$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$
$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$
$$\mathrm{tg}\,2\alpha = \frac{2\,\mathrm{tg}\,\alpha}{1-\mathrm{tg}^2\alpha}$$
$$\mathrm{ctg}\,2\alpha = \frac{\mathrm{ctg}^2\alpha - 1}{2\,\mathrm{ctg}\,\alpha}$$
$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$$
$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$
$$\mathrm{tg}\frac{\alpha}{2} = \frac{1-\cos\alpha}{\sin\alpha} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$$
$$\mathrm{ctg}\frac{\alpha}{2} = \frac{1+\cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1-\cos\alpha} = \pm\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}}$$

$$(kx+n)' = k$$
$$(C)' = 0$$
$$(C\cdot f)'(x) = C\cdot f'(x)$$
$$\left(\sum_{i=1}^{n} f_i\right)'(x) = \sum_{i=1}^{n} f_i'(x)$$
$$(x^n)' = n\cdot x^{n-1}; n\in R$$
$$(a^x)' = a^x\cdot \ln a; a>0$$
$$(a^x)^{(n)} = a^x\cdot \ln^n a; a>0$$

$$(e^x)' = e^x$$
$$x = e^{\ln x}$$
$$(\ln x)' = \frac{1}{x}$$
$$(\ln x)^{(n)} = \frac{(n-1)!}{x^n}$$
$$(\sin x)' = \cos x$$
$$(\cos x)' = -\sin x$$

$$(\mathrm{tg}\,x)' = \ldots$$
$$(\mathrm{ctg}\,x)' = - \ldots$$
$$(\arcsin x)' \ldots$$
$$(\arccos x)' \ldots$$

$$(f\pm g)'(x) = f'(x) \pm g'(x)$$
$$(f\cdot g)'(x) = f'(x)\cdot g(x) + f(x)\cdot g'(x)$$
$$\left(\frac{f}{g}\right)'(x) = \frac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{[g(x)]^2}; g(x)\neq 0$$

T: f in g sta odvedljivi funkciji.

$$(g\circ f)'(x) = g'(f(x))\cdot f' \ldots$$

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}; u = g(x), y \ldots$$

T: f je odvedljiva in bijektivna. Potem je f^{-1} tudi odvedljiva in velja:

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

DIFERENCIAL: $dy = f'(x_0)dx$

$$\int (f\pm g)dx = \int f\,dx \pm \int g\,d\ldots$$
$$\int C\cdot f\,dx = C\cdot\int f\,dx$$
$$\int u\,dv = uv - \int v\,du \qquad \mathrm{P}\ldots$$
$$\int\frac{dx}{x} = \ln|x| + C$$
$$\int\frac{f'}{f}dx = \ln|f| + C$$
$$\int x^n dx = \frac{x^{n+1}}{n+1} + C; n\neq 1$$
$$\int a^x dx = \frac{a^x}{\ln a} + C$$
$$\int a^{kx}dx = \frac{a^x}{k\cdot\ln a} + C$$
$$\int e^x dx = e^x + C$$
$$\int e^{kx}dx = \frac{1}{k}e^x + C$$
$$\int \sin x\,dx = -\cos x + C$$
$$\int \cos x\,dx = \sin x + C$$
$$\int \mathrm{sh}\,x\,dx = \mathrm{ch}\,x + C$$
$$\int \mathrm{ch}\,x\,dx = \mathrm{sh}\,x + C$$
$$\int\frac{dx}{\sin^2 x} = -\mathrm{ctg}\,x + C$$
$$\int\frac{dx}{\cos^2 x} = \mathrm{tg}\,x + C$$
$$\int\frac{dx}{\sqrt{1-x^2}} = \arcsin x + C$$
$$\int\frac{dx}{1+x^2} = \mathrm{arctg}\,x + C$$

$$\int\frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$$
$$\int\frac{dx}{a^2+x^2} = \frac{1}{a}\mathrm{arctg}\frac{x}{a} + C$$
$$\int\frac{dx}{\sqrt{x^2+k}} = \ln\left|x+\sqrt{x^2+k}\right| + C$$
$$\int\frac{p^{(n)}(x)}{\sqrt{ax^2+bx+c}}dx = q^{(n-1)}(x)\sqrt{ax^2+bx+c}\ldots$$
$$\int\frac{Ax+B}{(x^2+px+q)^n}dx = \frac{T^{(2n-3)}(x)}{(x^2+px+q)^{n-1}} + \int\ldots$$
$$\int\frac{S^{(m)}(x)}{(x-k)^n\sqrt{ax^2+bx+c}}, m<n: x-k = \frac{1}{t}$$
$$t = \mathrm{tg}\frac{x}{2} \quad \sin x = \frac{2t}{1+t^2} \quad \cos x = \frac{1-t^2}{1+t^2} \quad dx\ldots$$

$$\int_a^b (f(x)\pm g(x))dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$$
$$\int_a^b C\cdot f(x)dx = C\cdot\int_a^b f(x)dx$$
$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx \quad a<c<\ldots$$
$$\int_a^b f(x)dx = -\int_b^a f(x)dx$$
$$\int_a^a f(x)dx = 0$$

P: (**DRUGI OSNOVNI IZREK INTEGRALSKEGA RAČUNA**): Če je $G'(x) = f(x)\ \forall x$, je $\int_a^b f(x)dx = G(x)\Big|_a^b = G(b) - G(a)$

$$\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du$$

T: Če je $\varphi: [\alpha,\beta] \to [a,b]$ monotona odvedljiva funkcija in f zvezna, potem velja $\int_a^b f(x)dx = \int_\alpha^\beta f(\varphi(t))\varphi'(t)dt$, če je

$\varphi(\alpha) = a$ in $\varphi(\beta) = b$.

DVOJNI INTEGRAL:

$$z = f(x,y),\ \text{zvezna, definirana na } (D\cup\partial D)\subseteq R^2,\ D \text{ je omejena} \iint_D f(x,y)dp = \lim_{\Delta P\to 0}\sum_K f(\xi_K,\eta_K)\Delta_K p = \int_a^b dx \int_{\varphi(x)}^{\psi(x)}\ldots \quad \Delta P = \max\{\Delta_K x, \Delta_K y\}$$

Polarne koordinate:

$$x = r\cos\varphi \qquad r \geq 0$$
$$y = r\sin\varphi \qquad \varphi \in [0, 2\pi)$$
$$dp = r \cdot dr \cdot d\varphi \qquad (x,y) \in D \Rightarrow (r,\varphi) \in \Delta$$
$$\iint_D f(x,y)dp = \iint_\Delta f(r\cos\varphi, r\sin\varphi) r \cdot dr \cdot d\varphi$$

¤ $\int_0^\infty e^{-x^2}dx = \frac{\sqrt{\pi}}{2}$

Ploščina, masa in težišče lika:

$$S = \iint_D dxdy$$
$$m = \iint_D \rho(x,y)dxdy$$
$$x_T = \frac{1}{m}\iint_D xdxdy \quad y_T = \frac{1}{m}\iint_D ydxdy$$

Volumen cilindričnega telesa:

$$z = z(x,y) \quad V = \iint_D z(x,y)dxdy$$

TROJNI INTEGRAL:

$$D: x \in [a,b], y \in [\psi_1(x), \psi_2(x)], z \in [\varphi_1(x,y), \varphi_2(x,y)]$$
$$\iiint f(x,y,z)dV = \lim_{\substack{\Delta_K x \to 0 \\ \Delta_K y \to 0 \\ \Delta_K z \to 0}} \sum_K f(x_K, y_K, z_K)\Delta_K V = \int_a^b dx \int_{\psi_1(x)}^{\psi_2(x)} d$$

Volumen, masa in težišče lika:

$$V = \iiint_D dxdydz$$
$$m = \iiint_D \rho(x,y,z)dxdydz$$
$$x_T = \frac{1}{m}\iiint_D xdxdydz \quad y_T = \frac{1}{m}\iiint_D ydxdydz \quad z_T = \frac{1}{m}$$

Cilindrične koordinate:

$$x = r\cos\varphi \qquad r \geq 0$$
$$y = r\sin\varphi \qquad \varphi \in [0, 2\pi)$$
$$z = z \qquad z \in \mathbb{R}$$
$$dV = r \cdot dr \cdot d\varphi dz \qquad (x,y,z) \in D \Rightarrow (r,\varphi,z) \in \Delta$$
$$\iiint_D f(x,y,z)dV = \iiint_\Delta f(r\cos\varphi, r\sin\varphi, z) r \cdot dr \cdot d\varphi dz$$

Sferične koordinate:

$$x = R\sin\vartheta\cos\varphi \qquad R \geq 0$$
$$y = R\sin\vartheta\sin\varphi \qquad \vartheta \in [0, \pi)$$
$$z = R\cos\vartheta \qquad \varphi \in [0, 2\pi)$$
$$dV = R^2\sin\vartheta \cdot dR \cdot d\vartheta \cdot d\varphi \qquad (x,y,z) \in D \Rightarrow (r,\vartheta,\varphi) \in \Delta$$
$$\iiint_D f(x,y,z)dV = \iiint_\Delta f(R\sin\vartheta\cos\varphi, R\sin\vartheta\sin\varphi, R\cos\vartheta$$

Splošne koordinate:

$$x = x(u,v) \qquad y = y(u,v) \qquad dp = |J| \cdot du \cdot dv \qquad J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$
$$(x,y) \in D, \ (u,v) \in \Delta$$
$$\iint_D f(x,y,z)dV = \iint_\Delta f(x(u,v), y(u,v))|J| \cdot du \cdot dv$$

$$x = x(u,v,w) \qquad y = y(u,v,w) \qquad z = z(u,v,w) \qquad dp = |J| \cdot du \cdot dv \cdot dw$$
$$J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{vmatrix}$$
$$(x,y,w) \in D, \ (u,v,w) \in \Delta$$
$$\iiint_D f(x,y,z)dV = \iiint_\Delta f(x(u,v,w), y(u,v,w), z(u,v,w))|J$$

T: INTEGRALI S PARAMETROM: Naj bo f zvezna funkcija na pravokotniku $[a,b] \times [c,d]$. Potem je $I(y) = \int_a^b f(x,y)dx$ zvezna funkcija spremenljivke $y \in [c,d]$. Če $\exists \frac{\partial f}{\partial y}(x,y)$ in je zvezen, potem je I odvedljiva in $I'(y) = \int_a^b \frac{\partial f}{\partial y}(x,y)dx$.

$$\frac{\partial}{\partial y}\sum_i f(x,y) = \sum_i \frac{\partial f}{\partial y}(x,y).$$

D: Integral $\int_a^b \frac{\partial f}{\partial y}(x,y)dx$ je **ENAKOMERNO KONVERGENTEN** za $y \in A$, če

$$(\forall \varepsilon > 0)(\exists M \in \mathbb{R})\left(b \geq M \Rightarrow \left|\int_a^b f(x,y)dx - I(y)\right| < \varepsilon \ \forall y \in A\right).$$

I: Naj bo f zvezna funkcija na $P = [a,\infty) \times [c,d]$. Če je integral $I(y) = \int_a^\infty f(x,y)dx$ enakomerno konvergenten za $y \in [c,d]$, potem je I zvezna funkcija. Če je integral $I(y) = \int_a^\infty f(x,y)dx$ konvergenten, $\frac{\partial f}{\partial y}$ zvezen na P, integral $\int_a^\infty \frac{\partial f}{\partial y}(x,y)dx$ pa enakomerno konvergenten, velja $I'(y) = \int_a^\infty \frac{\partial f}{\partial y}(x,y)dx$.

I: Če je f zvezna na $P = [a,b] \times [c,d]$, potem je $\int_c^d \int_a^b f(x,y)dxdy = \int_a^b \int_c^d f(xy)dxdy = \iint_P f(x,y)dp$.

I: f je zvezna na $P=[a,\infty)\times[c,d]$. Če je integral $I(y)=\int_a^{\infty} f(x,y)\,dx$ enakomerno konvergenten za $y\in[c,d]$, potem je

$$\int_c^d\int_a^{\infty} f(x,y)\,dx\,dy=\int_a^{\infty}\int_c^d f(xy)\,dx\,dy$$

GAMA FUNKCIJA: $\Gamma(x)=\int_0^{\infty} t^{x-1}e^{-t}\,dt \quad x>0$

$$\Gamma(x+1)=x\cdot\Gamma(x)$$

$$\Gamma(n)=(n-1)!$$

$$\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$$

$$\Gamma(x)\Gamma(1-x)=\frac{\pi}{\sin \pi x}$$

$$\operatorname{erf}(x)=\frac{2}{\sqrt{\pi}}\cdot\int_0^{\infty} e^{-t^2}\,dt \Rightarrow \operatorname{erf}(\infty)=1$$

BETA FUNKCIJA:

$$B(x,y)=\int_0^1 t^{x-1}(1-t)^{y-1}\,dt \quad x,y>0$$

$$B(x,y)=B(y,x)$$

$$B(x,y)=\frac{\Gamma(x)\cdot\Gamma(y)}{\Gamma(x+y)}$$

$$B(x,y)=2\int_0^{\frac{\pi}{2}} \sin^{2x-1}\varphi\cos^{2y-1}\varphi\,d\varphi$$

Stirlingova formula: $n!\doteq\sqrt{2\pi n}\left(\frac{n}{e}\right)^n$

D: **VEKTORSKO POLJE** na množici $D\subseteq R^{2(3)}$ je preslikava $F:D\to R^{2(3)}$, ki vsaki točki te množice priredi vektor.

D: Vektorsko polje je **POTENCIALNO**, če obstaja taka funkcija u, ki je **POTENCIAL ALI PRIMITIVNA FUNKCIJA** za F, da je $F=\operatorname{grad}u$

D: $u=u(x,y)$, $\vec{F}=(M(x,y),N(x,y))$, $M=\frac{\partial u}{\partial x}$, $N=\frac{\partial u}{\partial y}$. **TOTALNI DIFERENCIAL** je

$M(x,y)dx+N(x,y)dy.$

I: Potrebni pogoj, da je $M(x,y)dx+N(x,y)dy$ totalni diferencial kake funkcije, je $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$.

D: $u=u(x,y,z)$, $\vec{F}=(P(x,y,z),Q(x,y,z),R(x,y,z))$, $P=\frac{\partial u}{\partial x}$, $Q=\frac{\partial u}{\partial y}$, $R=\frac{\partial u}{\partial z}$. **TOTALNI DIFERENCIAL** je

$Pdx+Qdy+Rdz$

I: Potrebni pogoj, da je $Pdx+Qdy+Rdz$ totalni diferencial kake funkcije, je $\frac{\partial R}{\partial y}=\frac{\partial Q}{\partial z}\wedge\frac{\partial R}{\partial x}=\frac{\partial P}{\partial z}\wedge\frac{\partial Q}{\partial x}=\frac{\partial P}{\partial y}$.

KRIVULJNI INTEGRAL: – ***vektorsko polje***:

$F:R^3\to R^3$, $F(x,y,z)=(P(x,y,z),Q(x,y,z),R(x,y,z))$, $K:\vec{r}=(x,y,z)=\vec{r}(t)=(x(t),y(t),z(t))$, $\dot{\vec{r}}=(\dot{x},\dot{y}$

$$\int_K \vec{F}d\vec{r}=\int_a^b \vec{F}\circ\dot{\vec{r}}\,dt$$.

– ***skalarno polje***: $u:R^3\to R^3$, $K: x=x(t), y=y(t), z=z(t)$, $t\in[a,b]$; $\int_K u\,ds=\int_a^b u\cdot\sqrt{\dot{x}^2+\dot{y}^2+\dot{z}^2}\,dt$.

T: Če je F potencialno polje, $F=\operatorname{grad}u$, je krivuljni integral $\int_K \vec{F}d\vec{r}=u(B)-u(A)$. Torej je krivuljni integral potencialnega polj enak razliki potenciala v končni in začetni točki. Krivuljni integral potencialnega polja je zato neodvisen od krivulje, ki povezuje dve dani očki. Velja tudi obratno: če je F tako vektorsko polje, da je za vsaki dve točki njegov krivuljni integral enak za vse krivulje, ki povezujejo ti dve točki, je to polje potencialno.

D: Krivulja, parametrizirana z enačbo $r=r(t)$, $t\in[a,b]$, je sklenjena, če je $r(a)=r(b)$.

T: Vektorsko polje F je potencialno natanko takrat, ko je njegov krivuljni integral $\oint_K \vec{F}d\vec{r}=0$ za vsako sklenjeno krivuljo K.

GREENOVA FORMULA: Naj bo D območje v ravnini, omejeno s končno mnogo sklenjenimi krivuljami, ki se dajo parametrizirati z odsekoma zvezno odvedljivimi funkcijami. Naj bosta M in N zvezni funkciji dveh spremenljivk, definirani na $D\cup\partial D$. Potem velja:

$F=(M,N)$, $\partial D:\vec{r}=\vec{r}(t), t\in[a,b]$, $\oint_{\partial D} M\,dx+N\,dy=\iint_D\left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right)dx\,dy$.

T: Če je $F=(M,N)$ vektorsko polje, definirano na območju D 'brez lukenj', potem je F potencialno natanko takrat, ko je $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$.

/rd/	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
/°/	0	30	45	60	90	180	270
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	−1

cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	–1	0
tg	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞	0	∞
ctg	∞	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	∞	0