$\Psi = \sum_i c_i \psi_i$ $\sim$ verjetnost, da se delec nahaja v ψ_i = $\boxed{|c_i|^2}$

$|\psi|^2 = \sum_i c_i^* \psi_i^* \sum_j c_j \psi_j$

$E_1 \ldots |c_1|^2$

$E_2 \ldots |c_2|^2$

POV. ENERGIJA:

$$\boxed{\langle \hat{H} \rangle = \langle \psi | \hat{H} | \psi \rangle}$$

$$\boxed{[\hat{A}, \hat{B}] = 0}$$

A in B imata ENAKE LASTNE FUNKCIJE.

$\hat{B} \,/\, \hat{A}\Psi = \lambda_A \Psi$

$\hat{A} \,/\, \hat{B}\psi = \lambda_B \psi$

$\Rightarrow$

$\hat{B}\hat{A}\psi = \hat{B}\lambda_A \Psi = \lambda_A \hat{B}\psi = \lambda_A \lambda_B \Psi$

$\hat{A}\hat{B}\psi = \hat{A}\lambda_B \psi = \lambda_B \hat{A}\psi = \lambda_B \lambda_A \Psi$

$$[\hat{B}, \hat{A}]\psi = \hat{B}\hat{A}\psi - \hat{A}\hat{B}\psi = \lambda_A \lambda_B \Psi - \lambda_B \lambda_A \psi = 0$$

$$\boxed{[\hat{B}, \hat{A}] = 0}$$

$[\hat{L}^2, \hat{L}_z] = 0$

$\Downarrow$

$\hat{L}_z = m \hbar Y_{lm}$

$\hat{L}^2 = l(l+1) \hbar Y_{lm}$

Prosti delec

Na ta delec ne deluje nobena sila.

$\boxed{V=0}$ potencial je 0

$v =$ konst.

$x = vt \qquad W_k = \frac{1}{2}mv^2 = \frac{p^2}{2m}$

$$\hat{H}\psi = E\psi$$

$$\hat{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \cancel{V}$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} = E\psi \quad / \cdot \frac{-2m}{\hbar^2}$$

$$\frac{\partial^2\psi}{\partial x^2} + \frac{2mE}{\hbar^2}\psi = 0 \qquad \boxed{\frac{2mE}{\hbar^2} = k^2}$$

$$\psi'' + k^2\psi = 0$$

$$\psi = Ae^{ikx} + Be^{-ikx}$$

$$\Psi(x,t) = \left(Ae^{ikx} + Be^{-ikx}\right)e^{-\frac{iE}{\hbar}t}$$

$$\Psi(x,t) = Ae^{ikx-\frac{iE}{\hbar}t} + Be^{-ikx-\frac{iE}{\hbar}t}$$

ravno valovanje:

$$S(x,t) = S_0 \cdot e^{ikx-i\omega t}$$

ta delec se obnaša kot VALOVANJE

$$\boxed{\vec{j} = \frac{\hbar}{2mi}\left(\psi^*\vec{\nabla}\psi - \psi\vec{\nabla}\psi^*\right)}$$

$$j_x = \frac{\hbar}{2mi}\left(\psi^*\frac{\partial\psi}{\partial x} - \psi\frac{\partial\psi^*}{\partial x}\right) = \frac{\hbar}{2mi}\Big(\left(A^*e^{-ikx} + B^*e^{ikx}\right)\left(Aike^{ikx} + B(-ik)e^{-ikx}\right) - \left(Ae^{ikx} + Be^{-ikx}\right)\left(A^*(-ik)e^{-ikx} + B^*ike^{ikx}\right)\Big) =$$

$$\psi = Ae^{ikx} + Be^{-ikx}$$

$$\frac{\partial\psi}{\partial x} = Aike^{ikx} + B(-ik)e^{-ikx}$$

$$\psi^* = A^*e^{-ikx} + B^*e^{ikx}$$

$$\frac{\partial\psi^*}{\partial x} = A^*(-ik)e^{-ikx} + B^*(ik)e^{ikx}$$

$$= \frac{\hbar ik}{2mi}\Big(\big(|A|^2 - \cancel{A^*Be^{-2ikx}} + \cancel{B^*Ae^{2ikx}} - |B|^2\big) - \big(-|A|^2 + \cancel{AB^*e^{2ikx}} - \cancel{BA^*e^{-2ikx}} + |B|^2\big)\Big) =$$

$$= \frac{\hbar k \cancel{i}}{\cancel{2}m\cancel{i}}\left(\cancel{2}|A|^2 \cancel{ik} - \cancel{2}|B|^2 \cancel{ik}\right) =$$

$$j_x = \frac{\hbar k}{m}\left(|A|^2 - |B|^2\right)$$

$$\Psi = Ae^{ikx} + Be^{-ikx}$$

$$j_x = \frac{\hbar k}{m}|A|^2 \longrightarrow \qquad j_x = \frac{\hbar k}{m}|B|^2 \longleftarrow$$

$$\rho = \Psi^*\Psi = A^* e^{-ikx} A e^{ikx} = |A|^2$$

$$\boxed{\begin{array}{l}\langle \hat{x} \rangle = 0 \\ \langle \hat{x}^2 \rangle = \infty\end{array}}$$

$$\langle \hat{x} \rangle$$

$$\int_{-\infty}^{\infty} x \rho \, dx$$

$$\lim_{x_0 \to \infty} \int_{x_0}^{x} x \rho \, dx = 0$$

E - energija

$$E = \frac{p^2}{2m} \Rightarrow p = \sqrt{2mE}$$

$$\boxed{\Delta p = 0} \Rightarrow \boxed{\Delta x \Delta p \geq \hbar}$$

$$\Delta x \geq \frac{\hbar}{\Delta p} \Rightarrow$$

$$\Delta x = \infty \; ???$$

ne vemo, kje je delec

$$\boxed{-\infty < x < \infty}$$

Δx - povprečje položajev, kjer je delec lahko

$$\bar{x} = \frac{\sum x_i}{N}$$

$$s^2(\sigma^2) = \frac{\sum (x_i - \bar{x})^2}{N}$$

$N-1$; majhno št. meritev

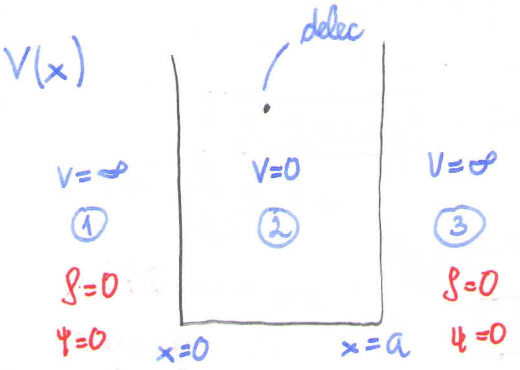
$$\boxed{\Delta x^2 = s^2 = \overline{x^2} - \bar{x}^2}$$

$$\langle \hat{x}^2 \rangle = \int x^2 \rho \, dx$$

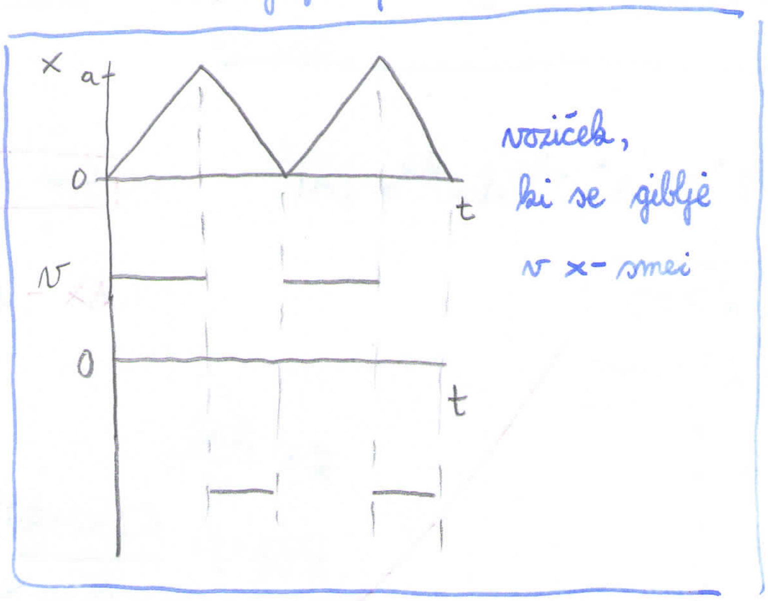
$$\Delta x^2 = \langle \hat{x}^2 \rangle - \langle \hat{x} \rangle^2 = $$

$$= \infty$$

Delec v neskončni potencialni jami



ven delec ne more priti zaradi zunanjega potenciala



$$\boxed{V(x) = \begin{cases} 0\,; & 0 \leq x \leq a \\ \infty\,, & (x>a) \vee (x<0) \end{cases}}$$

1, 3: $\hat{H}\psi = E\psi$

$$\hat{H} = -\frac{\hbar^2}{2m} + \frac{\partial^2}{\partial x^2} + \infty$$

$$\Rightarrow -\frac{\hbar^2}{2m} + \frac{\partial^2 \psi}{\partial x^2} + \infty\psi = E\psi \Rightarrow \underline{\psi = 0}$$

- $\rho = 0$ v 1 in 3, ker delec tja ne more priti $\Rightarrow \psi = 0$ (dokaz z računom in logiko).

2: $V = 0$

$$-\frac{\hbar}{2m}\frac{\partial^2\psi}{\partial x^2} = E\psi$$

$$\psi'' + k^2\psi = 0$$

$\Downarrow$ delec je vezan $\Rightarrow$ sin|cos

$$\underline{\psi = A\sin kx + B\cos kx}$$

Določiti moramo A, B in k!

$$\left.\begin{array}{l} \psi(0) = 0 \\ \psi(a) = 0 \end{array}\right\} \text{pogoj za zveznost}$$

$$\psi(0) = 0 = A\sin k0 + B\cos k0 = B$$

$$\underline{B = 0}$$

$$\psi = A\sin kx$$

$$\psi(a) = 0 = A\sin kA$$

ni fiz. rešitev

1. $A = 0 \Rightarrow \underline{\psi = 0}$ nimamo delca, saj je $\rho = 0$
2. $\sin kx = 0$ $\boxed{ka = n\pi}$ $n \in \mathbb{N}$

$k = \frac{n\pi}{a} \Rightarrow \boxed{\psi_n = A_n \sin \frac{n\pi x}{a}}$

$$\rho_n = |\psi|^2 = A_n^2 \sin^2 \frac{n\pi x}{a}$$

en delec ima gostoto 1

$$1 = \int_0^a \rho_n \, dx = \int_0^a A_n^2 \sin^2 \frac{n\pi x}{a} dx = A_n^2 \int_0^a \frac{1}{2}\left(1 - \cos \frac{2n\pi x}{a}\right) dx =$$

$$= \frac{A_n^2}{2}\left(x - \frac{1}{\frac{2n\pi}{a}} \sin \frac{2n\pi x}{a}\right)\Big|_0^a =$$

$$\underline{\psi_n = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a}}$$

$$= \frac{A_n^2}{2}(a - 0) = 1$$

$$\frac{2mE_n}{\hbar^2} = k_n^2 \qquad \frac{a A_n^2}{2} = 1 \qquad \boxed{A_n = \sqrt{\frac{2}{a}}}$$

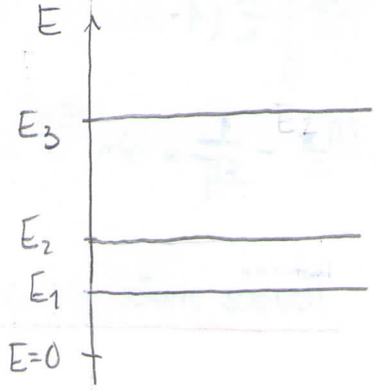
$$E_n = \frac{\hbar^2 k_n^2}{2m} = \frac{\hbar^2 n^2 \pi^2}{2ma^2} = \frac{h^2 n^2 \cancel{\pi^2}}{4\cancel{\pi^2} 2ma^2} = \frac{h^2 n^2}{8ma^2}$$

$$\boxed{E_n = \frac{n^2 h^2}{8ma^2}} \; ; \; n \in \mathbb{N}$$

naš delec se obnaša kot STOJEČE VALOVANJE

n	E_n
1	$\frac{h^2}{8ma^2} = E_1$
2	$\frac{4h^2}{8ma^2} = 4E_1$
3	$\frac{9h^2}{8ma^2} = 9E_1$
4	

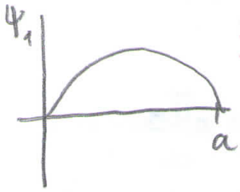
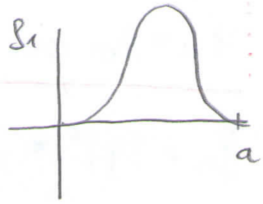
$$\boxed{E_1 : E_2 : E_3 : \ldots : E_n = 1 : 4 : 9 : \ldots : n^2 : \ldots}$$



Delec ne more imeti energije 0, saj bi potem imel $p = 0 \Rightarrow$ bi miroval, lahko bi mu določili x; vendar v skladu s Heisenbergovim principom nedoločljivosti to NI MOGOČE.

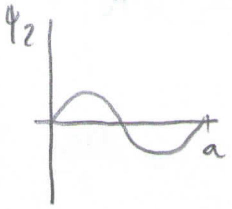
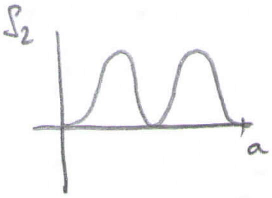
$\psi_1 = \sqrt{\frac{2}{a}} \sin \frac{\pi x}{a}$

$\rho_1 = \frac{2}{a} \sin^2 \frac{\pi x}{a}$

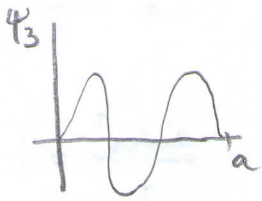
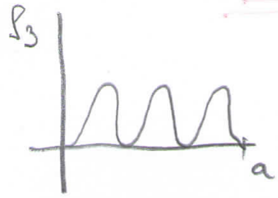
$\psi_2 = \sqrt{\frac{2}{a}} \sin \frac{2\pi x}{a}$

$\rho_2 = \frac{2}{a} \sin^2 \frac{2\pi x}{a}$

$\psi_3 = \sqrt{\frac{2}{a}} \sin \frac{3\pi x}{a}$

$\rho_3 = \frac{2}{a} \sin^2 \frac{3\pi x}{a}$

Pri več ničlah imamo večje nihanje; delec ima večjo energijo.

$$\langle \hat{x} \rangle = \int \psi^* \hat{x} \psi \, dx = \qquad \hat{x} = x.$$

$$= \int \psi^* x \psi \, dx = \int x \psi^* \psi \, dx = \int x |\psi|^2 dx$$

Potrebuješ Bronsteinov priročnik!

$$\rho_1 = \frac{2}{a} \sin^2 \frac{\pi x}{a}$$

$$\langle \hat{x} \rangle = \int_0^a x \frac{2}{a} \sin^2 \frac{\pi x}{a} dx = \frac{2}{a} \int_0^a \frac{x}{2} \left(1 - \cos \frac{2\pi x}{a}\right) dx = \frac{1}{a} \int_0^a \left(x - x \cos \frac{2\pi x}{a}\right) dx =$$

$$= \frac{1}{a} \left(\frac{x^2}{2} + \frac{1}{\frac{2\pi}{a}} \cos \frac{2\pi x}{a} - \frac{1}{\frac{2\pi}{a}} x \sin \frac{2\pi x}{a} \right) \Big|_0^a =$$

$$= \underline{\underline{\frac{a}{2}}}$$ – Težišče naše porazdelitve

$$\hat{p}_x = -i\hbar \frac{\partial}{\partial x}$$

$$\langle \hat{p}_x \rangle = \int_0^a \psi^* \hat{p}_x \psi \, dx = \int_0^a \psi^* \left(-i\hbar \frac{\partial \psi}{\partial x}\right) dx =$$

$$= -i\hbar \int_0^a \psi^* \frac{\partial \psi}{\partial x} dx = -i\hbar \int_0^a \sqrt{\frac{2}{a}} \sin \frac{\pi x}{a} \sqrt{\frac{2}{a}} \frac{\pi}{a} \left(\cos \frac{\pi x}{a}\right) dx =$$

$$= -i\hbar \frac{2}{a} \frac{\pi}{a} \int_0^a \sin \frac{\pi x}{a} \cos \frac{\pi x}{a} dx = \underline{\underline{0}}$$ – gib. količina je VEKTOR; delec ima enako verjetnost, da se giblje v pozitivne in negativne xe.

$\boxed{\text{KVANTNI DELCI NIKOLI NE MIRUJEJO}}$

$$\langle \hat{H} \rangle = \int \psi_1^* \hat{H} \psi_1 \, dx = \int \psi_1^* E_1 \psi_1 \, dx = E_1 \int \psi_1^* \psi_1 \, dx = \underline{\underline{E_1}}$$

$\boxed{\hat{H}\psi_1 = E_1\psi}$ po definiciji

kv. delec v OSNOVNEM STANJU ima HAMILTONOV OPERATOR enako vrednost kot ENERGIJA osnovnega stanja

$$\boxed{\langle \psi_i | \psi_j \rangle = \int_0^a \psi_i^* \psi_j \, dx = \begin{cases} 1; & i = j \\ 0; & i \neq j \end{cases}}$$

$$\psi = a\psi_1 + b\psi_2$$

$$\langle \psi | \psi \rangle = 1$$

$$\langle a\psi_1 + b\psi_2 | a\psi_1 + b\psi_2 \rangle = 1$$

$$\langle a\psi_1 | a\psi_1 \rangle + \langle a\psi_1 | b\psi_2 \rangle + \langle b\psi_2 | a\psi_1 \rangle + \langle b\psi_2 | b\psi_2 \rangle = 1 \iff \int (a\psi_1 + b\psi_2)(a\psi_1 + b\psi_2) \, dx$$

$$a^* a \underbrace{\langle \psi_1 | \psi_1 \rangle}_{1} + a^* b \underbrace{\langle \psi_1 | \psi_2 \rangle}_{0} + b^* a \underbrace{\langle \psi_1 | \psi_2 \rangle}_{0} + b^* b \underbrace{\langle \psi_1 | \psi_2 \rangle}_{1} = 1$$

ψ_1 in ψ_2 sta $\perp$ ENA NA DRUGO

$$|a|^2 + |b|^2 = 1$$

$|a|^2$ ... verjetnost

$$\psi = a\psi_1 + b\psi_2$$

izmerimo E_1 z verjetnostjo $|a|^2$, E_2 z verjetnostjo $|b|^2$

$$\langle \psi | \hat{H} | \psi \rangle = \langle \hat{H} \rangle$$

$$\underline{\langle \hat{H} \rangle = |a|^2 E_1 + |b|^2 E_2}$$