$\rho = |\psi|^2 \qquad P\left(0<x<\frac{a}{3}\right)$

$\psi = \sqrt{\frac{1}{a}}\cos\frac{\pi x}{2a}$

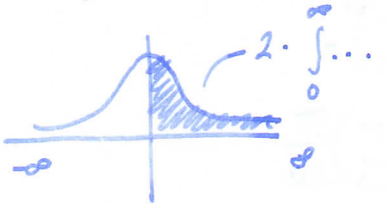
$$\Rightarrow P = \int_0^{\frac{a}{3}} \rho\, dx = \frac{1}{a}\int_0^{\frac{a}{3}} \cos^2\frac{\pi x}{2a} = \frac{1}{a}\left(\frac{x}{2} + \frac{\sin\frac{\pi x}{a}}{\frac{2\pi}{a}}\right)\Bigg|_0^{\frac{a}{3}} =$$

Kakšna je verjetnost, da najdemo delec na $0<x<\frac{a}{3}$?

$$= \frac{1}{a}\left(\frac{a}{6} + \frac{\sin\frac{\pi}{3}}{2\frac{\pi}{a}}\right) = \frac{1}{6} + \frac{\sin\frac{\pi}{3}}{2\pi} = \frac{1}{6} + \frac{\frac{1}{2}\sqrt{3}}{2\pi} = \frac{1}{6} + \frac{\sqrt{3}}{4\pi} = \boxed{0{,}31}$$

$\boxed{\psi = Ne^{-x^2}}$

$$\int_{-\infty}^{\infty} \psi^* \psi\, dx = 1$$



$$N^2\int_{-\infty}^{\infty} e^{-x^2} e^{-x^2} dx = 1$$

$$N^2\int_{-\infty}^{\infty} e^{-2x^2} dx = 1 \longrightarrow \quad \int_0^{\infty} e^{-ax^2} dx = \frac{\sqrt{\pi}}{2\sqrt{a}}$$

$\underline{a=2}$ iz $\int_0^\infty e^{-2x^2}dx$

$$\cancel{2} N^2 \frac{\sqrt{\pi}}{\cancel{2}\sqrt{2}} = 1$$

$$N^2 = \frac{\sqrt{2}}{\sqrt{\pi}} \Rightarrow \underline{N = \sqrt[4]{\frac{2}{\pi}}}$$ vrednost konst. normalizirane funkcije

---

Zanima nas verjetnost nahajanja (x-a) delca!

$\boxed{\psi^* = \psi = \sqrt[4]{\frac{2}{\pi}}e^{-x^2}}$

$\underline{\psi = \sqrt[4]{\frac{2}{\pi}}e^{-x^2}}$

$$\langle \hat{x} \rangle = \langle \psi | \hat{x} | \psi \rangle =$$

$$= \int_{-\infty}^{\infty} \psi^* \hat{x} \psi\, dx = \int_{-\infty}^{\infty} \left(\sqrt[4]{\frac{2}{\pi}}\right)^2 e^{-2x^2} x\, dx = \sqrt{\frac{2}{\pi}}\int_{-\infty}^{\infty} e^{-2x^2} \cdot x\, dx = \underline{\underline{0}}$$

$$\langle \hat{x}^2 \rangle = \langle \psi | \hat{x}^2 | \psi \rangle =$$

$$= \int_{-\infty}^{\infty} \psi^* \hat{x}^2 \psi = \sqrt{\frac{2}{\pi}} \int_{-\infty}^{\infty} e^{-2x^2} x^2 dx \quad \dots = \underline{\underline{\frac{1}{4}}}$$

$\langle \hat{p}_x^2 \rangle$

$\psi_1^* = \psi_1 = \sqrt{\frac{1}{a}} \cos\frac{\pi x}{2a}$

$\boxed{\hat{p}_x^2 = -\hbar^2 \frac{\partial^2}{\partial x^2}}$

$\langle \hat{p}_x^2 \rangle = \langle \psi_1 | \hat{p}_x^2 | \psi_1 \rangle$

$\langle \hat{p}_x^2 \rangle = \int_{-a}^{a} \psi_1^* \left(-\hbar^2 \frac{\partial^2}{\partial x^2}\right) \psi_1 \, dx =$

$= \int_{-a}^{a} \sqrt{\frac{1}{a}} \cos\frac{\pi x}{2a} \left(-\hbar^2 \frac{\partial^2}{\partial x^2}\right) \sqrt{\frac{1}{a}} \cos\frac{\pi x}{2a} \, dx =$

$= \frac{1}{a}(-\hbar^2) \int_{-a}^{a} \cos\frac{\pi x}{2a} \left(\cos\frac{\pi x}{2a}\right)'' dx =$

$\left(\cos\frac{\pi x}{2a}\right)'' = \left(\sin\frac{\pi x}{2a} \cdot \frac{\pi}{2a}\right)' =$

$= \left(-\cos\frac{\pi x}{2a} \cdot \frac{\pi}{2a}\right)\frac{\pi}{2a}$

$= -\left(\frac{\pi}{2a}\right)^2 \cos\frac{\pi x}{2a}$

$= \frac{1}{a}(-\hbar^2) \int_{-a}^{a} \cos\frac{\pi x}{2a} \left(-\frac{\pi^2}{4a^2} \cos\frac{\pi x}{2a}\right) dx =$

$= \frac{1}{a} \hbar^2 \frac{\pi^2}{4a^2} \int_{-a}^{a} \cos^2\frac{\pi x}{2a} \, dx =$ Bronštejn

$= \frac{1}{a} \hbar^2 \frac{\pi^2}{4a^2} \int_{-a}^{a} \frac{x}{2} + \frac{\sin\left(\frac{2 \cdot \pi x}{2a}\right)}{\frac{4 \cdot \pi}{2a}} dx =$

$= \frac{1}{a} \hbar^2 \frac{\pi^2}{4a^2} \int_{-a}^{a} \frac{x}{2} + \frac{\sin\left(\frac{\pi x}{a}\right)}{\frac{2\pi}{a}} dx =$

$= \frac{1}{a} \hbar^2 \left(\frac{\pi}{2a}\right)^2 \left(\frac{x}{2} + \frac{\sin\frac{\pi x}{a}}{\frac{2\pi}{a}}\right)\Bigg|_{-a}^{a} =$

$= \frac{1}{a} \hbar^2 \left(\frac{\pi}{2a}\right)^2 \left(\frac{a}{2} + \frac{\overset{0}{\sin \pi}}{\frac{2\pi}{a}} - \left(-\frac{a}{2} - \frac{\overset{0}{\sin \pi}}{\frac{2\pi}{-a}}\right)\right) =$

$= \frac{1}{a} \hbar^2 \left(\frac{\pi}{2a}\right)^2 \cdot a =$

$= \hbar^2 \left(\frac{\pi}{2a}\right)^2 = \left(\frac{\pi \hbar}{2a}\right)^2$

$\Delta p_x = \sqrt{\langle \hat{p}_x^2 \rangle - \langle \hat{p}_x \rangle^2} =$

$= \sqrt{\left(\frac{\pi \cdot \hbar}{2a}\right)^2} = \boxed{\frac{\pi \hbar}{2a}}$

$\langle p_x \rangle = 0$ (glej nazaj)

Heisenberg:

$\boxed{\Delta x \, \Delta p_x \geq \frac{1}{2}\hbar}$

natančno

$\boxed{\Delta x = a\sqrt{\frac{1}{3} - \frac{2}{\pi^2}}}$

$\Delta x \cdot \Delta p_x = \frac{\pi \hbar}{2a} \cdot a\sqrt{\frac{1}{3} - \frac{2}{\pi^2}} =$

$= \frac{\pi \hbar \sqrt{\frac{1}{3} - \frac{2}{\pi^2}}}{2} = \frac{\hbar}{2}\sqrt{\frac{\pi^2}{3} - 2} > \frac{\hbar}{2}$

$$\langle \hat{p}_x \rangle = \int_{-\infty}^{\infty} \psi^* \cdot \hat{p}_x \psi \, dx = \int_{-\infty}^{\infty} \sqrt[4]{\frac{2}{\pi}} e^{-x^2} \left(-i\hbar \frac{\partial}{\partial x}\right) \sqrt[4]{\frac{2}{\pi}} e^{-x^2} dx =$$

$$= -i\hbar \sqrt{\frac{2}{\pi}} \int_{-\infty}^{\infty} e^{-x^2} (e^{-x^2})' dx = \qquad u = e^{-x^2} \qquad du = (e^{-x^2})' dx$$

$$= -i\hbar \sqrt{\frac{2}{\pi}} \int_0^0 u \, du = \underline{\underline{0}}$$

$$\langle \hat{p}_x^2 \rangle = \int_{-\infty}^{\infty} \psi^* \cdot \hat{p}_x^2 \psi \, dx = \int_{-\infty}^{\infty} \sqrt[4]{\frac{2}{\pi}} e^{-x^2} \left(-\hbar^2 \frac{\partial^2}{\partial x^2}\right) \sqrt[4]{\frac{2}{\pi}} e^{-x^2} dx = -\hbar^2 \sqrt{\frac{2}{\pi}} \int_{-\infty}^{\infty} e^{-x^2} (4x^2 e^{-x^2} - 2e^{-x^2}) dx =$$

$$(e^{-x^2})'' = (-2x e^{-x^2})' = \underline{-2e^{-x^2} + 4x^2 e^{-x^2}}$$

$$= -\hbar^2 \sqrt{\frac{2}{\pi}} \int_{-\infty}^{\infty} (4x^2 e^{-2x^2} - 2e^{-2x^2}) =$$

$$= -\hbar^2 \sqrt{\frac{2}{\pi}} \int_{-\infty}^{\infty} (4x^2 - 2) e^{-2x^2} dx =$$

$$= -\hbar^2 \sqrt{\frac{2}{\pi}} \left(4 \int_{-\infty}^{\infty} x^2 e^{-2x^2} dx - 2 \int_{-\infty}^{\infty} e^{-2x^2} dx\right)$$

$$= -2 \cdot \hbar^2 \sqrt{\frac{2}{\pi}} \left(4 \int_0^{\infty} x^2 e^{-2x^2} dx - 2 \int_0^{\infty} e^{-2x^2} dx\right) \overset{\text{Bronštejn}}{=}$$

$$= -2 \cdot \hbar^2 \sqrt{\frac{2}{\pi}} \left(4 \cdot \frac{\sqrt{\pi}}{4 \cdot 2^{\frac{3}{2}}} - 2 \cdot \frac{\sqrt{\pi}}{2 \cdot \sqrt{2}}\right) =$$

$$= -2 \hbar^2 \sqrt{\frac{2}{\pi}} \left(-\frac{\sqrt{\pi}}{2\sqrt{2}}\right) = \underline{\underline{\hbar^2}}$$

$$\Delta x = \sqrt{\langle \hat{x}^2 \rangle - \langle \hat{x} \rangle^2} = \frac{1}{2} \qquad \langle \hat{x} \rangle = 0$$

$$\Delta p_x = \sqrt{\langle \hat{p}_x^2 \rangle - \langle \hat{p}_x \rangle^2} = \hbar \qquad \langle \hat{p}_x \rangle = 0$$

$$\Delta x \cdot \Delta p_x \geq \frac{1}{2} \hbar$$

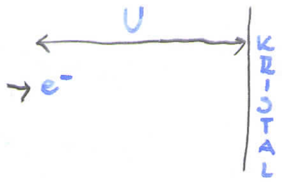
$$\boxed{\frac{1}{2} \hbar = \frac{1}{2} \hbar}$$

4: DELCI KOT VALOVANJE

8. naloga:

$U = 5\,kV$

$I = 10\,mA$



$A = eU = W_k$

$$W_k = \frac{p^2}{2m}$$

$$p = \sqrt{2mW_k} = \sqrt{2meU} =$$

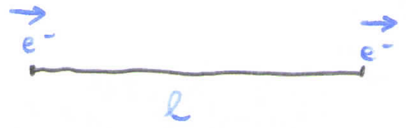
$$= \sqrt{2 \cdot 9{,}1 \cdot 10^{-31}\,kg\ 1{,}6 \cdot 10^{-19}\,As \cdot 50 \cdot 10^{3}\,V} =$$

$$= 3{,}8 \cdot 10^{-23}\,\frac{kg\,m}{s}$$

$$\lambda = \frac{h}{p} = \frac{6{,}62 \cdot 10^{-34}\,Js}{3{,}8 \cdot 10^{23}\,\frac{kg\,m}{s}} =$$

$$= 17{,}36\,pm$$

→ s tem e^- mikroskopom vidimo vse, kar je > 17,36 pm.



$$v = \frac{p}{m} = \frac{3{,}8 \cdot 10^{-23}\,\frac{kg\,m}{s}}{9{,}1 \cdot 10^{-31}\,kg} = 4{,}2 \cdot 10^{6}\,\frac{m}{s}\ ✓$$

$$l = v \cdot t$$

$$\Downarrow$$

$$I = \frac{\Delta e}{\Delta t} = \frac{e_0}{t} \Rightarrow$$

$$t = \frac{e_0}{I}$$

$$l = v \cdot \frac{e_0}{I} = 4{,}2 \cdot 10^{6}\,m\,s^{-1} \cdot \frac{1{,}6 \cdot 10^{-19}\,As}{10 \cdot 10^{-3}\,A} =$$

$$= 6{,}7 \cdot 10^{-5}\,m$$

$\psi = 3\psi_1 + 4\psi_2$ $\quad\langle \psi' | \psi' \rangle = 1$ $\quad \hat{H}\psi_1 = E_1\psi_1$ $\quad \hat{H}|\psi_1\rangle = |E_1 \psi_1\rangle$

$\psi' = a\psi_1 + b\psi_2$ $\quad \langle \psi | \psi' \rangle = 0$ $\quad \hat{H}\psi_2 = E_2\psi_2$

$$\langle a\psi_1 + b\psi_2 | a\psi_1 + b\psi_2 \rangle = 1$$

$$\langle a\psi_1 | a\psi_1 \rangle + \langle a\psi_1 | b\psi_2 \rangle + \langle b\psi_2 | a\psi_1 \rangle + \langle b\psi_2 | b\psi_2 \rangle =$$

$$= |a|^2 \underset{1}{\langle \psi_1 | \psi_1 \rangle} + a^* b \langle \psi_1 | \psi_2 \rangle + b^* a \langle \psi_2 | \psi_1 \rangle + |b|^2 \underset{1}{\langle \psi_2 | \psi_2 \rangle} =$$

$$= |a|^2 + |b|^2 = 1 \Rightarrow \boxed{a^2 + b^2 = 1} \quad a, b \in \mathbb{R}$$

$$\langle 3\psi_1 + 4\psi_2 | a\psi_1 + b\psi_2 \rangle = 0$$

$$\langle 3\psi_1 | a\psi_1 \rangle + \langle 3\psi_1 | b\psi_2 \rangle + \langle 4\psi_2 | a\psi_1 \rangle + \langle 4\psi_2 | b\psi_2 \rangle = 0$$

$$3a \underset{1}{\langle \psi_1 | \psi_1 \rangle} + 3b \underset{0}{\langle \psi_1 | \psi_2 \rangle} + 4a \underset{0}{\langle \psi_2 | \psi_1 \rangle} + 4b \underset{1}{\langle \psi_2 | \psi_2 \rangle} = 0$$

$$\boxed{3a + 4b = 0}$$

$3a = -4b$

$a = -\frac{4}{3} b$

$a^2 + b^2 = 1$

$\left(-\frac{4}{3} b\right)^2 + b^2 = 1$

$\frac{16}{9} b^2 + \frac{b^2}{1} = 1$

$\frac{25 b^2}{9} = 1$

$b^2 = \frac{9}{25}$

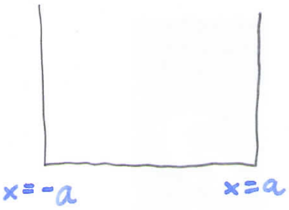
$b = \pm \frac{3}{5}$

$$\boxed{\begin{aligned} \psi_1' &= -\frac{4}{5}\psi_1 + \frac{3}{5}\psi_2 \\ \psi_2' &= \frac{4}{5}\psi_1 - \frac{3}{5}\psi_2 \end{aligned}}$$

$$E = \langle \psi_1 | \hat{H} | \psi_1 \rangle = c_i^2 E_1 + c_j^2 E_2 \Rightarrow$$

$$\boxed{E_{(1)} = \frac{16}{25} E_1 + \frac{9}{25} E_2 = E'_{(2)}}$$

izmerjena energija valovnih funkcij ψ_1' in ψ_2'



$$\psi_1 = \sqrt{\frac{1}{a}} \cos\frac{\pi x}{2a} \qquad E_1 = \frac{\hbar^2}{32ma^2}$$

$$\psi_2 = \sqrt{\frac{1}{a}} \sin\frac{\pi x}{a} \qquad E_2 = \frac{4\hbar^2}{32ma^2}$$

$\psi = \psi_1 + \psi_2$

$\langle \hat{x}(t) \rangle$

$$\psi_1(x,t) = \psi_1(x)\, e^{\frac{iE_1 t}{\hbar}}$$

$$\rho_1(x,t) = \psi_1^*(x,t)\psi_1(x,t) =$$

$$= \psi_1^*(x)\cdot e^{-\frac{iE_1 t}{\hbar}} \cdot \psi_1(x)\cdot e^{\frac{iE_1 t}{\hbar}}$$

$$\psi(x,t) = \psi_1(x)\cdot e^{\frac{iE_1 t}{\hbar}} + \psi_2(x)\, e^{\frac{iE_2 t}{\hbar}}$$

$$\psi^*(x,t) = \psi_1^*(x)\, e^{-\frac{iE_1 t}{\hbar}} + \psi_2^*(x)\, e^{-\frac{iE_2 t}{\hbar}}$$

$$\rho(x,t) = \psi^*(x,t)\,\psi(x,t)$$

NADALJEVANJE PREJŠNJE NALOGE

$\psi = \psi_1 + \psi_2$

$\psi_1 = \sqrt{\frac{1}{a}} \cos \frac{\pi x}{2a}$

$\psi_2 = \sqrt{\frac{1}{a}} \sin \frac{\pi x}{a}$

$\rho(t) = ?$

$\langle \hat{x}(t) \rangle = ?$

$\psi = (\psi_1 + \psi_2) N =$

$= \frac{1}{\sqrt{2}} (\psi_1 + \psi_2)$

Funkcijo moramo normirati!

$\langle \psi | \psi \rangle = 1$

$\langle N(\psi_1 + \psi_2) | N(\psi_1 + \psi_2) \rangle = 1$

$N^2 \langle (\psi_1 + \psi_2) | \psi_1 + \psi_2 \rangle = 1$

$N^2 (\overset{=1}{\langle \psi_1 | \psi_1 \rangle} + \overset{=0}{\langle \psi_1 | \psi_2 \rangle} + \overset{=0}{\langle \psi_2 | \psi_1 \rangle} + \overset{=1}{\langle \psi_2 | \psi_2 \rangle}) = 1$

$N^2 \cdot 2 = 1$

$\underline{N = \frac{1}{\sqrt{2}}}$

$\psi = \frac{1}{\sqrt{2}} \left(\psi_1(x) \cdot e^{-\frac{iE_1}{\hbar} t} + \psi_2(x) e^{-\frac{iE_2}{\hbar} t} \right)$

$\psi^* = \frac{1}{\sqrt{2}} \left(\psi_1(x) e^{\frac{iE_1}{\hbar} t} + \psi_2(x) e^{\frac{iE_2}{\hbar} t} \right)$

$\rho = \psi \cdot \psi^* = \left(\frac{1}{\sqrt{2}}\right)^2 \cdot \Big(\psi_1^2(x) \cdot e^0 + \psi_1(x) \cdot \psi_2(x) \cdot e^{\frac{iE_2 - iE_1}{\hbar} t} +$

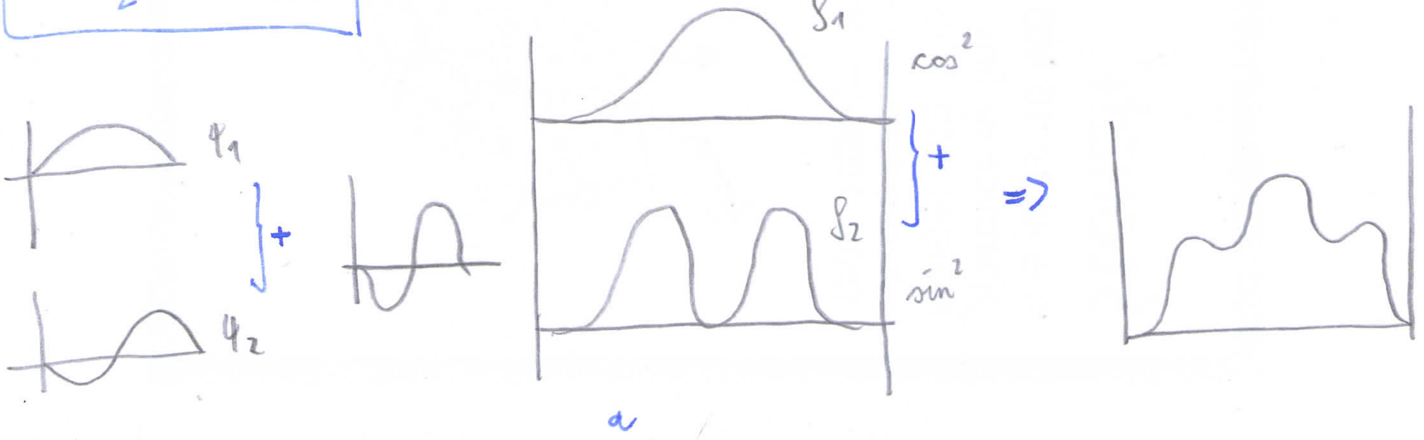
$\psi_2(x) \cdot \psi_1(x) \, e^{\frac{-iE_2}{\hbar} t + \frac{iE_1 t}{\hbar}} + \psi_2^2(x) \cdot e^0 \Big) =$

$= \frac{1}{2} \left(\psi_1^2 + \psi_2^2 + \psi_1 \psi_2 \left(e^{-\frac{it}{\hbar}(E_1 - E_2)} + e^{\frac{it}{\hbar}(E_1 - E_2)} \right) \right) =$

$= \frac{1}{2} \psi_1^2 + \frac{1}{2} \psi_2^2 + \psi_1 \psi_2 \cos\left((E_1 - E_2) \frac{t}{\hbar} \right)$

$$\boxed{\frac{e^{ix} + e^{-ix}}{2} = \cos x}$$

$\rho = \frac{1}{2} \rho_1 + \frac{1}{2} \rho_2 + \psi_1 \psi_2 \cos \frac{(E_1 - E_2) t}{\hbar}$



$$\langle \hat{x} \rangle = \int_{-a}^{a} x \rho \, dx = \int_{-a}^{a} x \left(\frac{1}{2}\psi_1^2 + \frac{1}{2}\psi_2^2 + \psi_1 \psi_2 \cos\left((E_1 - E_2) \frac{t}{\hbar} \right) \right) dx =$$

$$= \underbrace{\frac{1}{2} \int_{-a}^{a} x \psi_1^2 \, dx}_{0} + \underbrace{\frac{1}{2} \int_{-a}^{a} x \psi_2^2 \, dx}_{0} + \int_{-a}^{a} x \psi_1 \psi_2 \cos\left((E_1 - E_2) \frac{t}{\hbar} \right) dx =$$

liha funkcija

x - liha
ψ_1^2 - soda } liha

$$= \frac{1}{a} \cos (E_1 - E_2) \frac{t}{\hbar} \int_{-a}^{a} x \cos \frac{\pi x}{2a} \sin \frac{\pi x}{a} \, dx = \text{Bronštejn}$$

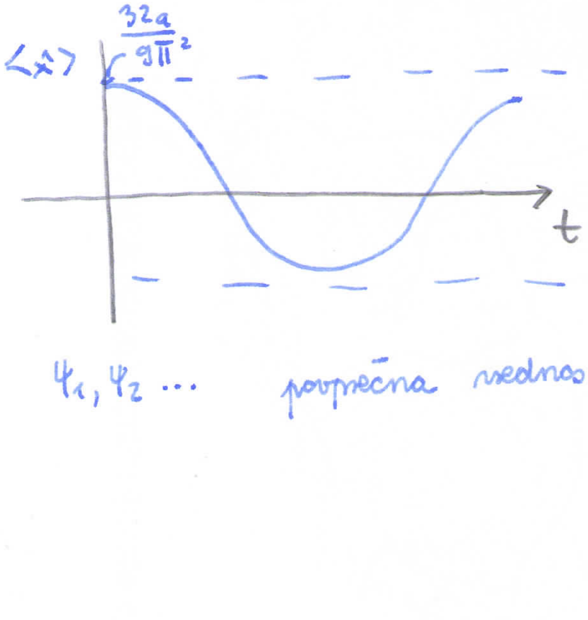
pride iz $\psi_1 = \sqrt{\frac{1}{a}} \cos \frac{\pi x}{2a}$, $\psi_2 = \sqrt{\frac{1}{a}} \sin \frac{\pi x}{a}$

$$= \left(\frac{2}{\left(\frac{\pi}{a} - \frac{\pi}{2a}\right)^2} - \frac{2}{\left(\frac{\pi}{a} + \frac{\pi}{2a}\right)^2} \right) \frac{1}{2a} \cos \left[(E_1 - E_2) \frac{t}{\hbar} \right] =$$

$$= \left(\frac{4a}{\pi^2} - \frac{4a}{9\pi^2} \right) \cos \left[(E_1 - E_2) \frac{t}{\hbar} \right] =$$

$$= \left(\frac{32a}{9\pi^2} \right) \cos \left[(E_1 - E_2) \frac{t}{\hbar} \right]$$

$$\langle \hat{x}(t) \rangle = \frac{32a}{9\pi^2} \cos \frac{(E_1 - E_2) t}{\hbar}$$



ψ_1, ψ_2 ... povprečna vrednost oz. $\langle \hat{x} \rangle$ se spreminja s časom

e^- v H se nahaja v stanju 4f. Val. funkcija, ki opiše stanje tega e^- ima obliko $\psi_{431} = R_{43} Y_{31}$. Kakšne so lastne vrednosti $\hat{H}$, $\hat{L}^2$ in $\hat{L}_z$?

ψ_{431} $n=4$, $l=3$, $m=1$

$$E_n = \frac{-13{,}6\,eV}{n^2} = \frac{-13{,}6\,eV}{16} = -0{,}85\,eV$$ } STAC. STANJE

$$\langle \hat{H} \rangle = \langle \psi_{431} | \hat{H} | \psi_{431} \rangle = \langle \psi_{431} | E_{n=4} \psi_{431} \rangle =$$

$$\hat{H}\psi_{431} = E_{n=4}\psi_{431} \qquad = E_{n=4} \underbrace{\langle \psi_{431} | \psi_{431} \rangle}_{1} =$$

$$= E_{n=4} = -0{,}85\,eV$$

STAC. STANJE ⇒ PRIČAKOVANA VREDNOST HAM. OPERATORJA JE ENAKA ENERGIJI

$$\langle \hat{L}^2 \rangle = \langle \psi_{431} | \hat{L}^2 | \psi_{431} \rangle = \langle \psi_{431} | 12\hbar^2 \psi_{431} \rangle = 12\hbar^2 \cdot 1 =$$
$$= 12\hbar^2$$

$$\hat{L}^2 \psi_{431} = (l+1) l \hbar^2 \psi_{431} = 12\hbar^2 \psi_{431}$$, saj je $l=3$

dolžina vrtilne količine $L = \sqrt{L^2} = \sqrt{12\hbar^2} = 2\sqrt{3}\hbar$

$$\langle \hat{L}_z \rangle = \langle \psi_{431} | \hat{L}_z | \psi_{431} \rangle = \langle \psi_{431} | \hbar \psi_{431} \rangle = \hbar \cdot 1 = \hbar$$

$$\hat{L}_z \psi_{431} = m\hbar \psi_{431} = \hbar \psi_{431}$$ ($m = 1$)

prvo kv. št. nam pove energijo stanja, v katerem se delec nahaja: $\langle \hat{H} \rangle$

2. kv. št. nam pove kvadrat velikosti L : $\langle \hat{L}^2 \rangle$ $L = \sqrt{\ }$

3. kv. št. nam pove, kolikšna je velika projekcija L na z-os: $\langle \hat{L}_z \rangle$

PAZI!! ni potrebno računati

c