Ali operatorja $\hat{H}$ in $\hat{p}_x$ komutirata?

$$\hat{H} = \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)$$

$$\hat{p}_x = -i\hbar\frac{\partial}{\partial x}$$

$$[\hat{H}, \hat{p}_x] = \hat{H}\hat{p}_x - \hat{p}_x\hat{H} = +i\hbar\frac{\partial V}{\partial x}\cancel{f} = \underline{+i\hbar\frac{\partial V}{\partial x}}$$

$$\hat{H}\hat{p}_x f = \left(\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)\right)\left(-i\hbar\frac{\partial}{\partial x}\right) f =$$

$$= \left(\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)\right)\left(-i\hbar\frac{\partial f}{\partial x}\right) =$$

$$= -i\hbar\left(\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)\right)\left(\frac{\partial f}{\partial x}\right) =$$

$$= \underline{-i\hbar\left(\cancel{-\frac{\hbar^2}{2m}\frac{\partial^3 f}{\partial x^3}} + \cancel{V(x)\frac{\partial f}{\partial x}}\right)}$$

$$\hat{p}_x\hat{H} f = \left(-i\hbar\frac{\partial}{\partial x}\right)\left(\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)\right) f =$$

$$= \left(-i\hbar\frac{\partial}{\partial x}\right)\left(\frac{-\hbar^2}{2m}\frac{\partial^2 f}{\partial x^2} + V(x) f\right) =$$

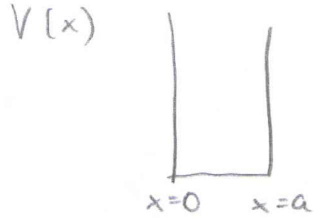
$$= \underline{-i\hbar\left(\cancel{\frac{-\hbar^2}{2m}\frac{\partial^3 f}{\partial x^3}} + \frac{\partial V}{\partial x} f + \cancel{V(x)\frac{\partial f}{\partial x}}\right)}$$

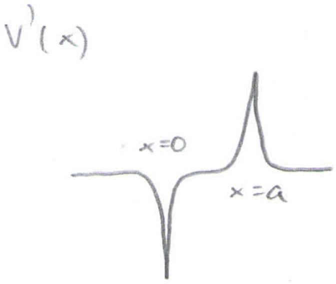
1) prosti delec

$\hat{H} = \frac{\hat{p}_x^2}{2m}$ $\quad [\hat{A}, \hat{A}^2] = 0$ $\quad$ <u>KOMUTIRATA</u>

2) nesk. pot. jama $\quad$ <u>NE KOMUTIRATA</u>

$[\hat{H}, \hat{p}_x] \neq 0$





$\psi = Ae^{ikx}$

$\hat{H}\psi = E\psi \qquad E = \frac{\hbar^2 k^2}{2m}$

$\hat{p}_x \psi = -i\hbar \frac{\partial \psi}{\partial x} = \cancel{-i\hbar} A e^{ikx} \cdot \cancel{i}k = \hbar k A e^{ikx} = \underline{\hbar k \psi}$

$\langle \hat{p}_x \rangle = \hbar k$ – <u>lastna vrednost</u>

pričakovana vrednost gib. količine

Kakšna je lastna vrednost in lastna funkcija $\hat{L}_z$?

$\hat{L}_z = -i\hbar \frac{\partial}{\partial \varphi} \qquad -i\hbar \frac{\partial \psi}{\partial \varphi} = \lambda \psi$

$\hat{L}_z \psi = \lambda \psi \qquad -i\hbar \frac{\partial \psi}{\psi} = \partial \varphi \lambda$

$$\int \frac{\partial \psi}{\psi} = -\frac{\partial \varphi \lambda}{i\hbar} = \int \frac{i\lambda \partial \varphi}{\hbar}$$

$$\ln \psi = \frac{i\lambda\varphi}{\hbar} + C$$

lastna funkcija $\quad \underline{\psi = De^{\frac{i\lambda\varphi}{\hbar}}} \quad$ = !!! $\psi(\varphi) = \psi(\varphi + 2k\pi)$

$$\cancel{D}\cancel{e^{\frac{i\lambda\varphi}{\hbar}}} = \cancel{D}e^{\frac{i\lambda(\varphi + 2z\pi)}{\hbar}} = \cancel{e^{\frac{i\lambda\varphi}{\hbar}}} \cdot e^{\frac{2z\pi\lambda i}{\hbar}}$$

$$1 = e^{\frac{2z\pi\lambda i}{\hbar}}$$

$$1 = e^{i2k\pi} = e^{\frac{i\lambda 2z\pi}{\hbar}} \qquad z\ k \in \mathbb{Z}$$

$$2k\cancel{\pi} = \frac{\lambda z \cancel{\pi}}{\hbar}$$

$$\cancel{2}k\hbar = \cancel{2}\lambda z$$

lastna vrednost $\quad \boxed{\lambda = \frac{k\hbar}{z}}$

$\hat{p}_x$

$$\hat{p}_x \psi = p\psi$$

$$-i\hbar \frac{\partial}{\partial x}\psi = p\psi$$

$$\int \frac{\partial \psi}{\psi} = \int \frac{p}{-i\hbar}\partial x$$

$$\ln\psi = \frac{pi}{\hbar}x + C$$

$$\underline{\psi = D e^{\frac{ip}{\hbar}x}}$$ lastna funkcija

$$\psi = A e^{ikx}$$

$$k = \frac{p}{\hbar}$$

$$\boxed{p = \hbar k}$$ lastna vrednost

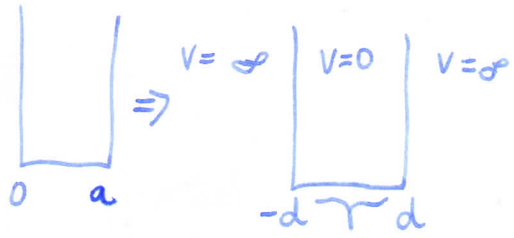
Kakšna je funkcija osnovnega in vzbujenega stanja v nesk. pot. jami?

$$V(x) = \begin{cases} 0, & -a < x < a \\ \infty, & (x>a) \vee (x<-a) \end{cases}$$

$E_1 \Rightarrow \psi_1$

$E_2 \Rightarrow \psi_2$

$$\boxed{E_n = \frac{n^2 h^2}{8ma^2}}$$



$\underline{a=2d}$

$$E_n = \frac{n^2 h^2}{8m4d^2} = \frac{n^2 h^2}{32md^2}$$

$$E_1 = \frac{h^2}{2^5 md^2} = \frac{h^2}{32md^2}$$

$$E_2 = \frac{2^2 h^2}{2^5 md^2} = \frac{h^2}{2^3 md^2} = \frac{h^2}{8md^2}$$

$$\hat{H}\psi = E\psi$$

$$\frac{-\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} = E\psi$$

$$\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + E\psi = 0 \quad \Big| \frac{2m}{\hbar^2}$$

$$\frac{\partial^2\psi}{\partial x^2} + \frac{E2m}{\hbar^2}\psi = 0$$

$$\psi'' + k^2\psi = 0$$

$$\psi = A\sin kx + B\cos kx$$

$$\psi(a) = 0$$

$$\psi(-a) = 0$$

$$\psi(a) = A\sin ka + B\cos ka = 0$$

$$\psi(-a) = A\sin k(-a) + B\cos k(-a) = 0$$

$$-A\sin ka + B\cos ka = 0$$

$$\underline{2B\cos ka = 0}$$

① $\underline{B=0}$

$\psi(a) = 0 \cdot \cos ka + A \sin ka = 0$ $\qquad z \in \mathbb{Z}$

$A \sin ka = 0$

① $A = 0$

$A = B = 0$

nesmiselna rešitev

② $ka = z\pi$

$k = \frac{z\pi}{a}$

$E = \frac{\hbar^2 k^2}{2m}$

$E(z) = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2 z^2 \pi^2}{2ma^2} = \frac{h^2}{8\cancel{\pi^2}} \frac{z^2 \cancel{\pi^2}}{ma^2} = \frac{h^2 z^2}{8ma^2}$

$$\boxed{\psi_z = A \sin \frac{z\pi x}{a}} \qquad k \qquad z \in \mathbb{Z}$$

$E(1) = \frac{h^2}{8ma^2}$ - prej E_2

② $\underline{ka = 0}$

$\psi(a) = B \cdot 0 + A \sin\left(\frac{\pi}{2} + j\pi\right) = 0$ $\qquad j \in \mathbb{Z}$

$A = 0$

$k = \frac{\pi}{2}a + j\frac{\pi}{a} = \frac{\pi}{2a}(2j+1)$

$E(j) = \frac{\hbar^2 \pi^2 (2j+1)^2}{4a^2 2m} = \frac{h^2 \cancel{\pi^2} (2j+1)^2}{4\cancel{\pi^2} 4a^2 2m} = \frac{h^2 (2j+1)^2}{32a^2 m}$

$$\boxed{\psi_j = B \cos \frac{\pi(2j+1)}{2a} x} \qquad k \qquad j \in \mathbb{Z}$$

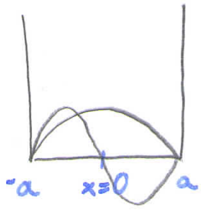
$j = 0, 1, 2 \ldots$

$E(0) = \frac{h^2}{32a^2 m}$ - prej E_1

POVZETEK:

$E_n = \frac{h^2 n^2}{32ma^2}$

n - lih	n - sod
$\psi_n = A \cos kx$	$\psi_n = B \sin kx$



$\psi_1 = \cos k_1 x$ $\qquad$ $\underline{\psi_1 = \cos\frac{\pi x}{2a}}$ $\qquad$ $\psi_3 = \cos k_3 x$

$\cos k_1 a = 0$

$k_1 a = \frac{\pi}{2} + \dots$

$k_1 = \frac{\pi}{2a}$ najnižja E

$\cos k_3 a = 0$

$k_3 a = \frac{\pi}{2} + j\pi$

$k_3 = \frac{3\pi}{2a}$

$\psi_2 = \sin k_2 x$ $\qquad$ $\underline{\psi_2 = \sin\frac{\pi x}{a}}$

$\sin k_2 a = 0$

$k_2 a = l\pi$ $\qquad$ $k_2 = \frac{\pi}{a}$

najnižja E

Pri lihih kv. št. imamo cos, pri sodih pa SIN.

$\psi_1 = N\cos\frac{\pi x}{2a}$

$\langle \psi_1 | \psi_2 \rangle = 1$

$\int_{-a}^{a} \psi_1^* \psi_1 \, dx = 1$

$$1 = N^2 \int_{-a}^{a} \cos^2\frac{\pi x}{2a} dx = N^2 \left(\frac{x}{2} - \frac{\sin 2\cdot\frac{\pi}{2a}x}{2k\cdot\frac{\pi}{2a}} \Bigg|_{-a}^{a} \right) =$$

$$= N^2 \left(\frac{a}{2} - \cancel{\frac{\sin \pi}{\frac{2\pi}{a}}} - \left(-\frac{a}{2} - \cancel{\frac{\sin(-\pi)}{\frac{2\pi}{a}}} \right) \right) =$$

$$1 = N^2 \left(\frac{a}{2} + \frac{a}{2} \right) = N^2 a$$

$$\underline{\underline{N = \sqrt{\frac{1}{a}}}}$$

Normiraj ψ_2!

n_x	n_y	n_z	E	št. stanj
1	1	3	$\frac{11h^2}{8ma^2}$	3
1	3	1		
3	1	1		
2	2	2	$\frac{12h^2}{8ma^2}$	1

GLEJ PREDAVANJE ŠT. !

$\langle \psi_1 | \psi_2 \rangle = 0$; $\perp$ funkciji ψ_1, ψ_2

$$\int_{-a}^{a} \psi_1^* \psi_2 \, dx = \frac{1}{a}\int_{-a}^{a} \cos\frac{\pi x}{2a} \sin\frac{\pi x}{a} \, dx = \frac{1}{a}\cdot\left[-\frac{\cos\left(\frac{\pi}{2a}-\frac{\pi}{a}\right)x}{2\cdot\left(\frac{\pi}{2a}-\frac{\pi}{a}\right)} - \frac{\cos\left(\frac{\pi}{2a}+\frac{\pi}{a}\right)x}{2\left(\frac{\pi}{2a}+\frac{\pi}{a}\right)}\right]\Bigg|_{-a}^{a}$$

$$= \frac{1}{a}\cdot\left[-\frac{\cos\left(\frac{-\pi}{2a}\right)x}{-\frac{\pi}{a}} - \frac{\cos\left(\frac{3\pi}{2a}\right)x}{\frac{3\pi}{a}}\right]\Bigg|_{-a}^{a} = \ldots = 0, \text{ saj je}$$

$\boxed{\text{sin a cos b LIHA; ploščina LIHE f. je } \underline{0}}$

a=f
b=f

Določi pov. vrednost, kjer se x nahaja!

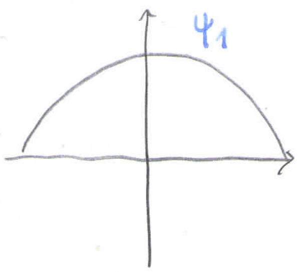
$$\langle \hat{x} \rangle = \langle \psi_1 | \hat{x} | \psi_1 \rangle = \int_{-a}^{a} \psi_1^* \underset{\overset{\parallel}{x}}{\hat{x}} \psi_1 \, dx = \int_{-a}^{a} \frac{1}{a}\left(\cos^2\frac{\pi x}{2a}\right) x \, dx =$$

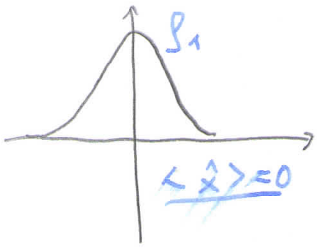
$$= \frac{1}{a}\int_{-a}^{a} x \cos^2\frac{\pi x}{2a} \, dx \underset{\text{g. Bronštejn}}{=} \frac{1}{a}\left(\frac{x^2}{4} + \frac{\cos\frac{2\pi}{2a}x}{8\left(\frac{\pi}{2a}\right)^2} + \frac{x \sin\frac{2\pi}{2a}x}{\frac{2\cdot 2\pi}{2a}}\right)\Bigg|_{-a}^{a} =$$

$$= \frac{1}{a}\left(\cancel{\frac{a^2}{4}} + \cancel{\frac{\cos\frac{\pi}{a}(+a)}{\frac{8\pi^2}{4a^2}}} + \cancel{\frac{a \sin\left(\frac{\pi}{a}(+a)\right)}{\frac{2\pi}{a}}} - \left(\cancel{\frac{a^2}{4}} + \cancel{\frac{\cos(-\pi)}{8\left(\frac{\pi}{2a}\right)^2}} + \cancel{\frac{-a \sin(-\pi)}{\frac{2\pi}{a}}}\right)\right) =$$

$$= \frac{1}{a}\cdot 0 = \underline{\underline{0}}$$

Pov. vrednost je pri 0.





$\Delta x = \sqrt{\langle \hat{x}^2 \rangle - \langle \hat{x} \rangle^2}$ Δx ... nedoločenost položaja

$$\langle \hat{x}^2 \rangle = \langle \psi_1 | \hat{x}^2 | \psi_1 \rangle = \int_{-a}^{a} \psi_1^* \hat{x}^2 \psi_1 \, dx = \int_{-a}^{a} \frac{1}{a} \left(\cos^2 \frac{\pi x}{2a}\right) x^2 \, dx =$$

$$= \frac{1}{a} \int_{-a}^{a} x^2 \cos^2 \frac{\pi x}{2a} \, dx \underset{\text{g. Bronštejn}}{=} \frac{1}{a} \left(\frac{x^3}{6} + \frac{x \cos \frac{2\pi}{2a} x}{4 \left(\frac{\pi}{2a}\right)^2} + \cancel{\frac{\left(-1 + 2 \cdot \left(\frac{\pi}{2a}\right)^2 x^2\right) \sin 2 \cdot \frac{\pi}{2a} x}{8 \cdot \left(\frac{\pi}{2a}\right)^3}} \right) \Bigg|_{-a}^{a} =$$

$$= \frac{1}{a} \left(\frac{a^3}{6} - \frac{a \, a^2}{\pi^2} - \left(\frac{-a^3}{6} + \frac{-a \cdot a^2 \cdot (-1)}{\pi^2} \right) \right) =$$

$$= \frac{1}{a} \left(\frac{2a^3}{6} - \frac{2a^3}{\pi^2} \right) = \frac{a^3}{a} \left(\frac{1}{3} - \frac{2}{\pi^2} \right) = a^2 \left(\frac{1}{3} - \frac{2}{\pi^2} \right)$$

$$\Delta x = \sqrt{a^2 \left(\frac{1}{3} - \frac{2}{\pi^2} \right)} = \boxed{a \sqrt{\frac{1}{3} - \frac{2}{\pi^2}}}$$

odmik

$$\langle \hat{p}_x \rangle = \langle \psi_1 | \hat{p}_x | \psi_1 \rangle = \int_{-a}^{a} \psi_1^* \left(-i\hbar \frac{\partial}{\partial x} \right) \psi_1 \, dx = -i\hbar \int_{-a}^{a} \psi_1 \frac{\partial \psi_1}{\partial x} \, dx =$$

$$= -i\hbar \int_{-a}^{a} \psi \psi' \, dx = -i\hbar \int_{-a}^{a} u \, du = -i\hbar \int_{0}^{0} u \, du = \underline{\underline{0}}$$

$u = \psi_1$
$du = \psi_1' \, dx$

$\psi_1(-a) = 0$
$\psi(a) = 0$

to nam pove, da je enaka verjetnost, da se delec giblje v smer $\longrightarrow$ in $\longleftarrow$