Heisenbergove neenakosti

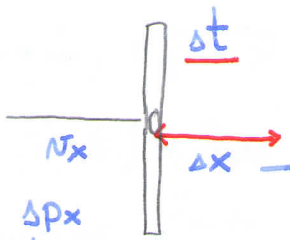
$$\boxed{\begin{aligned} \Delta x \Delta p_x &\geq h \\ \Delta y \Delta p_y &\geq h \\ \Delta z \Delta p_z &\geq h \end{aligned}}$$

PO DIMENZIJAH

- učbenik str. 17

$\Delta p_x \to 0$

$\Delta x = \dfrac{h}{\Delta p_x} \to \infty$



$\to \Delta x = v_x \Delta t$

poznamo: $m \cdot \Delta v_x$

$\Delta x \cdot \Delta p_x = v_x \Delta t \, m \Delta v_x =$

$= \Delta t \, m v_x \Delta v_x$

$W_k = \frac{1}{2} m v_x^2$

$dW_k = m v_x dv_x$

$\Delta W_k = m v_x \Delta v_x$

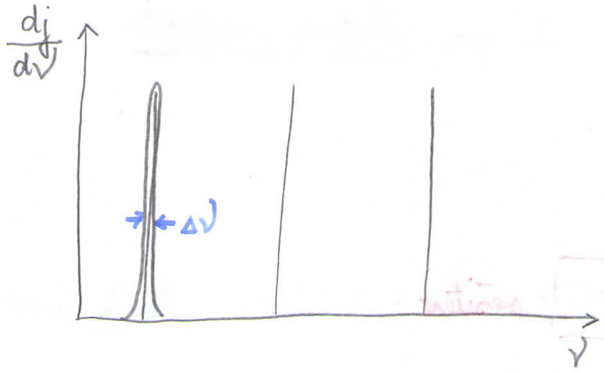
$\boxed{\Delta t \cdot \Delta W_k \geq h}$

$\boxed{\Delta t \Delta W \geq h}$ SPLOŠNA ENAČBA

$W = h\nu$

$\Delta W = h \Delta \nu$

$\boxed{\Delta t \Delta \nu \geq 1}$ - razlog, da imajo spektralne črte neko KONČNO ŠIRINO:



$\Delta p_x \Delta x \geq h$

$\Downarrow$

$\boxed{\Delta L \Delta \varphi \geq h}$ ENAČBA, VKLJUČUJOČ VRTILNO KOLIČINO

KVANTNA MEHANIKA

Imamo „nek" PRINCIP NEDOLOČENOSTI ; npr. pri e^- curku ne moremo ločiti e^-.
V kv. meh. „nadev" ne moremo ponavljati. Z nebo statistično vrednostjo lahko določimo rezultate.

PRINCIPI KV. MEH:

- stanje v kv. meh. sistemu je popolnoma določeno z VALOVNO FUNKCIJO $\boxed{\psi(\vec{r},t)}$ OSI, ki je odvisna od KRAJA in ČASA; ima lastnost, da je $\boxed{|\psi|^2 d\vec{r}\, dt}$ verjetnost, da se KV. DELEC nahaja v volumskem elementu $d\vec{r}\, dt$ ob času t na položaju r
- vsaki količini iz klas. mehanike v kv. meh. pripada LINEARNI HERMITSKI OPERATOR
- za vsako meritev količine, ki ji ustreza OPERATOR A, lahko vedno vidimo le LASTNO VREDNOST OPERATORJA $\boxed{\hat{A}\psi = a\psi}$
- če je delec v stanju, ki ga opisuje $\psi(\vec{r},t)$, potem je pov. vrednost količine, ki ji ustreza vrednost A taka:

$$\boxed{\langle \hat{A} \rangle = \int \psi^* \hat{A} \psi \, dV}$$

- val. funkc. sistema se v času obnaša glede na Schrödingerjevo enačbo

$$\boxed{\hat{H}\psi = -i\hbar \frac{\partial \psi}{\partial t}}$$

- celotna funkc. mora biti ASIMETRIČNA glede na zamenjavo FERMIONOV (delci, ki imajo polovični spin)

Klasična | !! Kvantna !! | $\wedge$ - OPERATOR | - množenje $\cdot$

$i^2 = -1$

x — $\hat{x} = x\cdot$

y — $\hat{y} = y\cdot$

z — $\hat{z} = z\cdot$

$\vec{r}$ — $\hat{\vec{r}} = (x, y, z)\cdot$

p_x — $\hat{p}_x = -i\hbar \frac{\partial}{\partial x}\cdot$

p_y — $\hat{p}_y = -i\hbar \frac{\partial}{\partial y}\cdot$

p_z — $\hat{p}_z = -i\hbar \frac{\partial}{\partial z}\cdot$

p — $\hat{\vec{p}} = -i\hbar\vec{\nabla}\cdot = -i\hbar\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$

W_k — $\hat{W}_k = \frac{1}{2}\frac{\hat{p}^2}{m} = \frac{1}{2m}(i\hbar\vec{\nabla})^2 = \frac{-\hbar^2}{2m}\vec{\nabla}^2\cdot$

1D : $\vec{\nabla}^2 = \frac{\partial^2}{\partial x^2}$

3D : $\vec{\nabla}^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

V — $\hat{V}(\hat{\vec{r}}) = V(\vec{r})\cdot$

$V(x) = \frac{1}{2}kx$ — $\hat{V} = \frac{1}{2}kx^2$

Hamiltonov operator :

$$\hat{H} = -\frac{\hbar^2}{2m}\vec{\nabla}^2 + V$$

$\vec{L} = \vec{r}\times\vec{p}$ — $\hat{\vec{L}} = \hat{\vec{r}}\times\hat{\vec{p}} = -i\hbar\begin{vmatrix} \vec{i}, & \vec{j}, & \vec{k} \\ x, & y, & z \\ \frac{\partial}{\partial x}, & \frac{\partial}{\partial y}, & \frac{\partial}{\partial z} \end{vmatrix}\begin{matrix} \vec{i} & \vec{j} \\ x & y \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \end{matrix} =$

$$\hat{\vec{L}} = \left(\vec{i}\,y\frac{\partial}{\partial z} + \vec{j}\,z\frac{\partial}{\partial x} + \vec{k}\,x\frac{\partial}{\partial y} - \vec{i}\,z\frac{\partial}{\partial y} - \vec{j}\,x\frac{\partial}{\partial z} - \vec{k}\,y\frac{\partial}{\partial x}\right)(-i\hbar) =$$

$$= -i\hbar\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y},\ z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z},\ x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right) \Downarrow$$

$$\hat{L}_x = -i\hbar\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right)$$
$$\hat{L}_y = -i\hbar\left(z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z}\right)$$
$$\hat{L}_z = -i\hbar\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right)$$

$$\hat{\vec{L}} = (\hat{L}_x, \hat{L}_y, \hat{L}_z)$$

$$\hat{L}^2 = \hat{L}_x^2 + \hat{L}_y^2 + \hat{L}_z^2$$

$$[\hat{x}, \hat{p}_x]f = (\hat{x}\hat{p}_x - \hat{p}_x\hat{x})f = \left(x\cdot\left(-i\hbar\frac{\partial}{\partial x}\right) + i\hbar\frac{\partial}{\partial x}x\cdot\right)f =$$
$$= x\left(-i\hbar\frac{\partial}{\partial x}\right)f + i\hbar\frac{\partial}{\partial x}xf =$$
$$= \cancel{-i\hbar x\frac{\partial f}{\partial x}} + i\hbar\left(1f + \cancel{x\frac{\partial f}{\partial x}}\right) =$$
$$[\vec{x}, \vec{p}_x]f = i\hbar f$$
$$[\vec{x}, \hat{p}_x] = i\hbar$$

ne komutirata
$\Downarrow$
istočasno dveh operatorjev ne moremo poznati (Heisenberg)

$$[\hat{x}, \hat{y}] = \hat{x}\hat{y} - \hat{y}\hat{x} = xy - yx = \underline{\underline{0}}$$
} poznamo obe koordinati hkrati

KOMUTIRATA

$$[\hat{p}_x, \hat{p}_y]f = -i\hbar\frac{\partial}{\partial x}\left(-i\hbar\frac{\partial}{\partial y}\right)f - \left(-i\hbar\frac{\partial}{\partial y}\right)\left(-i\hbar\frac{\partial}{\partial x}\right)f =$$

KOMUTIRATA

$$= -\hbar^2\frac{\partial^2 f}{\partial x\partial y} + \hbar^2\frac{\partial^2 f}{\partial y\partial x} = \underline{\underline{0}}$$

$$[\hat{y}, \hat{p}_x] = \underline{\underline{0}}$$

V kv. meh. imamo LINEARNE OPERATORJE, velja:

$$\underline{A(af + bg) = aAf + bAg}$$