Glej prejšnjo nalogo! ($Be^{3+}$)

$ze_0$ $\xleftarrow{F_e}$ $-e_0$

$$\frac{z e_0^2}{4\pi\varepsilon_0 r_n^{\cancel{2}}} = m_e \frac{v_n^2}{\cancel{r_n}}$$

$r_n = n^2 a_0$

$$m_e v_n r_n = n\hbar$$

$$v_n = \frac{n\hbar}{m_e r_n}$$

$\Downarrow$

5. vzb. stanje => $\underline{n=6}$

$$\frac{z e_0^2}{4\pi\varepsilon_0 \cancel{r_n}} = \cancel{m_e} \frac{n^2\hbar^2}{m_e^{\cancel{2}} r_n^{\cancel{2}}}$$

$$r_n = \frac{n^2 \underbrace{\hbar^2 4\pi\varepsilon_0}_{a_0}}{m_e\, z\, e_0^2} = \frac{a_0 n^2}{z} =$$

$$= \frac{6^2 \cdot a_0}{z} = \frac{\overset{9}{\cancel{36}} \cdot 0{,}529\,\mathring{A}}{\cancel{4}} =$$

$$\underline{\underline{r_n = 4{,}761\,\mathring{A}}}$$

$\lambda$ $\quad$ $\underset{n=2}{1.\,vz.\,st.} \rightarrow \underset{m=3}{2.\,vz.\,st.}$

$\Delta E = E_m - E_n$

$$E_n = \frac{-13{,}6\,eV \cdot z^2}{n^2} = \frac{-13{,}6\,eV \cdot 4^{\cancel{2}}}{\cancel{2^2}} = -54{,}4\,eV$$

$$E_m = \frac{-13{,}6\,eV \cdot 4^2}{3^2} = -24{,}18\,eV$$

$$\underline{\Delta E = 30{,}22\,eV}$$

$\Delta E = h\nu$

$$\nu = \frac{\Delta E}{h} = \frac{30{,}22\,eV}{6{,}62 \cdot 10^{-34}\,Js} = \frac{30{,}22 \cdot 1{,}6 \cdot 10^{-19}\,\cancel{J}}{6{,}62 \cdot 10^{-34}\,\cancel{J}s} = 7{,}3 \cdot 10^{15}\,s^{-1}$$

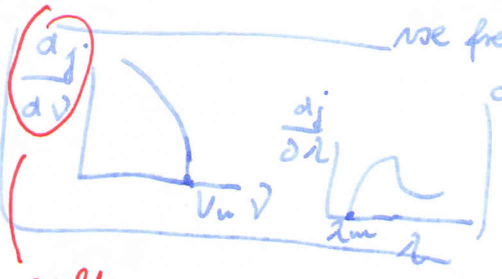
$$\lambda = \frac{c}{\lambda} = \frac{3 \cdot 10^8\,m\,\cancel{s}}{7{,}3 \cdot 10^{15}\,\cancel{s}} = 4{,}1 \cdot 10^{-8}\,m = \underline{\underline{41\,nm}} - UV$$

Rentg. cev deluje s 60 kV. Kakšne fotone oddaja ta cev? Ali lahko opazimo K črto, če je elektroda iz volframa?

$E = eU \Rightarrow \boxed{eU \geq h\nu}$ $\qquad \nu = \frac{eU}{h} = \frac{1{,}6\cdot 10^{-19}\,As \cdot 60\cdot 10^{3}\,V}{6{,}62\cdot 10^{-34}\,Js} = \underline{\underline{1{,}45\cdot 10^{19}\,s^{-1}}}$

$U = 60\,kV \Rightarrow eU = 60\,keV$

$\lambda_m = \frac{c}{\nu_m} = \frac{3\cdot 10^{8}}{1{,}45\cdot 10^{19}} = 2{,}1\cdot 10^{-11}\,m$




spekter

vse frekvence manjše od $1{,}45\cdot 10^{19}\,s^{-1}$

Fotone, ki imajo $\nu < 1{,}45\cdot 10^{19}\,s^{-1}$ oz. $\lambda > 2{,}1\cdot 10^{-11}\,m$.

$\qquad\qquad\qquad\qquad\qquad\qquad \| $

$\qquad\qquad\qquad\qquad\qquad\qquad 21\,pm$

$\frac{1}{\lambda_K} = \frac{(z-1)^2}{\lambda_0}$ $\qquad \lambda_0 = 121{,}6\,nm$

$z(W) = 74$

$\lambda_K = \frac{\lambda_0}{(z-1)^2} = \frac{121{,}6\,nm}{(74-1)^2} = \underline{\underline{0{,}0228\,nm}} = \underline{\underline{22{,}8\,pm}}$

$\Downarrow$

$\underline{\lambda_K > \lambda_M}$

lahko opazimo K črto.

---

Ione $He^+$ obsevamo s sv. dolžine 30,4 nm, 20 nm in 35 nm. Kaj se zgodi?

$E_n = \frac{-13{,}6\,eV}{n^2} z^2$

$(30{,}4)\; E_1 = h\nu = h\frac{c}{\lambda} = 6{,}62\cdot 10^{-34}\,Js \cdot \frac{3\cdot 10^{8}\,m\,s^{-1}}{30{,}4\cdot 10^{-9}\,m} = 6{,}53\cdot 10^{-18}\,J$

$(20)\; E_2 = 9{,}83\cdot 10^{-18}\,J$

$(35)\; E_3 = 5{,}67\cdot 10^{-18}\,J$

se absorbira

$E_1 = \underline{40{,}83\,eV}$ $\qquad E_n = \frac{-13{,}6\,eV\cdot 4}{1} = -54{,}4\,eV$

$\Delta E = E_n - E_2 = \underline{\underline{40{,}8\,eV}}$

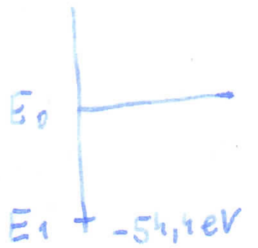
$E_2 = \underline{62{,}06\,eV} \rightarrow E_2 > E_n \Rightarrow$ pride do IONIZACIJE, $e^-$ ima $W_k = E_2 - E_n = \underline{\underline{7{,}66\,eV}}$

$E_3 = \underline{35{,}46\,eV}$

E fotona je premajhna

$\searrow E_3 < E_n \Rightarrow$ $\qquad \cancel{\lambda = \frac{c}{\nu} = \frac{ch}{E} = 3{,}6}$

fotoni gredo mimo, če ne dajo take E kot je potrebna za preskoke $e^-$ na višje E. stanje




kakšna je $\lambda$ izbitih $e^-$: $W_k = \frac{p^2}{2m}$ $\qquad p = \sqrt{W_k 2m} = 2{,}16\cdot 10^{-26}\,m\,s\,kg$ $\qquad \lambda = \frac{h}{p} = \cancel{30\,nm}$ $\underline{\underline{0{,}443\,nm}}$

$A = \sin(\ )$

$B = x\frac{\partial}{\partial x}$

$A(af+bg) = aAf + bAg$ linearnost

$aAf = a\sin f$
$bAg = b\sin f$

$\sin(af+bg) = \sin af \cos bg + \sin bg \cos af \neq a \sin f + b \sin g$

ni linearen

Dokaži LINEARNOST!

$$x\frac{\partial}{\partial x}(af+bg) = x\cdot\frac{\partial af}{\partial x} + x\frac{\partial}{\partial x}bg =$$

$$= ax\frac{\partial f}{\partial x} + bx\frac{\partial g}{\partial x} =$$

$$= aAf + bAg \quad \text{linearen}$$

---

Izračunaj $[\hat{L}_x, \hat{L}_y] = ?$

$\hat{L}_x = -i\hbar\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right)$

$\hat{L}_y = -i\hbar\left(z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z}\right)$

$$[\hat{L}_x, \hat{L}_y]f = \hat{L}_x\hat{L}_y f - \hat{L}_y\hat{L}_x f =$$

$$= \left(-i\hbar\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right)\cdot(-i\hbar)\left(z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z}\right)\right)f -$$

$$(-i\hbar)\left(z\frac{\partial}{\partial \cancel{y}\,x} - x\frac{\partial}{\partial z}\right)\cdot\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right)f =$$

$$= (-i\hbar)^2\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right)\left(z\frac{\partial f}{\partial x} - x\frac{\partial f}{\partial z}\right) -$$

$$(-i\hbar)^2\left(z\frac{\partial}{\partial \cancel{y}\,x} - x\frac{\partial}{\partial z}\right)\left(y\frac{\partial f}{\partial z} - z\frac{\partial f}{\partial y}\right) =$$

$$= (-i\hbar)^2\left(y\frac{\partial}{\partial z}\left(z\frac{\partial f}{\partial x} - x\frac{\partial f}{\partial z}\right) - z\frac{\partial}{\partial y}\left(z\frac{\partial f}{\partial x} - x\frac{\partial f}{\partial z}\right) - z\frac{\partial}{\partial \cancel{y}\,x}\left(y\frac{\partial f}{\partial z} - z\frac{\partial f}{\partial y}\right) + x\cdot\frac{\partial}{\partial z}\left(y\frac{\partial f}{\partial z} - z\frac{\partial f}{\partial y}\right)\right) =$$

$$= (-i\hbar)^2\left(y\left(\frac{\partial f}{\partial x} + \cancel{z\frac{\partial^2 f}{\partial z\partial x}} - \cancel{x\frac{\partial^2 f}{\partial z^2}}\right) - z\left(\cancel{z\frac{\partial^2 f}{\partial x\partial y}} - \cancel{x\frac{\partial^2 f}{\partial y\partial z}}\right) - z\left(\cancel{y\frac{\partial^2 f}{\partial x\partial z}} - \cancel{z\frac{\partial^2 f}{\partial x\partial y}}\right) + x\left(\cancel{y\frac{\partial^2 f}{\partial z^2}} - 1\frac{\partial f}{\partial y} - \cancel{z\cdot\frac{\partial^2 f}{\partial y\partial z}}\right)\right) = (-i\hbar)^2\left(y\frac{\partial f}{\partial x} - x\frac{\partial f}{\partial y}\right) =$$

$$= \cancel{L_z}\ (-i\hbar)^2\frac{\hat{L}_z}{i\hbar} = i\hbar\hat{L}_z f$$

$$\boxed{[\hat{L}_x, \hat{L}_y] = i\hbar\hat{L}_z}$$

ne poznamo vseh komponent

z, x, y
y, z, x
x, y, z

$[\hat{L}_i, \hat{L}_j] = i\hbar\varepsilon_{ijk}\hat{L}_k$

$\varepsilon_{ijk} = 1$

$= -1$ — xzy, zyx, yxz

Hrošček je zaprt v prostoru dimenzije 6 m. Teža hroška je 50 kg. Kakšna je nedoločenost njegove gib. količine in hitrosti? Kakšna je ta nedoločenost v primerjavi s pov. hitrostjo 36 km/h?

$dx \cdot dp_x \geq h$ ~ nedoločenost položaja je 6 m $\Rightarrow$ $dx = 6\,m$

$dx \cdot dp_x = h$

$6\,m \cdot dp_x = 6{,}62 \cdot 10^{-34}\,Js$

$dp_x = \underline{\underline{1{,}1 \cdot 10^{-34}}}\ \frac{\cancel{J}\cancel{s}\,kg\,m^{\cancel{2}}}{\cancel{m}\,s^{\cancel{2}}} = \frac{kg\,m}{s}$

$dv = \frac{dp_x}{m} = \frac{1{,}1 \cdot 10^{-34}\,kg\,m}{50\,\cancel{kg}\,s} = \underline{\underline{2{,}2 \cdot 10^{-36}\,m/s}}$

$v_x = 10\,m/s$ v primerjavi z $dv = 2{,}2 \cdot 10^{-36}\,m/s$ $\Rightarrow$ zelo majhna nedoločenost

---

Elektron je omejen na prostor $\pm 10^{-10}\,m$ premera. Kolikšna je nedoločenost njegove gib. kol. in hitrosti – $10^6\,m/s$ je tipična hitrost $e^-$ $\rightarrow$ kakšna je ta nedoločenost v primerjavi s hitrostjo?

$dx \cdot dp_x \geq h$ $\qquad m_e = 9{,}1 \cdot 10^{-31}\,kg$

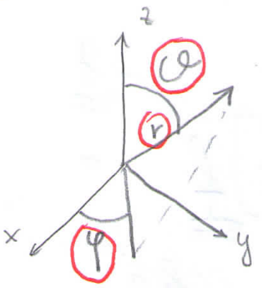
$dx \cdot dp_x = h$

$dp_x = \frac{h}{dx} = \frac{6{,}62 \cdot 10^{-34}\,Js}{10^{-10}\,m} = \underline{\underline{6{,}62 \cdot 10^{-24}\,\frac{kg\,m}{s}}}$

$dv = \frac{dp_x}{m_e} = \frac{6{,}62 \cdot 10^{-24}\,\cancel{kg}\,m}{9{,}1 \cdot 10^{-31}\,s\,\cancel{kg}} = \underline{\underline{7{,}27 \cdot 10^{6}\,m/s}}$

v primerjavi z $10^6\,m/s$ je velika nedoločenost

$[\hat{L}^2, \hat{L}_x] = [\hat{L}^2, \hat{L}_y] = [\hat{L}^2, \hat{L}_z] = 0$ $\quad$ $\hat{L}^2$ komutira z $\hat{L}_x, \hat{L}_y, \hat{L}_z$




SFERNI KOORDINATNI SISTEM

$-\infty < x < \infty$

$-\infty < y < \infty$

$-\infty < z < \infty$

$$\boxed{\begin{array}{l} 0 \le r < \infty \\ 0 \le \vartheta \le 180^\circ \\ 0 \le \varphi < 360^\circ \end{array}}$$

$$dV = dx\,dy\,dz$$

$$dV = r^2 dr \sin\vartheta \, d\vartheta \, d\varphi$$

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\right) + \frac{1}{r^2 \sin\vartheta}\frac{\partial}{\partial\vartheta}\left(\sin\vartheta \frac{\partial}{\partial\vartheta}\right) + \frac{1}{r^2 \sin^2\vartheta}\frac{\partial^2}{\partial\varphi^2}$$

$$\hat{L}_z = -i\hbar \frac{\partial}{\partial\varphi}$$

$$\hat{L}_x = -i\hbar\left(-\sin\varphi \frac{\partial}{\partial\vartheta} - \operatorname{ctg}\vartheta \cos\varphi \frac{\partial}{\partial\varphi}\right)$$

$$\hat{L}_y = -i\hbar\left(\cos\varphi \frac{\partial}{\partial\vartheta} - \operatorname{ctg}\vartheta \sin\varphi \frac{\partial}{\partial\varphi}\right)$$

$$\hat{L}^2 = -\hbar\left[\frac{1}{\sin\vartheta}\frac{\partial}{\partial\vartheta}\left(\sin\vartheta\frac{\partial}{\partial\vartheta}\right) + \frac{1}{\sin^2\vartheta}\frac{\partial^2}{\partial\varphi^2}\right]$$

Dokaži, da $\hat{L}^2$ in $\hat{L}_z$ komutirata!

$$[\hat{L}^2, \hat{L}_z] f = \hat{L}^2 \hat{L}_z f - \hat{L}_z \hat{L}^2 f$$

$$\hat{L}^2 \hat{L}_z f = -\hbar^2\left[\frac{1}{\sin\vartheta}\frac{\partial}{\partial\vartheta}\left(\sin\vartheta\frac{\partial}{\partial\vartheta}\right) + \frac{1}{\sin^2\vartheta}\frac{\partial^2}{\partial\varphi^2}\right]\left(-i\hbar\frac{\partial}{\partial\varphi}\right) f =$$

$$= +i\hbar^3\left[\frac{1}{\sin\vartheta}\frac{\partial}{\partial\vartheta}\left(\sin\vartheta\frac{\partial}{\partial\vartheta}\right) + \frac{1}{\sin^2\vartheta}\frac{\partial^2}{\partial\varphi^2}\right]\frac{\partial f}{\partial\varphi} =$$

$$= +i\hbar^3\left[\frac{1}{\sin\vartheta}\frac{\partial}{\partial\vartheta}\left(\sin\vartheta\frac{\partial^2 f}{\partial\vartheta\partial\varphi}\right) + \frac{1}{\sin^2\vartheta}\frac{\partial^3 f}{\partial\varphi^3}\right] =$$

$$= +i\hbar^3\left[\frac{1}{\sin}\left(\cos\vartheta\frac{\partial^2 f}{\partial\vartheta\partial\varphi} + \sin\vartheta\frac{\partial^3 f}{\partial\vartheta^2\partial\varphi}\right) + \frac{1}{\sin^2\vartheta}\frac{\partial^3 f}{\partial\varphi^3}\right] =$$

$$= i\hbar^3 \left[ \cot\vartheta \frac{\partial^2 f}{\partial\vartheta\partial\varphi} + \frac{\partial^3 f}{\partial\vartheta^2\partial\varphi} + \frac{1}{\sin^2\vartheta}\frac{\partial^3 f}{\partial\varphi^3}\right] !$$

$$\hat{L}_z \hat{L}^2 f = -i\hbar \frac{\partial}{\partial\varphi}\cdot\left(-\hbar^2\left[\frac{1}{\sin\vartheta}\cdot\frac{\partial}{\partial\vartheta}\left(\sin\vartheta\frac{\partial}{\partial\vartheta}\right) + \frac{1}{\sin^2\vartheta}\frac{\partial^2}{\partial\varphi^2}\right]f\right) = g$$

$$= i\hbar^3 \frac{\partial}{\partial\varphi}\left[\frac{1}{\sin\vartheta}\cdot\frac{\partial}{\partial\vartheta}\left(\sin\vartheta\frac{\partial g}{\partial\vartheta}\right) + \frac{1}{\sin^2\vartheta}\frac{\partial^2 g}{\partial\varphi^2}\right] =$$

$$= i\hbar^3 \frac{\partial}{\partial\varphi}\left[\frac{1}{\sin\vartheta}\left(\cos\vartheta\frac{\partial g}{\partial\vartheta} + \sin\vartheta\frac{\partial^2 g}{\partial\vartheta^2}\right) + \frac{1}{\sin^2\vartheta}\frac{\partial^2 g}{\partial\varphi^2}\right] =$$

$$= i\hbar^3 \frac{\partial}{\partial\varphi}\left[\cot\vartheta\frac{\partial g}{\partial\vartheta} + \frac{\partial^2 g}{\partial\vartheta^2} + \frac{1}{\sin^2\vartheta}\frac{\partial^2 g}{\partial\varphi^2}\right] =$$

$$= i\hbar^3\left(\cot\vartheta\frac{\partial^2 g}{\partial\varphi\partial\vartheta} + \frac{\partial^3 g}{\partial\vartheta^2\partial\varphi} + \frac{1}{\sin^2\vartheta}\frac{\partial^3 g}{\partial\varphi^3}\right) !$$

$$\hat{L}^2\hat{L}_z - \hat{L}_z\hat{L}^2 = 0$$