I_t = \sum_{i=1}^{n} \left(b_i t_i^3 / 3\right)$$



Približna formula za odprte prereze, ki ne upošteva globalnega sodelovanja elementov (10 – 30%).

Škatlasti prerezi



I_t zelo velik → ni nevarnosti bočne zvrnitve

POZOR!

Rešitev ravnotežne enačbe

Elastični kritični moment bočne zvrnitve

$$M_{cr} = \frac{\pi}{l}\sqrt{E\,I_z\,G\,I_t\left(1+\frac{\pi^2\,E\,I_\omega}{l^2\,G\,I_t}\right)} \qquad ;\ \chi = \frac{\pi^2\,E\,I_\omega}{l^2\,G\,I_t}$$

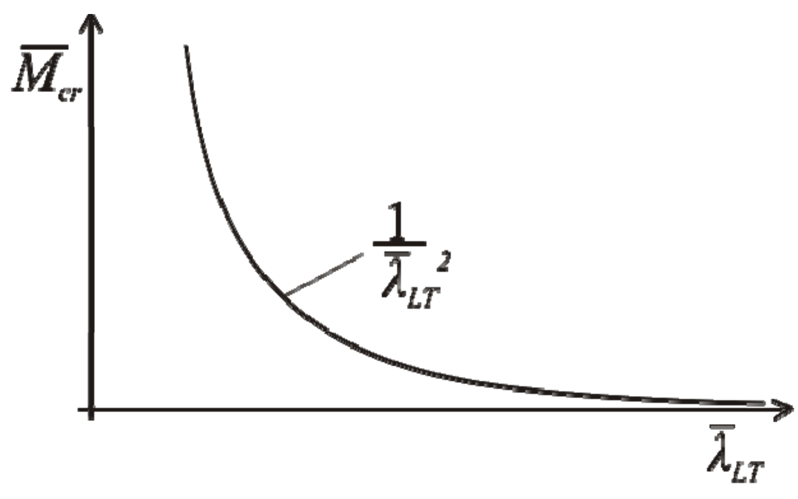
Vitkost:

$$\bar{\lambda}_{LT} = \sqrt{\frac{M_{el}\left(M_{pl}\right)}{M_{cr}}} = \sqrt{\frac{1}{\bar{M}_{cr}}}$$

$$M_{el} = W_{el,y}\,f_y \quad \rightarrow 3.\ \text{R. K.}$$

$$M_{pl} = W_{pl,y}\,f_y \quad \rightarrow 1.,\ 2.\ \text{R. K.}$$

$$\bar{M}_{cr} = \frac{1}{\bar{\lambda}_{LT}^2}$$



Možna primera v praksi

$$\chi = \frac{\pi^2 E I_\omega}{l^2 G I_t}$$

- <u>prevladuje ovirana torzija</u> $\boxed{\chi >> 1}$

Za I – profile velja:

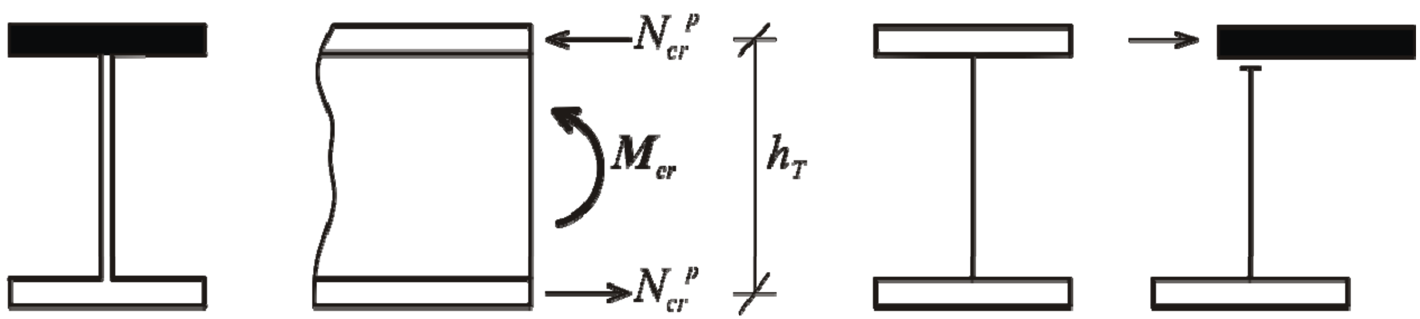
$$M_{cr} = \frac{\pi}{l}\sqrt{\frac{\pi^2 E^2 I_z I_\omega}{l^2}} = \frac{\pi^2}{l^2} E \sqrt{I_z I_\omega} = \frac{\pi^2}{l^2} \frac{E I_z h_T}{2} \approx \frac{\pi^2}{l^2} E I_p h_T = N_{cr}^p h_T$$

$I_z \approx 2 I_p$ ... zanemarimo vztrajnostni moment stojine okoli šibke osi.

Metoda tlačene pasnice

- Zanemarjen delež enakomerne torzije;
- uporabna za hitro kontrolo;
- vsebovana v predpisih.

Obnašanje tlačenega pasu pasnice, kot da le ta ne sodeluje z ostalim delom profila.



$$M_{cr} = h_T \, N_{cr}^{p}$$

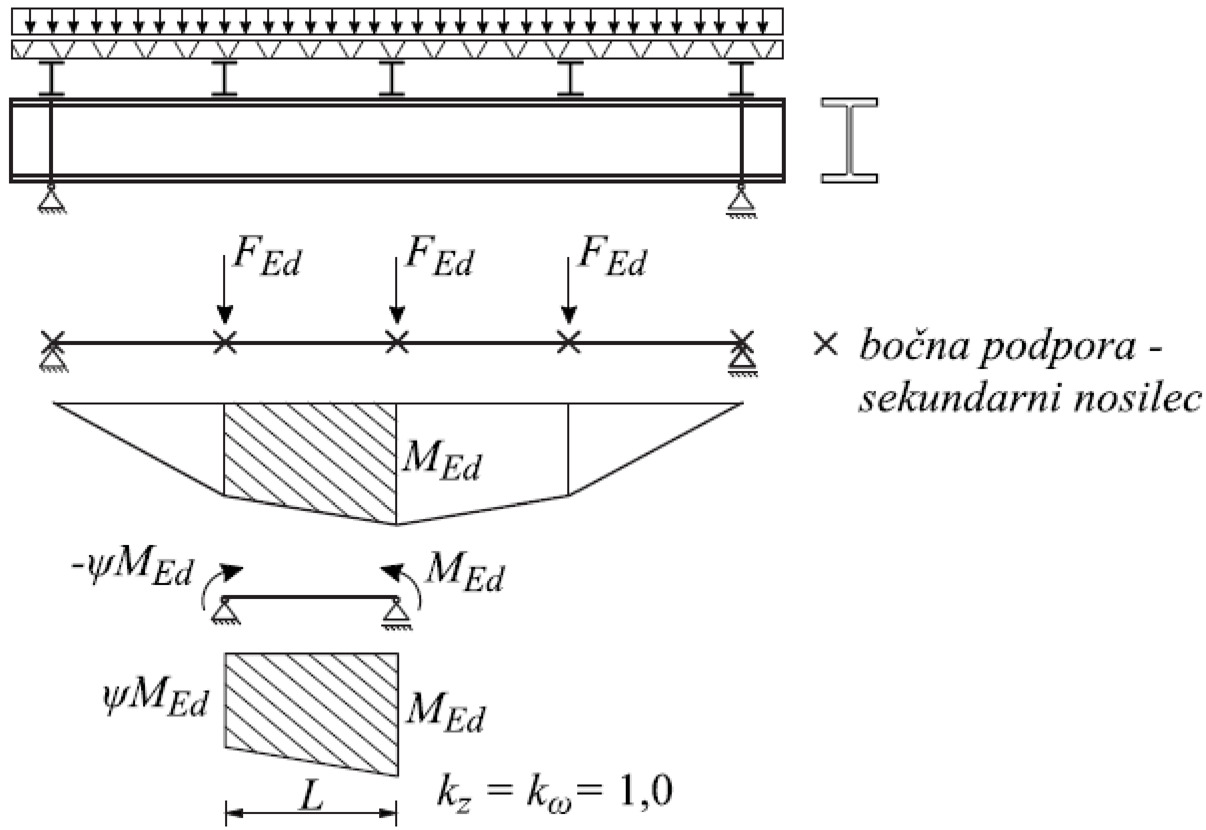
- prevladuje neovirana torzija $\boxed{\chi << 1}$

$$M_{cr} = \frac{\pi}{l}\sqrt{EI_z GI_t}$$ ... upoštevamo samo enakomerno torzijo

Primerno za:

- polni profili;
- veliki razponi.

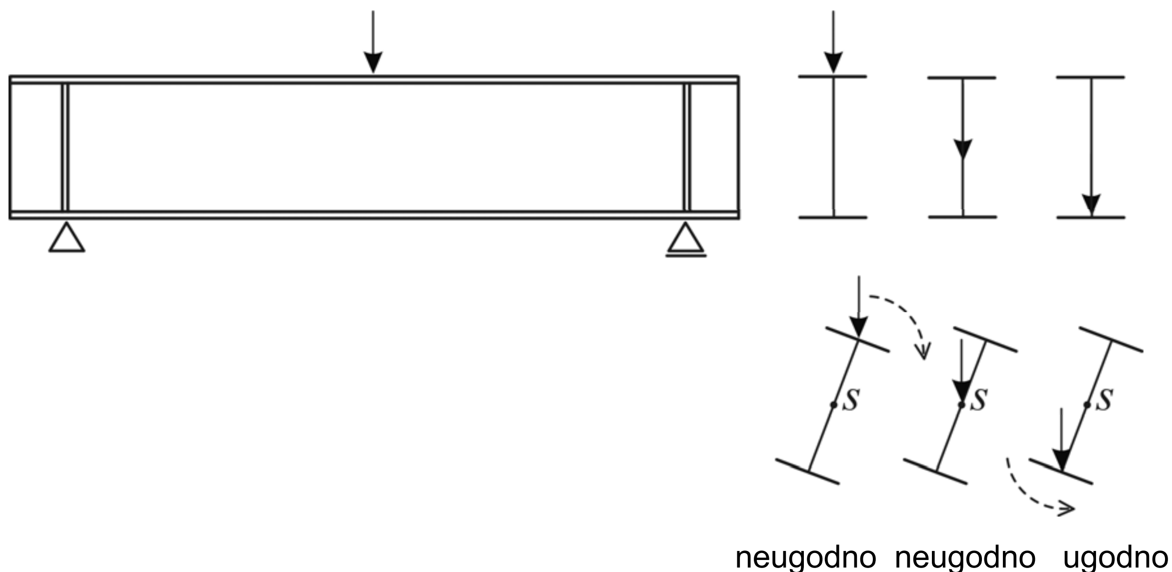
Kontrola bočne zvrnitve za nosilce z vmesnimi bočnimi podporami




yvonne mARTejevs

- Vpliv oblike momentne linije;

- vpliv prijemališča obtežbe;

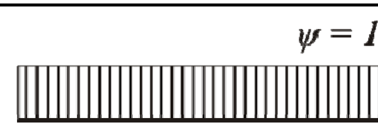
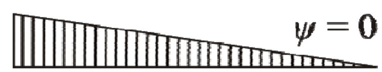


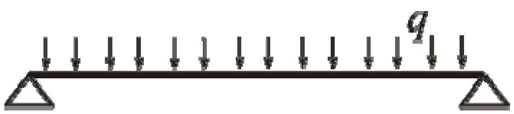
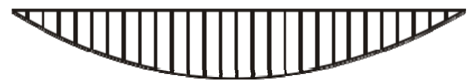
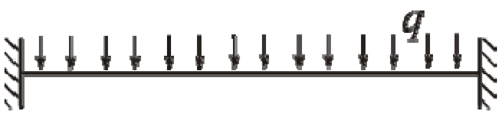
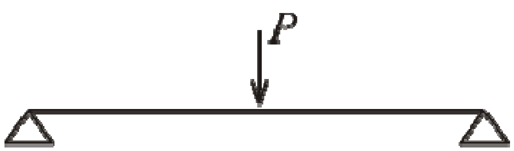
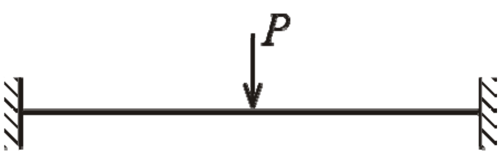
- vpliv robnih pogojev.

Nekonstanten potek upogibnega momenta

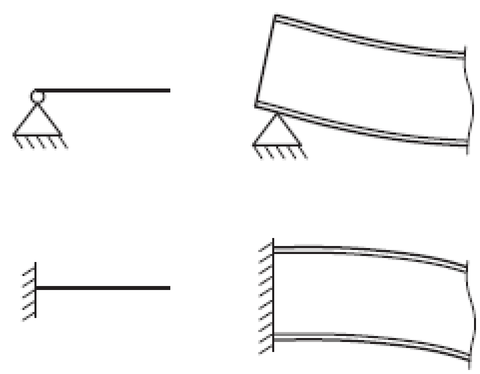
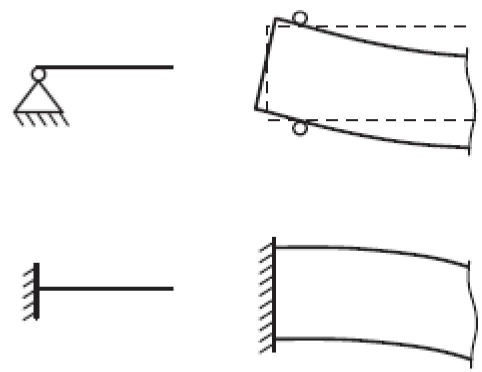
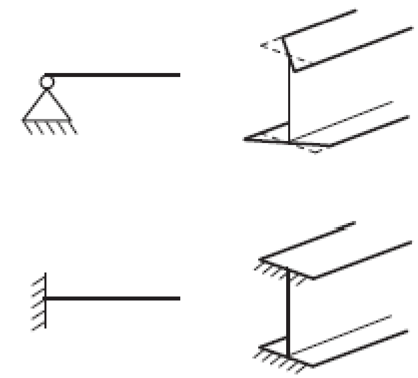
$$M_{cr}\left(M \neq konst.\right) = C_1 \cdot M_{cr}\,(M = konst.)$$

$C_1 \geq 1.0$ ... koeficient oblike momentne linije

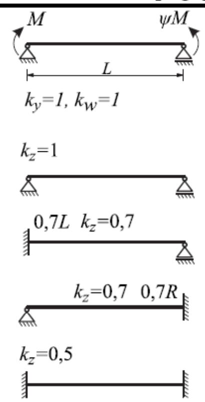
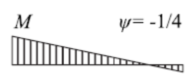
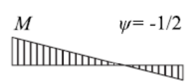
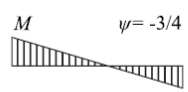
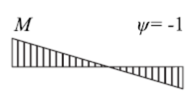
OBTEŽBA IN ROBNI POGOJI	POTEK MOMENTOV	C_1
		1,00
		1,31
		1,77
		2,33

OBTEŽBA IN ROBNI POGOJI	POTEK MOMENTOV	C_1
		1,13
		2,58
		1,35
		1,68

Pr. 3–1. Uklonski koeficienti

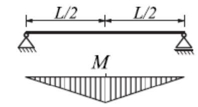
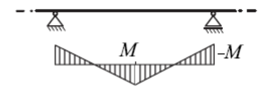
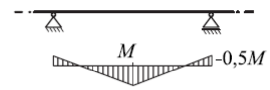
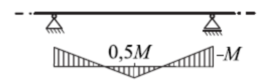
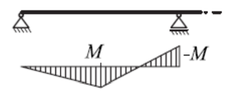
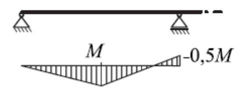
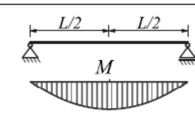
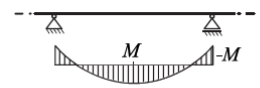
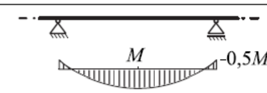
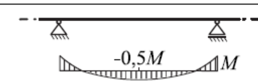
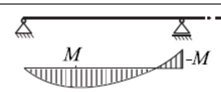
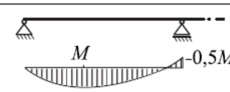
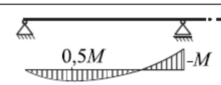
k_y – uklon okoli močne osi		k_z – uklon okoli šibke osi		k_ω – uklon okoli močne osi	
					
	$k_y = 1{,}0$		$k_z = 1{,}0$		$k_\omega = 1{,}0$
	$k_y = 0{,}7$		$k_z = 0{,}7$		$k_\omega = 0{,}7$
	$k_y = 0{,}5$		$k_z = 0{,}5$		$k_\omega = 0{,}5$

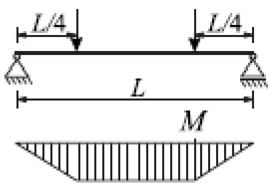
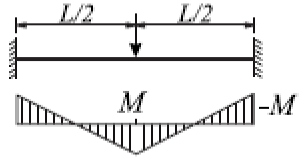
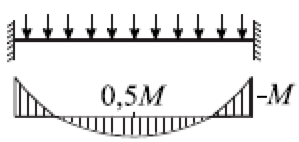
Pr. 3–2. Vrednosti koeficienta C_1 – nosilec obremenjen z robnimi momenti

obtežba in robni pogoji	potek momentov	k_z [1]	C_1
		1,0	1,00
		0,7L	1,02
		0,7R	1,02
		0,5	1,00
		1,0	1,14
		0,7L	1,21
		0,7R	1,11
		0,5	1,14
		1,0	1,31
		0,7L	1,48
		0,7R	1,21
		0,5	1,31
		1,0	1,52
		0,7L	1,85
		0,7R	1,33
		0,5	1,52
		1,0	1,77
		0,7L	2,33
		0,7R	1,45
		0,5	1,75
		1,0	2,05
		0,7L	2,83
		0,7R	1,58
		0,5	2,00
		1.0	2,33
		0,7L	3,08
		0,7R	1,71
		0,5	2,23
		1,0	2,55
		0,7L	2,59
		0,7R	1,83
		0,5	2,35
		1,0	2,56
		0,7L	1,92
		0,7R	1,92
		0,5	2,22

[1] 0,7L = levi rob vpet, 0,7R = desni rob vpet

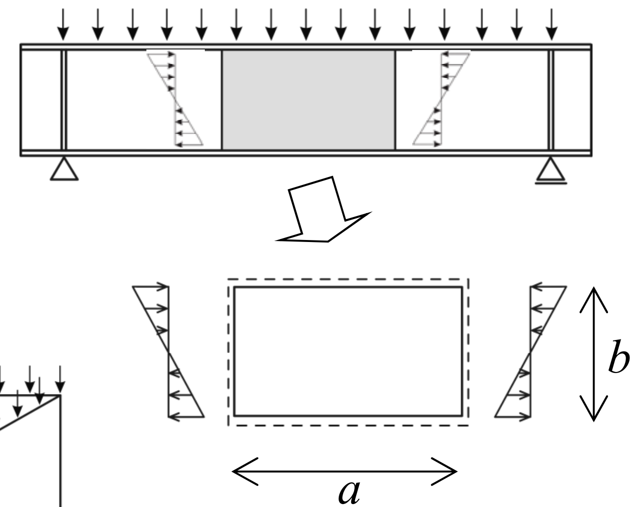
Pr. 3–3. Vrednosti koeficientov C_1 in C_2 - prečno obremen nosilec

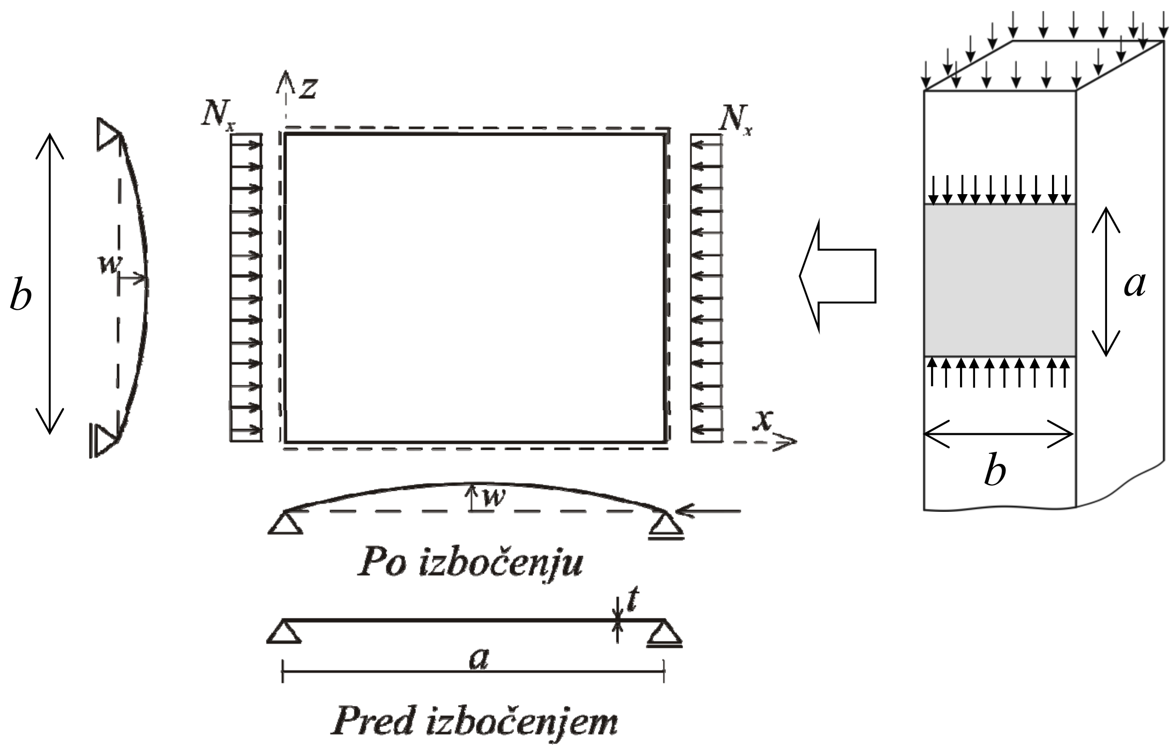
potek momentov in robni pogoji	k_y	k_z	C_1	C_2
	1	1	1,35	0,55
	1	0,5	1,03	0,45
	1	1	1,70	0,50
	1	1	1,60	0,50
	1	1	2,50	0,50
	1	1	1,52	0,50
	1	1	1,48	0,50
	1	1	1,13	0,46
	1	0,5	0,95	0,41
	1	1	1,39	0,50
	1	1	1,27	0,50
	1	1	2,50	0,50
	1	1	1,29	0,50
	1	1	1,23	0,50
	1	1	2,50	0,50

potek momentov in robni pogoji	k_y	k_z	C_1	C_2
	1	1	2,40	0,50
	1	1	1,04	0,43
	1	0,5	0,92	0,40
	0,5	1	1,68	1,39
	0,5	0,5	0,94	0,76
	0,5	1	2,58	1,56
	0,5	0,5	1,49	0,90

7.4 Lokalno izbočenje vitkih pločevin

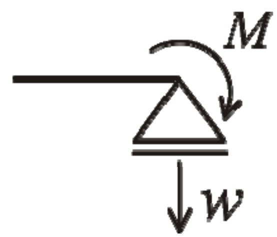
- ❑ **Elastična** in **idealno ravna pločevina**;
- ❑ konstantna debelina pločevine *t*;
- ❑ vrtljivo podprta pločevina na vseh robovih;
- ❑ obremenitev v lastni ravnini.





Ravnotežna enačba po teoriji drugega reda (linearna teorija izbočenja)

Robni pogoj:



$$w = 0$$
$$M = 0 \quad \rightarrow \quad w_{,xx} = 0$$

Ravnotežje v deformirani legi (TDR):

$$\boxed{w_{,xxxx} + 2w_{,xxyy} + w_{,yyyy} + \frac{N}{D} w_{,xx} = 0}$$

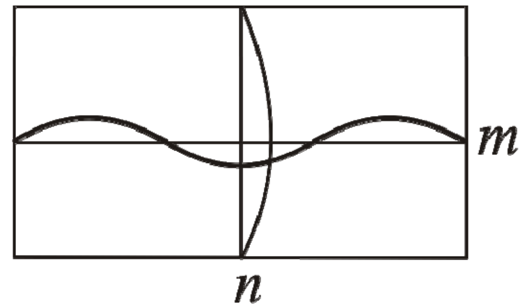
Analogija z nosilcem

$$w_{,xxxx} + \frac{N}{EI} w_{,xx} = 0$$

$$D = \frac{E\,t^3}{12\left(1-\nu^2\right)}$$ ... upogibna togost na enoto širine plošče.

Predpostavimo rešitev za w, ki avtomatično zadošča robnim pogojem:

$$w(x,y) = A\sin\frac{m\pi x}{a}\cdot\sin\frac{n\pi y}{b}$$



m ... število polvalov v vzdolžni smeri.

n ... število polvalov v prečni smeri.

$$\bar{m} = \frac{m\pi}{a} \qquad \bar{n} = \frac{n\pi}{b}$$

$$w_{,xxxx} = \bar{m}^4\, w$$

$$w_{,yyyy} = \bar{n}^4\, w$$

$$w_{,xxyy} = \bar{m}^2\,\bar{n}^2\, w$$

$$w_{,xx} = -\bar{m}^2\, w$$

$$w\left(\bar{m}^4 + 2\bar{m}^2\,\bar{n}^2 + \bar{n}^4\right) = \bar{m}^2\, w\frac{N}{D}$$

$$w\left(\bar{m}^4 + 2\bar{m}^2\,\bar{n}^2 + \bar{n}^4 - \bar{m}^2\frac{N}{D}\right) = 0$$

Netrivialna rešitev: $w \neq 0 \quad \rightarrow \quad \left(\bar{m}^2 + \bar{n}^2\right)^2 - \bar{m}^2\frac{N}{D} = 0$

$$N_{cr} = \frac{\left(\overline{m}^2 + \overline{n}^2\right)}{\overline{m}^2} D \qquad ; \alpha = \frac{a}{b}$$

$$N_{cr} = D \frac{\pi^2}{b^2} \left(\frac{m}{\alpha} + n^2 \frac{\alpha}{m} \right)^2$$

$$\sigma_{cr} = \frac{N_{cr}}{t} = \underbrace{\left(\frac{m}{\alpha} + n^2 \frac{\alpha}{m} \right)^2}_{k_\sigma} \underbrace{\frac{\pi^2 E t^2}{12\left(1-\nu^2\right) b^2}}_{\sigma_E}$$

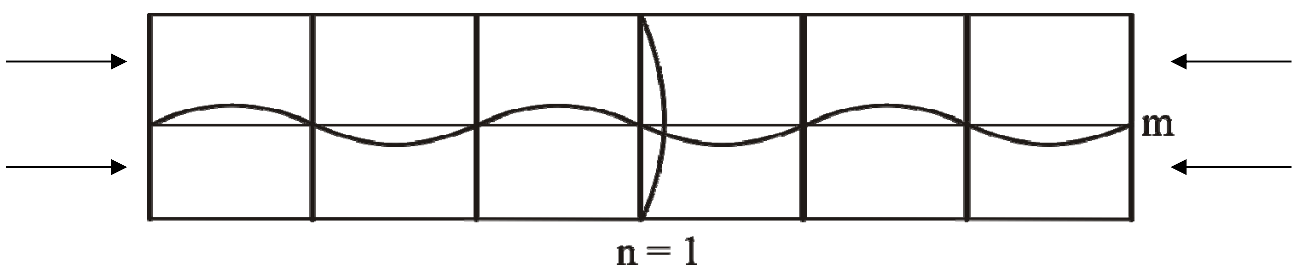
$$\boxed{k_\sigma = \left(\frac{m}{\alpha} + n^2 \frac{\alpha}{m} \right)^2}$$

Koeficient lokalnega izbočenja

$$\boxed{\sigma_E = \frac{\pi^2 E t^2}{12\left(1-\nu^2\right) b^2}}$$

$$k_{\sigma,MIN}(n) = ? \quad \rightarrow \quad n = 1 \quad \rightarrow \quad k_{\sigma,MIN}(n=1) = \left(\frac{m}{\alpha} + \frac{\alpha}{m} \right)^2$$

V primeru, da prečne obremenitve ni, nastane v prečni smeri le en polval:



$$\frac{dk_\sigma}{dm} = 2\underbrace{\left(\frac{m}{\alpha}+\frac{\alpha}{m}\right)}_{\neq 0}\underbrace{\left(\frac{1}{\alpha}-\frac{\alpha}{m^2}\right)}_{=0} = 0$$

$$\left(\frac{1}{\alpha}-\frac{\alpha}{m^2}\right) = 0 \quad \rightarrow \quad \boxed{m = \alpha}$$

$$k_{\sigma,MIN}\left(n=1, m=\alpha\right) = \left(\frac{1}{1}+\frac{1}{1}\right)^2 = 4$$

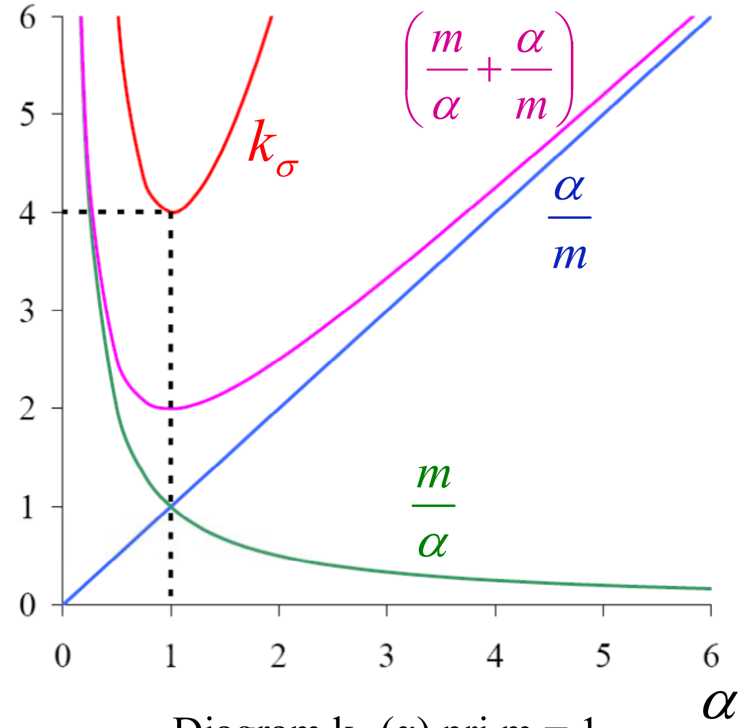
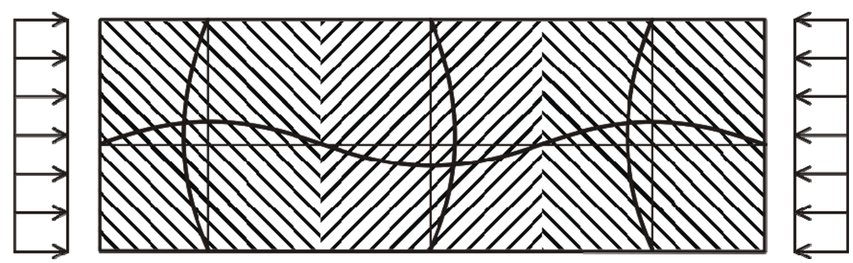


Diagram k_σ (α) pri m = 1.

Tvorijo se približno kvadratni paneli:



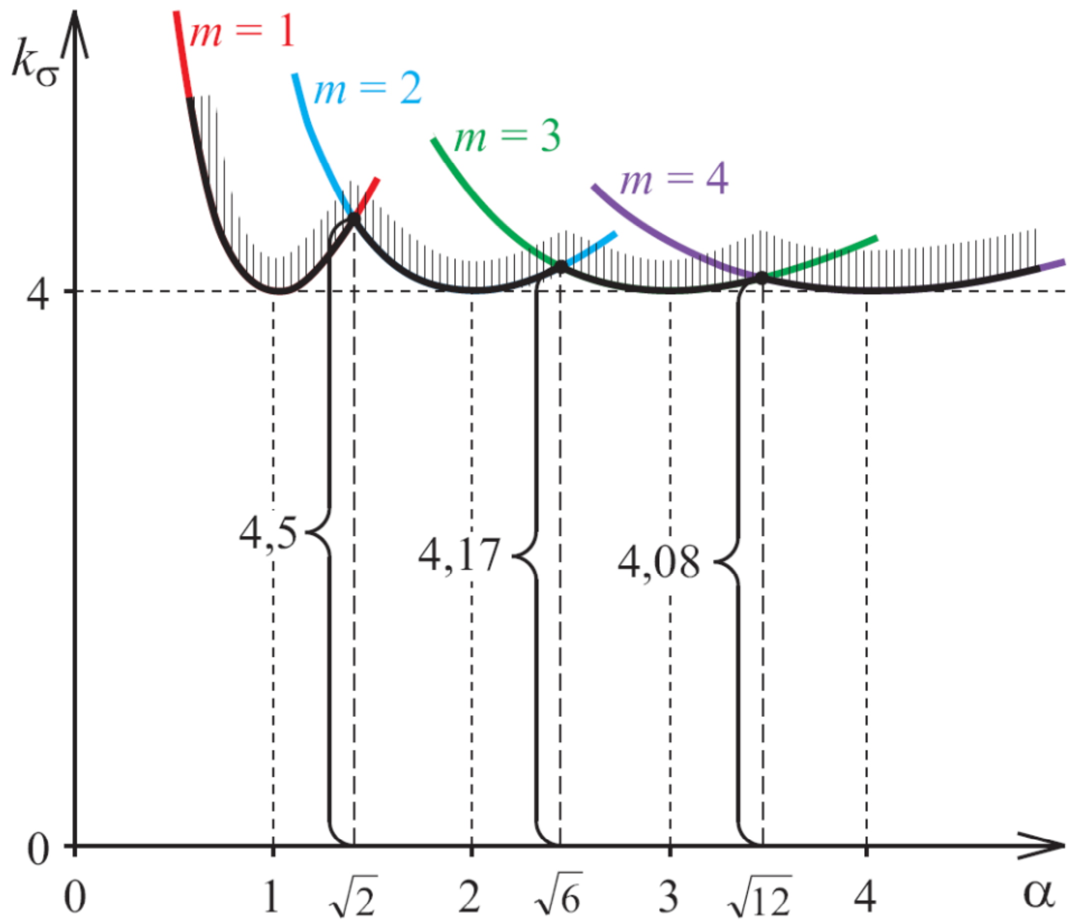
$$k_\sigma(m) = k_\sigma(m+1)$$

$$\frac{m}{\alpha} + \frac{\alpha}{m} = \frac{m+1}{\alpha} + \frac{\alpha}{m+1}$$

$$-\frac{1}{\alpha} = \frac{\alpha(m-m-1)}{m(m+1)}$$

$$\alpha^2 = m(m+1)$$

$$\rightarrow \quad \alpha_{mej} = \sqrt{m(m+1)}$$



$$m = 1, \alpha_{mej} = \sqrt{2} \quad \rightarrow \quad k_\sigma = \left(\frac{1}{\sqrt{2}} + \frac{\sqrt{2}}{1}\right)^2 = 4{,}5$$

$$m = 2, \alpha_{mej} = \sqrt{6} \quad \rightarrow \quad k_\sigma = \left(\frac{2}{\sqrt{6}} + \frac{\sqrt{6}}{2}\right)^2 = 4{,}17$$

$$m = 3, \alpha_{mej} = \sqrt{12} \quad \rightarrow \quad k_\sigma = \left(\frac{3}{\sqrt{12}} + \frac{\sqrt{12}}{3}\right)^2 = 4{,}08$$

K_σ pri različne obremenitvah pločevin:

- **<u>Normalne napetosti:</u>**

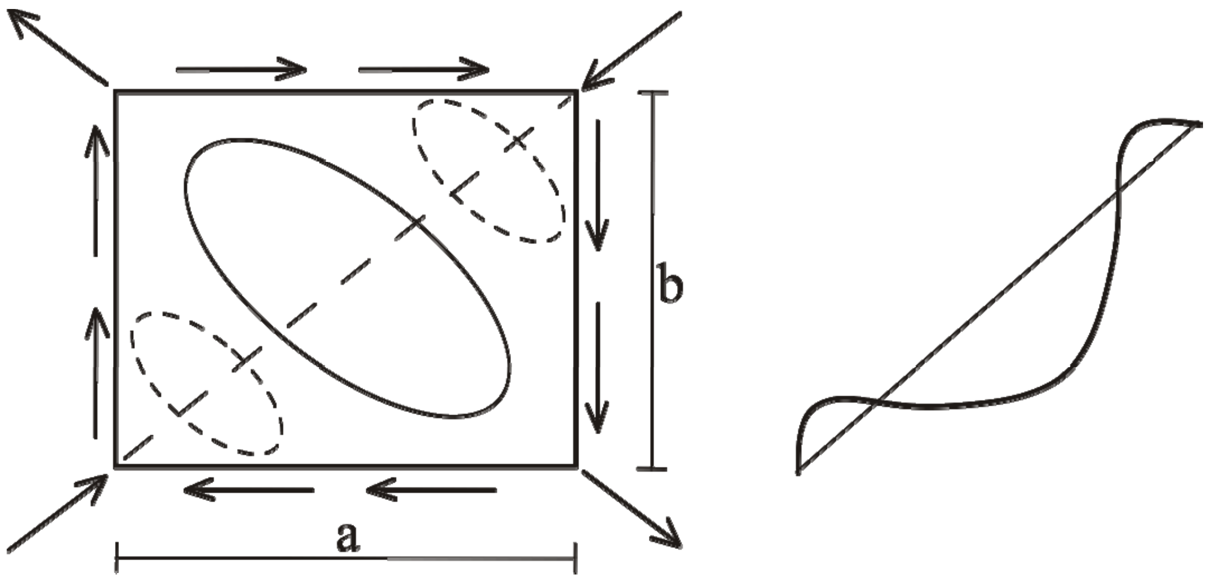
$$k_\sigma = 4$$

$$k_\sigma = 23,9$$

$$k_\sigma = 7,64$$

$$k_\sigma = 0,45$$

- **Strig:**

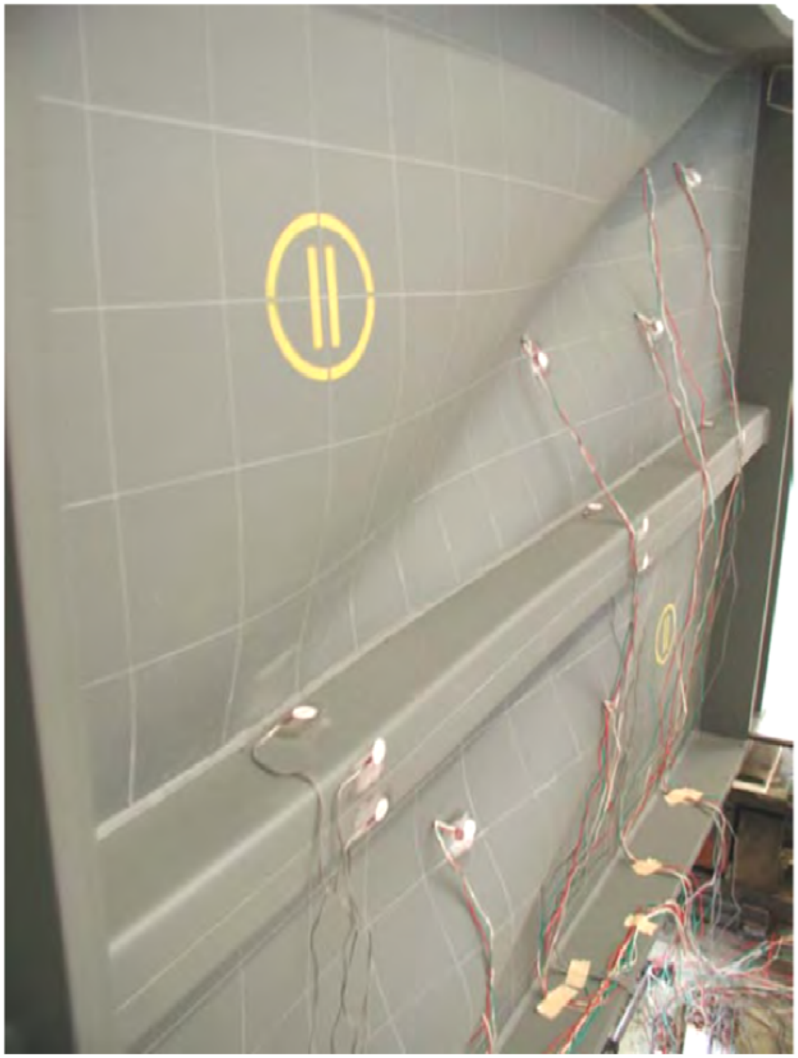


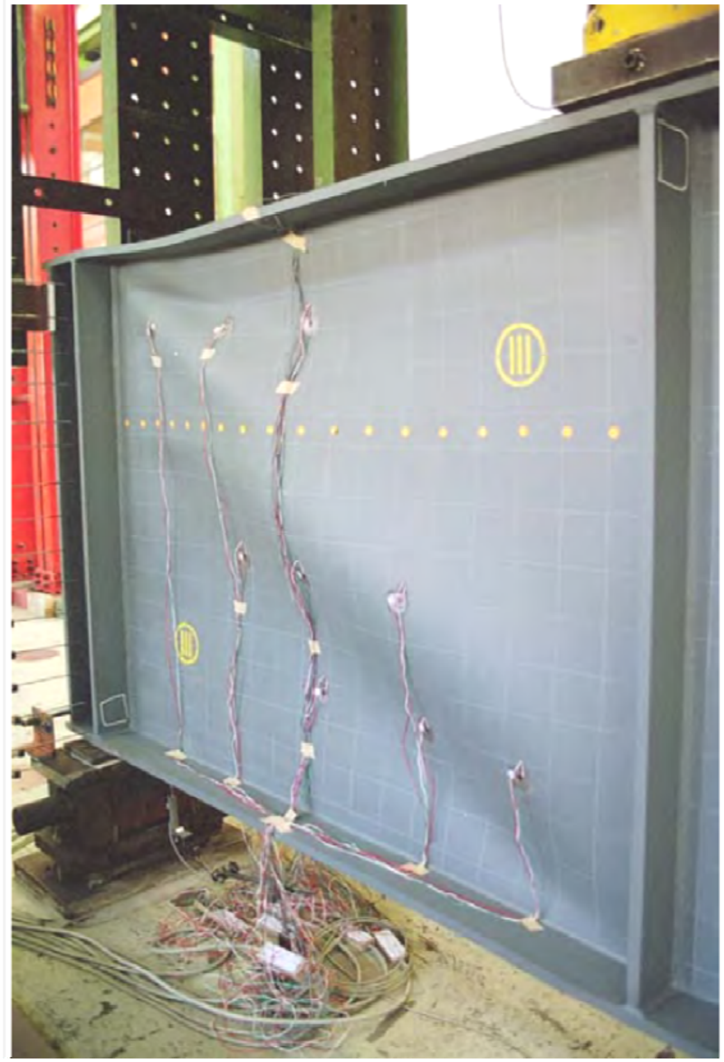
$$\sigma_{cr} = \sigma_E k_\sigma \ , \quad \boxed{\tau_{cr} = \sigma_E k_\tau}$$

$$k_\tau = 5,34 + \frac{4}{\alpha^2} \quad ; \alpha > 1,0 \qquad \alpha = \frac{a}{b}$$

$$k_\tau = 4 + \frac{5,34}{\alpha^2} \quad ; \alpha < 1,0$$

- Izbočenje strižno obremenjenega panela polnostenskega nosilca (levo: s togo vzdolžno ojačitvijo, desno: s podajno vzdolžno ojačitvijo v zgornji tretjini)

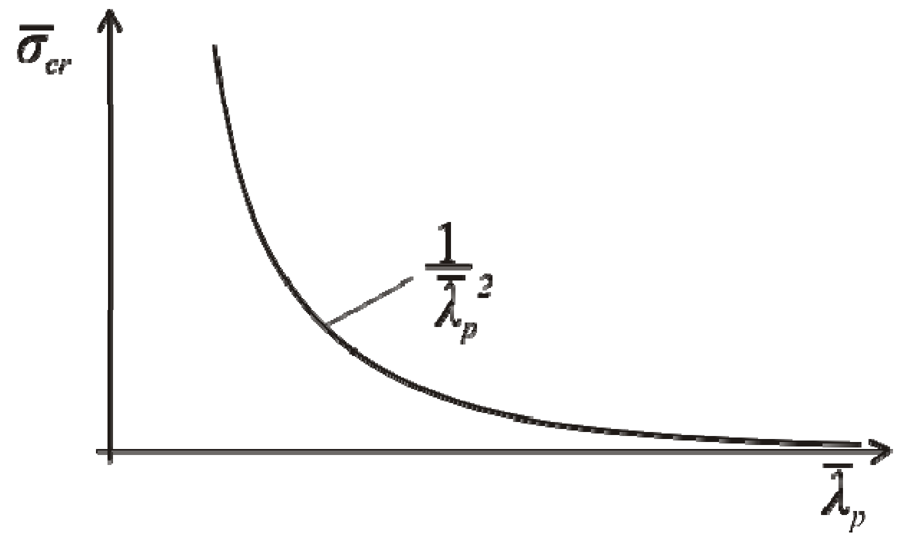




Vitkost:

Normalne napetosti:

$$\bar{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{cr}}} \qquad \frac{\sigma_{cr}}{f_y} = \bar{\sigma}_{cr} = \frac{1}{\bar{\lambda}_p^{\,2}}$$

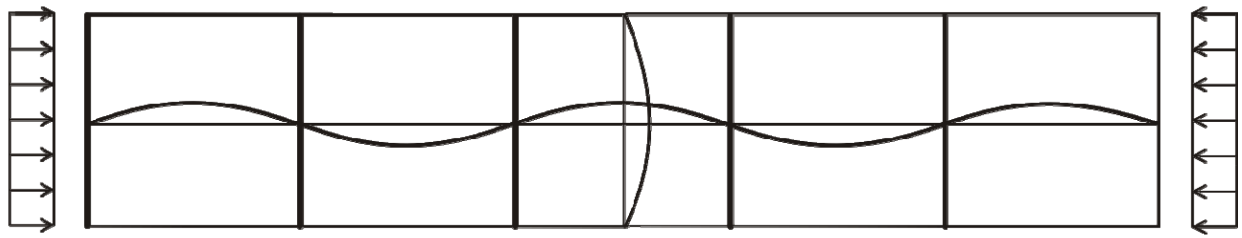


Strig:

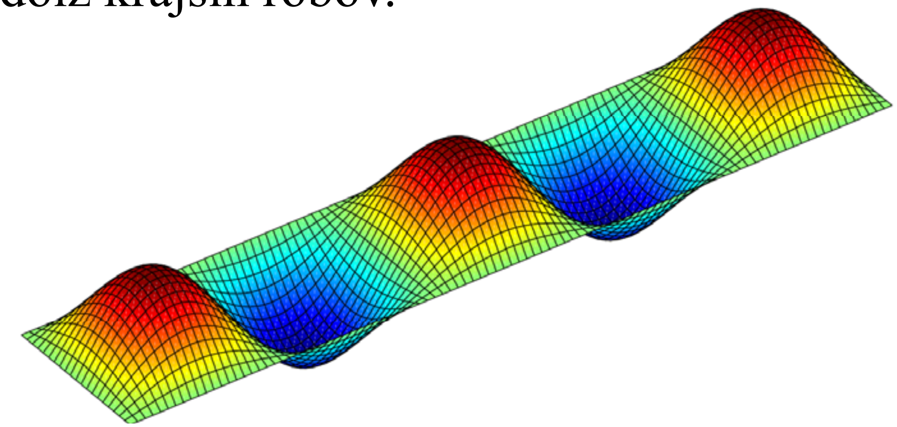
$$\bar{\lambda}_{p\tau} = \sqrt{\frac{f_y / \sqrt{3}}{\tau_{cr}}}$$

Toge ojačitve

- Prečne ojačitve vplivajo na **povečanje k_τ in τ_{cr}**, k_σ in σ_{cr} pa se bistveno ne povečata.

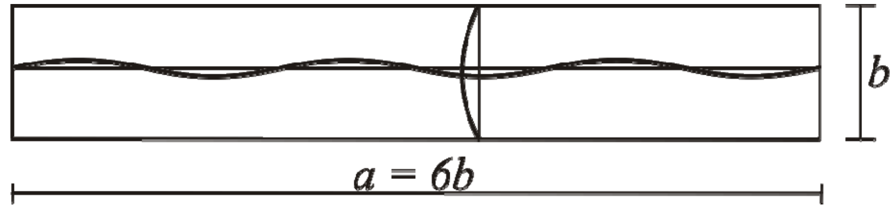


Pločevina brez ojačitev obremenjena s konstantnim potekom tlačnih napetosti vzdolž krajših robov.



- Pločevina brez ojačitev:

$$\boxed{\alpha = \frac{a}{b} = 6} \qquad \boxed{m = \alpha = 6}$$

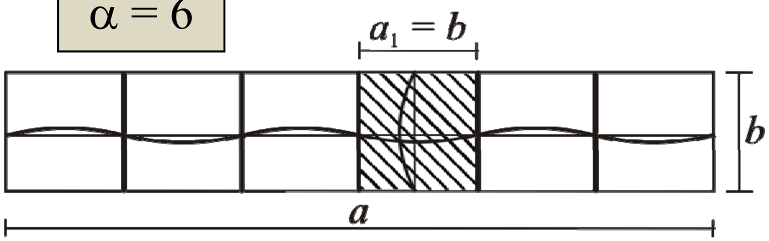


$$n = 1$$

$$k_\sigma = \left(\frac{m}{\alpha} + \frac{\alpha}{m}\right)^2 = \left(\frac{6}{6} + \frac{6}{6}\right)^2 = 4$$

- Pločevina s togimi prečnimi ojačitvami:

$$\boxed{\alpha = 6}$$

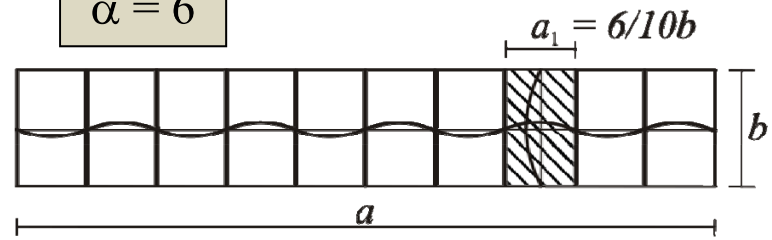


$$5 \text{ ojačitev}, 6 \text{ polj} \rightarrow \boxed{m = 6}$$

$$n = 1$$

$$k_\sigma = \left(\frac{m}{\alpha} + \frac{\alpha}{m}\right)^2 = \left(\frac{6}{6} + \frac{6}{6}\right)^2 = 4$$

$$\boxed{\alpha = 6}$$



$$9 \text{ ojačitev}, 10 \text{ polj} \rightarrow \boxed{m = 10}$$

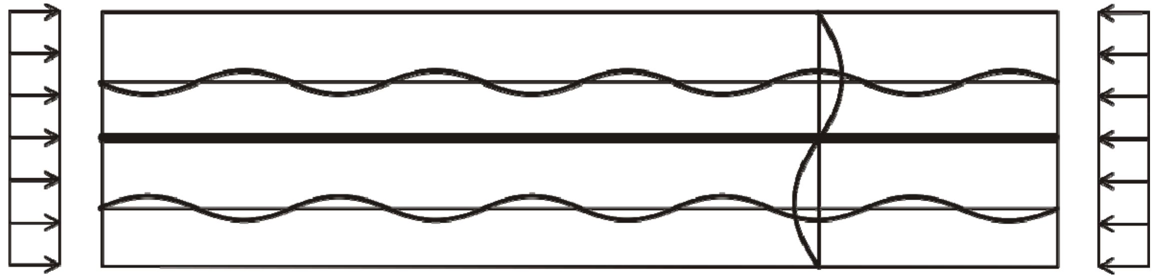
$$\alpha_1 = \frac{a_1}{b} = \frac{6}{10}$$

$$n = 1$$

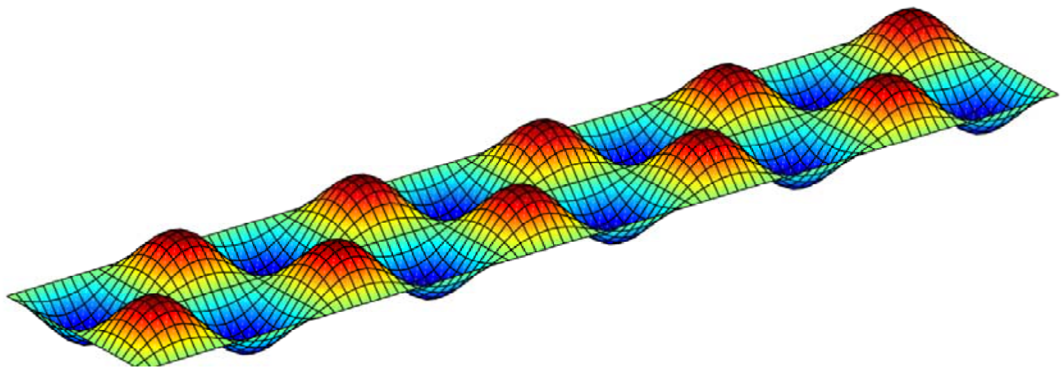
$$k_\sigma = \left(\frac{m}{\alpha} + \frac{\alpha}{m}\right)^2 = \left(\frac{10}{6} + \frac{6}{10}\right)^2 = 5{,}14$$

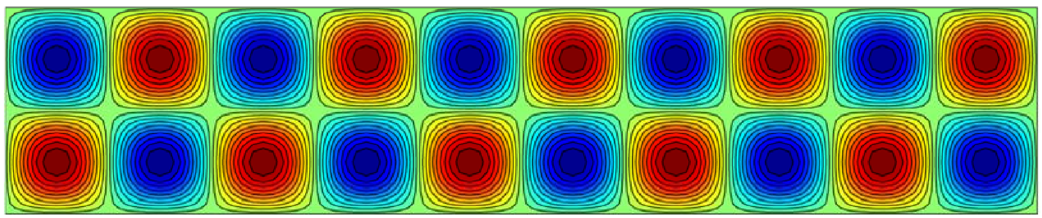
k_σ se bistveno ne poveča!

- k_σ **in** σ_{cr} **povečamo z vzdolžnimi ojačitvami**, saj pločevino prisilimo, da se prečno izboči v več kot enem polvalu.



Pločevina s togo vzdolžno ojačitvijo obremenjena s konstantnim potekom tlačnih napetosti vzdolž krajših robov.

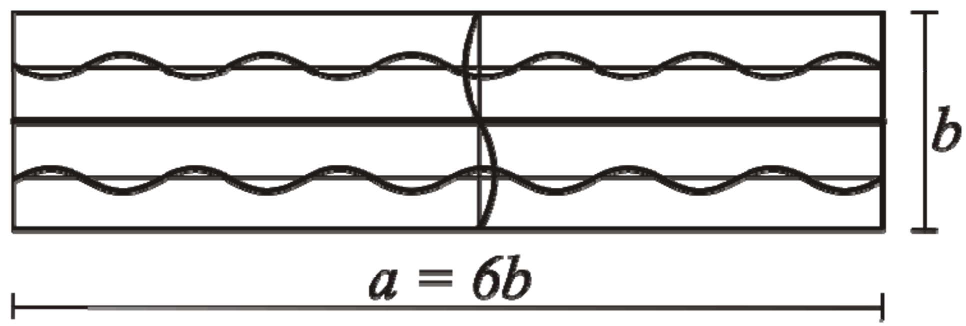




- Pločevina s togo vzdolžno ojačitvijo

$n = 2$

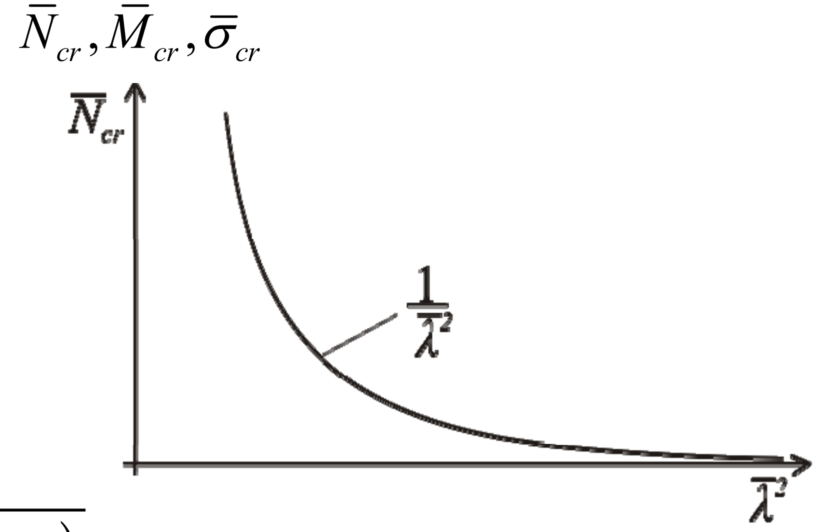
$$\boxed{\alpha = \frac{a}{b} = 6} \qquad \alpha_1 = \frac{a}{b/2} = \frac{6b \cdot 2}{b} = 12 \quad \Rightarrow \quad \boxed{m = 12}$$



$$k_\sigma = \left(\frac{m}{\alpha} + n^2 \frac{\alpha}{m} \right)^2 = \left(\frac{12}{6} + 2^2 \cdot \frac{6}{12} \right)^2 = 16$$

k_σ se bistveno poveča (×4)!

Kratek povzetek



Upogibni uklon:

Tlačna osna sila: $N_{cr},\quad \bar{\lambda} = \sqrt{\frac{N_{pl}}{N_{cr}}}$

Bočna zvrnitev:

Upogibni moment: $M_{cr},\quad \bar{\lambda}_{LT} = \sqrt{\frac{M_{el}\left(M_{pl}\right)}{M_{cr}}}$

Lokalno izbočenje pločevin:

Tlačna napetost v ravnini pločevine: $\sigma_{cr},\quad \bar{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{cr}}}$