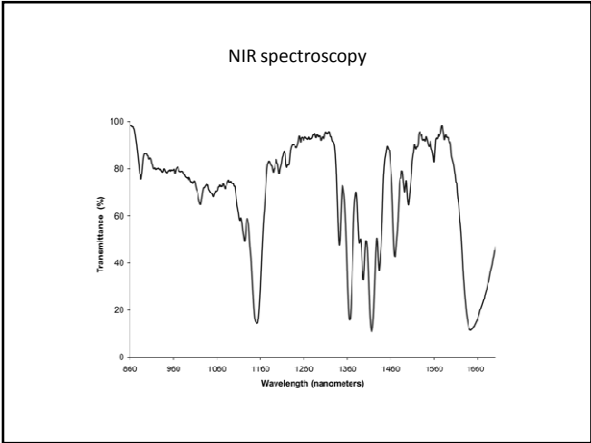


Multivariate modeling of spectroscopic data



- Aim**
- To find a correlation between spectroscopic information and some physico-chemical properties of the system
 - Simple linear regression can not be used
 - Some multivariate procedure should be used to address this problem.

Multivariate linear regression (MLR)

$$\begin{aligned}
 y_1 &= b_0 + b_1u_{11} + b_2u_{12} + \dots + b_pu_{1p} \\
 y_2 &= b_0 + b_1u_{21} + b_2u_{22} + \dots + b_pu_{2p} \\
 &\dots \\
 y_n &= b_0 + b_1u_{n1} + b_2u_{n2} + \dots + b_pu_{np}
 \end{aligned}$$

Matrix form:

$$\begin{bmatrix} y_1 \\ y_2 \\ \dots \\ y_n \end{bmatrix} = \begin{bmatrix} 1 & u_{11} & u_{12} & \dots & u_{1p} \\ 1 & u_{21} & u_{22} & \dots & u_{2p} \\ \dots & \dots & \dots & \dots & \dots \\ 1 & u_{n1} & u_{n2} & \dots & u_{np} \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ \dots \\ b_p \end{bmatrix}$$

Vector form:

$$\mathbf{y}^T = \mathbf{U} \mathbf{b}^T$$

The number of examples must be equal or greater than the number of coefficients

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Problems of MLR procedure

- The number of measurements should be greater or equal to the number of descriptors
- The colinearity of the descriptors.

Solution

- The reduction of the number of the descriptors should be performed

Principal Components Analysis - PCA

Tasks

- Multivariate projection technique
- Dimensionality reduction
- Graphical overview

Advantages

- Plot data in K-Dimensional space
- Directions of maximum variation
- Orthogonal components
- Projection of data onto lower dimensional planes

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Clustering using PCA

- PCA gives overview of data

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Partial Least Squares Regression PLS

It is multivariate regression technique.

- Models structure in X and relationship to Y
- Handles
 - Correlation in both X and Y
 - Short and wide data tables
 - $nVar \gg nObs$
 - Missing data
- Applications in
 - Spectroscopy
 - QSAR
 - Genomics Proteomics
 -

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Goal of PLS regression

- The goal of PLS regression is to predict **Y from X and to describe** their common structure.
- Unlike PCA, the PLS technique works by successively extracting factors from both predictive and target variables such that **covariance** between the extracted factors is maximized.

Partial Least Squares (PLS)

Description of the technique

Assume X is a $n \times p$ matrix and Y is a $n \times q$ matrix. PLS method can work with multivariate response variables (i.e. when Y is a $n \times q$ vector with $q > 1$). However in the simplest case we can have just a single response (target).

PLS technique tries to find a linear decomposition of X and Y such that

$$X = TP^T + E$$

$$Y = TQ^T + F,$$

T $n \times r$ = X-scores U $n \times r$ = Y-scores
 P $p \times r$ = X-loadings Q $1 \times r$ = Y-loadings
 E $n \times p$ = X-residual F $n \times 1$ = Y-residual

A PLS model will try to find the multidimensional direction in the X space that explains the maximum multidimensional variance direction in the Y space.

Comparison of PCA and PLS

Two major common effects of using PCA or PLS

- Convert a group of correlated predictive variables to a group of independent variables
- Construct a small number of "strong" predictive variable from several "weaker" predictive variables

Major difference between PCA and PLS

- PCA is performed without a consideration of the target variable. So PCA is an unsupervised analysis
- PLS is performed to maximize the correlation between the target variable and the predictive variables. So PLS is a supervised analysis

Prediction of methanol level using NIR spectroscopy

On-line measurements utilizing fiber optics → minimize the processing time

A partial least squares (PLS) calibration is built by running a number of small chemical reactions under identical conditions. Spectra are collected in real time and small aliquots are simultaneously removed from the reaction mix to perform an off-line HPLC analysis. The results of the HPLC analysis and the corresponding spectral data are input into a commercial software package, thus, creating a PLS prediction model.

S. Walker* et. Al., *Analytica Chimica Acta* 395 (1999) 335-341

