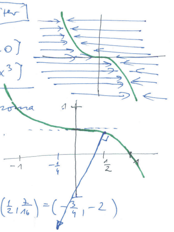
1. kolokvij iz MATEMATIKE 2 za kemike z dne 12.12.2008; rešitev



1. naloga • Narisano definicijsko območje: $D_f = \{(x,y) \mid x^3+2y \neq 0\} = \{(x,y) \mid y \neq -\frac12 x^3\}$

• Nivojnica: Najprej $f(\frac12,\frac{7}{16}) = \frac{1}{\frac18+\frac78} = 1$; gre za nivojnico $\frac{1}{x^3+2y} = 1$ oziroma

$$y = \frac12 - \frac{x^3}{2}.$$

$$\frac{\partial f}{\partial x} = -\frac{3x^2}{(x^3+2y)^2}, \quad \frac{\partial f}{\partial y} = -\frac{2}{(x^3+2y)^2}$$

$$\frac{\partial f}{\partial x}\left(\frac12,\frac{7}{16}\right) = -\frac34, \quad \frac{\partial f}{\partial y}\left(\frac12,\frac{7}{16}\right) = -2, \quad (\operatorname{grad} f)\left(\frac12,\frac{7}{16}\right) = \left(-\frac34,-2\right)$$

2. naloga $\frac{\partial f}{\partial x} = 3x^2+6y, \quad \frac{\partial f}{\partial y} = 6x+2y$

Stacionarne točke: $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = 0$:

$$3x^2+6y=0 \iff x^2+2y=0$$
$$6x+2y=0 \iff y=-3x$$

Vstavimo $y=-3x$ v $x^2+2y=0$ in dobimo $x^2-6x=0$.

$\Rightarrow x_1 = 0 \quad (y_1=0)$ in $x_2 = 6 \quad (y_2=-18)$

$$Hf(x,y) = \begin{bmatrix} 6x & 6 \\ 6 & 2 \end{bmatrix};$$

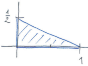
• $\det Hf(0,0) = \begin{vmatrix} 0 & 6 \\ 6 & 2 \end{vmatrix} = -36 < 0 \Rightarrow$ v $(0,0)$ ni ekstrema, ampak je sedlo

• $\det Hf(6,-18) = \begin{vmatrix} 36 & 6 \\ 6 & 2 \end{vmatrix} = +36 > 0 \Rightarrow$ ekstrem

$\operatorname{sled} Hf(6,-18) = 38 > 0 \Rightarrow$ v $(6,-18)$ imamo lokalni minimum

3. naloga

• $$\int_0^1 dx \int_0^{\frac{1-x}{2}} dy = \iint_{\text{trikotnik}} = \int_0^{\frac12} dy \int_0^{1-2y} dx$$



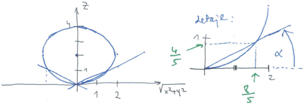
(namreč $\frac{1-x}{2} = y \iff x = 1-2y$)

• Izračunamo $$\int_0^1 dx \int_0^{\frac{1-x}{2}} \frac{dy}{(1+3x+2y)^2} = \int_0^1 \left(-\frac12 \cdot \frac{1}{1+3x+2y}\right)\Bigg|_{y=0}^{\frac{1-x}{2}} dx =$$

$$= -\frac12 \int_0^1 \left(\frac{1}{1+3x+1-x} - \frac{1}{1+3x}\right) dx = -\frac14 \int_0^1 \frac{dx}{1+x} + \frac12 \int_0^1 \frac{dx}{1+3x} = -\frac14 \log(1+x)\Big|_0^1 + \frac16 \log(1+3x)\Big|_0^1 =$$

$$= -\frac14 \log 2 + \frac16 \log 4 = \left(\frac13 - \frac14\right)\log 2 = \frac{1}{12}\log 2.$$

4. naloga Telo je vrtenina (rotacijsko simetrično). Skica prereza pri danem kotu φ (katerem koli):



Presečišča sfere in stožca: $x^2+y^2+(z-2)^2=4$ in $x^2+y^2=4z^2$

$\Rightarrow 4z^2+(z-2)^2=4$ oziroma $5z^2-4z=0 \Rightarrow z=0$ ali $z=\frac45$

Od tod $\sqrt{x^2+y^2} = \frac85$ in $\operatorname{tg}\alpha = \frac12$

Spodnja meja za krogelni r je vedno 0, zgornjo pa dobimo iz enačbe $x^2+y^2+z^2=4z$ (to je le preoblikovana enačba sfere) v krogelnih koordinatah $r^2 = 4r\sin\vartheta$ oziroma $r_{max} = 4\sin\vartheta$.

Tako $$V = \int_{\varphi=0}^{2\pi} d\varphi \int_{\vartheta=\operatorname{arctg}\frac12}^{\frac{\pi}{2}} d\vartheta \int_{r=0}^{4\sin\vartheta} r^2\cos\vartheta \, dr = \frac{2\pi}{3}\cdot 4^3 \int_{\operatorname{arctg}\frac12}^{\frac{\pi}{2}} \sin^3\vartheta\cos\vartheta\, d\vartheta \underset{[u=\sin\vartheta]}{=} \frac{2\pi}{3}\cdot 4^3 \int_{\sin(\operatorname{arctg}\frac12)}^{1} u^3\, du =$$

$$= \frac{2\pi}{3}\cdot 4^3 \cdot \frac14\left(1-\left(\sin(\operatorname{arctg}\tfrac12)\right)^4\right) = \frac{2\pi}{3}\cdot 16\cdot\left(1-\left(\frac{\frac14}{1+\frac14}\right)^2\right) = \frac{2\pi}{3}\cdot 16\cdot\frac{24}{25} = \frac{256\pi}{25}$$