$m_{O_2} = 5\,g$

$m_{H_2} = 10\,g$

$T = 25°C$

$p = 0{,}750\ bar$

a) $X_{O_2}, X_{H_2} = ?$

b) $p_{O_2}, p_{H_2} = ?$

c) $V = ?$

a) $$X_{H_2} = \frac{n_{H_2}}{n_{H_2} + n_{O_2}} = \frac{\frac{m_{H_2}}{M_{H_2}}}{\frac{m_{H_2}}{M_{H_2}} + \frac{m_{O_2}}{M_{O_2}}} = \frac{\frac{10\,g}{2{,}02\,g/mol}}{\frac{10\,g}{2{,}02\,g/mol} + \frac{5\,g}{32{,}0\,g/mol}} = 0{,}969 = \underline{96{,}9\,\%}$$

$$X_{O_2} = 1 - X_{H_2} = 0{,}031 = \underline{3{,}1\,\%}$$

b) $$p_{O_2} = X_{O_2} \cdot p = 0{,}969 \cdot 0{,}750\ bar = \underline{0{,}727\ bar}$$

$$p_{H_2} = X_{H_2} \cdot p = 0{,}031 \cdot 0{,}750\ bar = \underline{0{,}023\ bar}$$

c) $$pV = nRT$$

$$pV = (n_{H_2} + n_{O_2})RT$$

$$V = \frac{(n_{H_2} + n_{O_2})RT}{p} = \frac{5{,}11\ mol \cdot 8{,}314\ J \cdot 298{,}15\ K}{0{,}750 \cdot 10^5\ Pa\ mol\ K} =$$

$$= 0{,}169\ m^3 =$$

$$= \underline{169\ L}$$

Kabeljau (Cu)

$T = 293\ K$

$\varkappa_T = 7{,}35 \cdot 10^{-7}\ atm^{-1}$

$\Delta p = ?$

$\Delta V = 0{,}1\,\%$

$$\rho = \frac{m}{V} \Rightarrow d\rho = \frac{\partial \rho}{\partial V} dV = m(-1 \cdot V^{-2})\,dV =$$

$$= -\frac{m}{V^2}\,dV = -\rho\,\frac{dV}{V}$$

$$\frac{d\rho}{\rho} = -\frac{dV}{V}$$

$$\varkappa_T = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)$$

$$\frac{dV}{V} = -\varkappa_T\,dp$$

$$\int_{\rho_1}^{\rho_2} \frac{d\rho}{\rho} = \int_{p_1}^{p_2} \varkappa_T\,dp$$

$$\int_{p_1}^{p_2} dp = \bar{\varkappa}_T^{-1} \cdot \int_{\rho_1}^{\rho_2} \frac{d\rho}{\rho}$$

$$\Delta p = \frac{1}{\varkappa_T} \cdot \ln\frac{\rho_2}{\rho_1} =$$

$$= \frac{1}{\varkappa_T} \cdot \ln\frac{1{,}001 \cdot \rho_1}{\rho_1} =$$

$$= \frac{1\ atm}{7{,}35 \cdot 10^{-7}} \cdot \ln 1{,}001 =$$

$$\Delta p = \underline{1360\ atm}$$

$\alpha, \beta, \varkappa_T = ?$

$\alpha', \beta', \varkappa_T' = ?$

IDEALNI PLIN

REALNI PLIN

$pV = nRT$

$$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p = \frac{1}{V}\frac{\partial}{\partial T}\left(\frac{nRT}{p}\right) = \frac{1}{V}\frac{nR}{p}\left(\frac{T}{\partial T}\right) = \frac{nR}{Vp}$$

$$= \frac{nR}{Vp} = \frac{nRp}{nRTp} = \frac{1}{T}$$

$$\beta = \frac{1}{p}\left(\frac{\partial p}{\partial T}\right)_V = \frac{1}{p}\frac{\partial}{\partial T}\frac{nRT}{V} = \frac{nR}{V}\cdot\frac{1}{p} =$$

$$= \frac{nR}{Vp} = \frac{nRV}{V\cdot nRT} = \frac{1}{T}$$

$$\varkappa_T = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T = -\frac{1}{V}\frac{\partial}{\partial p}\left(\frac{nRT}{p}\right) = \frac{-nRT}{V}\frac{1}{\partial p}\frac{\partial p^{-1}}{\partial p} = \frac{nRT}{V}p^{-2} =$$

$$= \frac{nRT}{Vp^2} =$$

$$= \frac{nRTp}{nRTp^2} = \frac{1}{p}$$

$\left(p + \frac{a}{V_m^2}\right)(V_m - b) = RT$

$p = \frac{RT}{V_m - b} - \frac{a}{V_m^2}$

$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p$ - ne moremo določiti V

$$\beta = \frac{1}{p}\left(\frac{\partial p}{\partial T}\right)_{V_m} = \frac{1}{p}\frac{\partial}{\partial T}\left(\frac{RT}{V_m - b} - \frac{a}{V_m^2}\right) =$$

$$= \frac{1}{p}\left(\frac{R}{V_m - b}\right) = \frac{1}{\frac{RT}{V_m - b} - \frac{a}{V_m^2}}\cdot\frac{R}{V_m - b} =$$

$$\beta = \frac{RV_m^2}{RTV_m^2 - a(V_m - b)}$$

$$\varkappa_T = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T = -\frac{1}{V_M}\left(\frac{\partial p}{\partial V_M}\right)_T^{-1} = \frac{-1}{V_M}\left(-\frac{RT}{(V_m - b)^2} + \frac{2a}{V_m^2}\right)^{-1}$$

$$\varkappa_T = \frac{V_M^2(V_M - b)^2}{RTV_M^3 - 2a(V_M - b)^2}$$

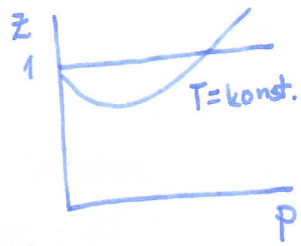
$\alpha = \beta \varkappa_T p$ - izračunan α $\Rightarrow$ $\alpha = \frac{RV_M^2(V_M - b)}{RTV_M^3 - 2a(V_M - b)^2}$

$p(V_M - b) = RT$

Izpelji Z_p!

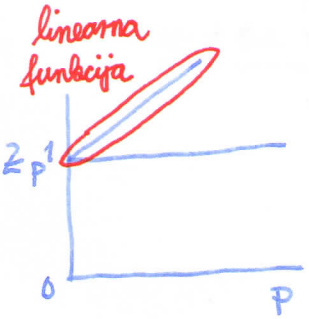
$V_M = \frac{RT + pb}{p}$

$V_M = \frac{RT}{p} + b$



$Z = \frac{p \cdot V_M}{RT}$

$Z = \frac{p}{RT}\left(\frac{RT}{p} + b\right) = 1 + \frac{bp}{RT}$ $\Rightarrow$ graf



Pri kateri T in p je H_2 v korespodentnem stanju s CH_4?

$T_{(CH_4)} = 500{,}0\ K$

$p_{(CH_4)} = 2{,}00\ bar$

$T, p = ?$

$T_K(H_2) = 33{,}2\ K$

$p_K(H_2) = 13{,}0\ bar$

$T_K(CH_4) = 190{,}6\ K$

$p_K(CH_4) = 46{,}0\ bar$

$T_R(H_2) = T_R(CH_4)$

$\frac{T(H_2)}{T_K(H_2)} = \frac{T(CH_4)}{T_K(CH_4)}$

$T_{(H_2)} = \frac{T_{(CH_4)} \cdot T_K(H_2)}{T_K(CH_4)} = \frac{33{,}2\ K \cdot 500{,}0\ K}{190{,}6\ K} =$

$T_{(H_2)} = \underline{\underline{87{,}1\ K}}$

$\frac{p(H_2)}{p_K(H_2)} = \frac{p(CH_4)}{p_K(CH_4)} \Rightarrow p(H_2) = 0{,}565\ bar$

Kakšno je mol. razmerje za O_2 in He na višini 5 km nad Zemljo pri $T = 0°C$?

$\frac{p}{p_0} = e^{-\frac{Mgz}{RT}}$

$pV = nRT$

$p = \frac{nRT}{V} = cRT$

$\frac{p}{p_0} = e^{\frac{-Mgz}{RT}}$

za O_2: $\frac{c}{c_0} = e^{-\frac{32 \cdot 10^{-3}\ kg \cdot 9{,}81\ m \cdot 5000\ m\ K\ mol}{mol\ s^2 \cdot 8{,}314\ J\ N\ m \cdot 273{,}15\ K}} = 0{,}501$

He: $\frac{c}{c_0} = 0{,}917$

$\Delta g = 0{,}01\ m/s^2 km \Rightarrow \frac{dg}{dz} = a = -10^{-2} m s^{-2} \cdot 10^{-3} m^{-1} =$

$= -10^{-5} s^{-2}$

$\int_{g_0}^{g} dg = \int_0^z a\, dz$

$g = az + g_0$

$\int_{p_0}^{p} \frac{dp}{p} = -\frac{M}{RT} \int_0^z (g_0 + az)\, dz$

$\ln(p : p_0) = -\frac{M}{RT}\left(g_0 z + \frac{az^2}{2}\right)$

$\frac{p}{p_0} = e^{-\frac{M}{RT}\left(g_0 z + \frac{az^2}{2}\right)}$

$p = p_0\, e^{-\frac{M}{RT}\left(g_0 z + \frac{az^2}{2}\right)}$

podatki:

N_2: $p_0 = 1\ atm$

$T = 25°C$

$z = 100\ km$

$p = 1{,}54 \cdot 10^{-5}\ atm$

$g = konst.$

$p = 2{,}71 \cdot 10^{-5}\ atm$

$g(z)$

Kolikšna je mol. masa plina pri 0°C, kat. hitrost je enaka 425 m/s.

$u_{rms} = ?$

$M = ?$

$u_{mp} = ?$

$\bar{u} = \sqrt{\frac{8RT}{\pi M}}$

$M = \frac{8RT}{\bar{u}^2 \cdot \pi} = \frac{8 \cdot 8{,}314\ J \cdot 273{,}15\ K \cdot kg\ m}{mol\ K \cdot (425)^2 m^2 \cdot \pi\ s^2} = 0{,}03202\ kg = 32\ g/mol$

O_2

$\sqrt{\overline{u^2}} = u_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \cdot 8{,}314\ J \cdot 273{,}15\ K\ mol}{K\ mol \cdot 32{,}0\ g}} =$

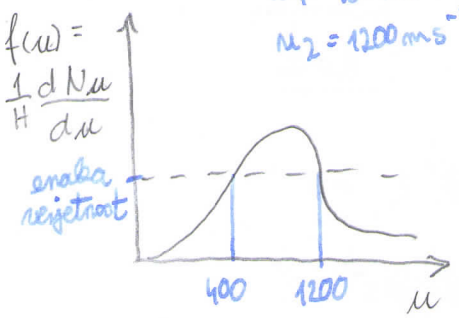
$\frac{u_{rms}}{\bar{u}} = \frac{\sqrt{\frac{3RT}{M}}}{\sqrt{\frac{8RT}{\pi M}}} = \sqrt{\frac{3\pi}{8}} = 1{,}0854$

$u_{rms} = \bar{u} \cdot 1{,}0854 = 461{,}4\ m/s$

$u_{mp} = \sqrt{\frac{2RT}{M}}$

$u_{mp} = u_{rms} \cdot \sqrt{\frac{2}{3}} = 376{,}7\ m/s$

Pri kateri T je verjetnost, da imajo molekule N_2 400 m/s in 1200 m/s, enaka?



$$f(u) = 4\pi\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} \cdot u^2 \cdot e^{-\frac{mu^2}{2kT}}$$

$$f(u_1) = f(u_2)$$

$$u_1^2 e^{-\frac{mu_1^2}{2kT}} = u_2^2 e^{-\frac{mu_2^2}{2kT}}$$

$$\left(\frac{u_1}{u_2}\right)^2 = e^{\frac{mu_1^2 - mu_2^2}{2kT}} = e^{\frac{m}{2kT}(u_1^2 - u_2^2)}$$

$$2\ln\frac{u_1}{u_2} = \frac{m}{2kT}(u_1^2 - u_2^2)$$

$$T = \frac{m(u_1^2 - u_2^2)\cdot N_A}{2\cdot 2kT\ 2\ln\frac{u_1}{u_2}} = \frac{M(u_1^2 - u_2^2)}{4RT\,2\ln\frac{u_1}{u_2}} =$$

$$= \frac{28\,kg\,mol^{-1}\cdot 10^{-3}\cdot(400^2 - 1200^2)\,\frac{K\,mol}{m^2}}{4\cdot 8{,}314\,J\,s^2\cdot 2\ln\frac{400}{1200}} =$$

$$T = 981\ K$$

Določi pov. kvadrat gib. kol. za delec z maso m in se giblje v smeri x!

$\overline{P^2} = ?$ $\quad g_x = m u_x$

m
x

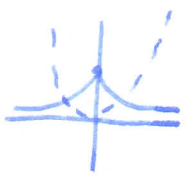
$$\bar{u} = \frac{\int u\,dN_u}{\int dN_u}$$ SPLOŠNA FORMULA

$$\frac{1}{N}\frac{dN_x}{du_x} = \left(\frac{m}{2\pi kT}\right)^{\frac{1}{2}} e^{-\frac{mu_x^2}{2kT}}$$

$$\overline{g_x^2} = \frac{1}{N}\int g_x^2\,dN_x = \frac{1}{N}\int (mu_x)^2\,dN_x = \frac{1}{N}\cdot m^2\int u_x^2\,dN_x =$$

$$= \frac{m^2}{N}\int_{0-\infty}^{+\infty} u_x^2\left(\frac{m}{2\pi kT}\right)^{\frac{1}{2}} e^{-\frac{mu_x^2}{2kT}} N\,du_x =$$

$$= m^2\left(\frac{m}{2\pi kT}\right)^{\frac{1}{2}}\int_{-\infty}^{+\infty} u_x^2\, e^{-\frac{mu_x^2}{2kT}}\,du_x$$



$$= m^2\left(\frac{m}{2\pi kT}\right)^{\frac{1}{2}} 2\int_0^{\infty} u_x^2\, e^{-\frac{mu_x^2}{2kT}}\,du_x$$

$$x = -\frac{mu_x^2}{2kT} \Rightarrow$$

$$u_x = \left(\frac{2kT}{m}\right)^{\frac{1}{2}} x^{\frac{1}{2}}$$

$$du_x = \left(\frac{2kT}{m}\right)^{\frac{1}{2}}\cdot\frac{1}{2}x^{-\frac{1}{2}}\,dx$$

$$\overline{g_x^2} = 2m^2\left(\frac{m}{2\pi kT}\right)^{\frac{1}{2}}\int_0^{\infty}\frac{2kT}{m}x\,e^{-x}\left(\frac{2kT}{m}\right)^{\frac{1}{2}}\frac{1}{2}x^{-\frac{1}{2}}dx$$

$$= m\left(\frac{1}{2\pi kT}\right)^{\frac{1}{2}}\left(\frac{2kT}{m}\right)^{\frac{1}{2}} 2kT\int_0^{\infty} x^{\frac{1}{2}}e^{-x}\,dx$$

$$= \frac{2mkT}{\pi^{\frac{1}{2}}}\int_0^{\infty} x^{\frac{3}{2}-1}e^{-x}\,dx =$$

$$\Gamma(n) = \int_0^{\infty} x^{n-1}e^{-x}\,dx$$

$$n = \frac{3}{2} \Rightarrow$$

$$\Gamma(n) = \frac{1}{2}\Gamma\left(\frac{1}{2}\right) =$$

$$= \frac{1}{2}\pi^{\frac{1}{2}}$$

$$\overline{g_x^2} = mkT$$

Nauči se [gama]] funkcijo!

Kolikšna hitrost je potrebna, da neko telo zapusti Zemljo?

$W(r) = \frac{m_1 m_2 G}{r}$ $\quad G = 6{,}67\ 10^{-11}\ \frac{J\,m}{kg^2}$ $\quad \boxed{h = R}$

$m_1 = m$

$m_2 = (M)$ masa Zemlje =

$M = 5{,}98 \cdot 10^{24}\ kg$

$R = 6371\ km$

$\frac{GmM}{R} = W_p(R) = mgR \Rightarrow$

$g = \frac{GM}{R^2} = 9{,}81\ ms^{-2}$

$W_k = W_p$

$\frac{mv^2}{2} = \frac{GmM}{R}$

$\bar{v} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2 \cdot 6{,}67 \cdot 10^{-11}\ J\,m\,kg^{-2} \cdot 5{,}98 \cdot 10^{24}\ kg}{s^2 \cdot 6{,}371 \cdot 10^6\ m}} =$

$\bar{v} = \sqrt{\frac{8RT}{\pi M}}$

$T = \frac{\bar{v}^2 \pi M}{8R} = \frac{\pi \cdot 28\ kg \cdot 10^{-3}\ \bar{v}^2\ mol\ K}{mol\ 8 \cdot 8{,}314\ J}$ $\quad \bar{v} = 11\,200\ m/s$ – mejna hitrost

$T = 1{,}66 \cdot 10^5\ K$ – mejna T

$Z_1 = ?$

$Z_{11} = ?$

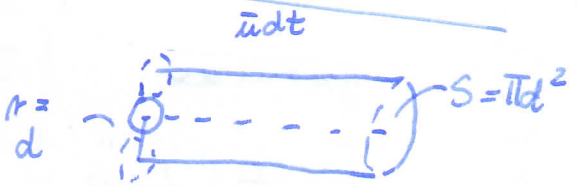
Ar

$T = 25°C$

$p = 1\ bar$

d (premer Ar) $= 3{,}96 \cdot 10^{-10}\ m$

$M = 39{,}95\ g\,mol^{-1}$



$\bar{v} = \sqrt{\frac{8RT}{M\pi}} = 397{,}5\ m\,s^{-1}$

$Z_1 = \sqrt{2} \cdot \rho \cdot \beta \cdot \bar{v} =$

$= \sqrt{2} \cdot 2{,}43 \cdot 10^{25}\ m^{-3} \cdot 4{,}93 \cdot 10^{-19}\ m^2 \cdot 397{,}5\ m\,s^{-1} =$

$Z_1 = 6{,}73 \cdot 10^{9}\ s^{-1}$

$\rho = \frac{N}{V}$

$\beta = \pi d^2 = \pi \cdot ((3{,}96 \cdot 10^{-10})\ m^{-10})^2 =$

$= 4{,}93 \cdot 10^{-19}\ m^2$

$Z_{11} = \frac{\sqrt{2}}{2} \rho^2 \cdot \beta\ \bar{v} =$

$Z_{11} = 8{,}18 \cdot 10^{34}\ s^{-1}\ m^{-3}$

$pV = nRT$ – idealno obnašanje argona

$p = \frac{NRT}{N_A V} = \rho \frac{RT}{N_A} \Rightarrow$

$\rho = \frac{pN_A}{RT} = \frac{10^5\ Pa \cdot 6{,}022 \cdot 10^{23}\ mol^{-1}}{8{,}314\ J\,mol^{-1}\,K^{-1} \cdot 288{,}15\ K}$

$= 2{,}43 \cdot 10^{25}\ m^{-3}$

$\lambda = 100\ \mu m$

$T = 20\,^\circ C$

$d = 2{,}73 \cdot 10^{-10}\ m$

$p = ?$

$$\lambda = \frac{1}{\sqrt{2}\,\pi d^2} = \frac{RT}{\sqrt{2}\,N_A\,p\,\pi d^2} \Rightarrow$$

$$p = \frac{RT}{\sqrt{2}\,N_A\,\lambda\,\pi d^2} = \frac{8{,}314\ J \cdot 293{,}15\ K\,mol}{K\,mol \cdot \sqrt{2} \cdot 6{,}022 \cdot 10^{23} \cdot 100 \cdot 10^{-6}\ m \cdot \pi \cdot (2{,}73 \cdot 10^{-10})^2\ m^2}$$

$$p = 122\ Pa = 1{,}22 \cdot 10^{-3}\ bar$$

N_{CH_4}

$M = 16\ g\,mol^{-1}$

$p = 1\ mL\ Hg$

$T = 298{,}15\ K$

$\Delta t = 1\ ms$

$S = 1\ cm^2$

$$\frac{1}{S} \cdot \frac{dZ_S}{dt} = \frac{1}{4} \int \bar{u} = \frac{1}{4} \frac{N}{V} \bar{u}$$

$$\int dZ_S = S \frac{1}{4} \int \bar{u} \int dt$$

$$Z_S = \frac{1}{4} \frac{N}{V} \bar{u} \cdot \Delta t \cdot S = \frac{S \cdot \Delta t}{4} \cdot \frac{p \cdot N_A}{RT} \left(\frac{8RT}{\pi M} \right)^{\frac{1}{2}} =$$

$$= \frac{10^{-4}\ m^2 \cdot 10^{-3}\ s}{4} \cdot \frac{1{,}013 \cdot 10^5\ Pa \cdot 6{,}022 \cdot 10^{23}\ mol\,K}{mol \cdot 8{,}314\ J \cdot 298{,}15\ K} \left(\frac{8 \cdot 8{,}314 \cdot 298{,}15}{\pi \cdot 16 \cdot 10^{-3}} \right)^{\frac{1}{2}} =$$

$$Z_S = 5{,}08 \cdot 10^{17}\ molekul$$

V steno L posode, kjer je Cl_2 pri $T = 300\ K$ izvrtamo luknjico premera $0{,}2\ mm$. V kolikšnem času pade tlak na $\frac{1}{2}$ začetne vrednosti tlaka?

$V = 1\ L$

$T = 300\ K$

$M(Cl_2) = 70{,}9\ g\,mol^{-1}$

$d = 0{,}2\ mm$

$t = ?$

$$\frac{1}{S} \cdot \frac{dZ_S}{dt} = \frac{1}{4} \frac{\bar{u} N}{V}$$

$$\frac{dZ_S}{dt} = -\frac{dN}{dt} = \frac{1}{4} \frac{S \cdot \bar{u} N}{V}$$

$$-\frac{dN}{N} = \frac{1}{4} \frac{S \bar{u}\, dt}{V}$$

$$-\int_{p_0}^{\frac{p_0}{2}} \frac{dp}{p} = \frac{1}{4} \frac{S \bar{u}}{V} \int_0^t dt$$

$$pV = NkT \Rightarrow$$

$$N = \frac{V}{kT} p$$

$$dN = \frac{V}{kT} dp$$

$$\Downarrow$$

$$\frac{dN}{N} = \frac{dp}{p}$$

$$\boxed{\frac{p_0}{2} = p}$$

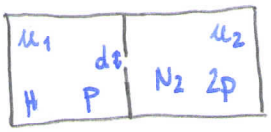
$$-\ln p \Big|_{p_0}^{\frac{p_0}{2}} = -\ln \frac{p}{p_0} = \frac{1}{4} \cdot \frac{S \bar{u}}{V} t$$

$$t = \frac{4V \ln \frac{p_0}{p}}{S \bar{u}} =$$

$$= \frac{4 \cdot 10^{-3}\ m^3 \ln 2}{\pi r^2 \sqrt{\frac{8RT}{\pi M}}} \sim \omega =$$

$$= \frac{4 \cdot 10^{-3}\ m^3 \ln 2}{\pi \cdot (0{,}1 \cdot 10^{-6}\ m)^2 \cdot \omega} =$$

$$t = 8{,}19 \cdot 10^4\ h = 3413\ dni = \boxed{9\ let\ 4\ meseci}$$



$d = 0{,}1\ \mu m$

$$\frac{1}{S}\frac{dz_S}{dt} = \frac{\rho \bar{u}}{4}$$

$$\frac{dz_S}{dt} = -\frac{dN}{dt} = \frac{S\rho\bar{u}}{4} = \nu$$

hitrost izteka molekul

$$\frac{\nu_{H_2}}{\nu_{N_2}} = \frac{\rho_{H_2}\cdot\bar{u}_{H_2}\cdot S/4}{\rho_{N_2}\cdot\bar{u}_{N_2}\cdot S/4} = \frac{\rho_{H_2}\bar{u}_{H_2}}{\rho_{N_2}\bar{u}_{N_2}} = \frac{p_{H_2}\bar{u}_{H_2}}{p_{N_2}\bar{u}_{N_2}} =$$

$pV = kNT$

$p = \rho kT$

$\rho = \frac{p}{kT}$

$$\bar{u} = \sqrt{\frac{8RT}{\pi M}}$$

$$\text{razmerje} = \frac{p\,\bar{u}_{H_2}}{2p\,\bar{u}_{N_2}} = \frac{1}{2}\frac{\bar{u}_{H_2}}{\bar{u}_{N_2}} =$$

$$= \frac{1}{2}\left(\frac{M_{N_2}}{M_{H_2}}\right)^{\frac{1}{2}} = \frac{1}{2}\sqrt{\frac{28}{2{,}02}} =$$

$$\text{razmerje} = 1{,}87 = \frac{\nu_{H_2}}{\nu_{N_2}}$$

med hitrostima izteka plinov