Kemijsko inženirstvo, 1. letnik

Matematika 1

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Zaporedja. Osnovni pojmi. Limita. Računanje z zaporedji. Število e. Konvergenca številske vrste, geometrijska vrsta.

1. Napiši nekaj členov zaporedja $a_n = \frac{n+2}{n}$. Ali je monotono naraščajoče, padajoče? Ali je konvergentno? Ima limito?

2. Dokaži monotonost naslednjih zaporedij:

   (a) $a_n = \frac{2^n}{n!}$,

   (b) $a_n = n^2$,

   (c) $a_n = \frac{2^n}{n}$.

3. Izračunaj limite:

   (a) $\lim\limits_{n\to\infty} \dfrac{2n+5}{3-n}$,

   (b) $\lim\limits_{n\to\infty} \dfrac{5n^2-7n+4}{n^3-4n}$,

   (c) $\lim\limits_{n\to\infty} \dfrac{n+\sqrt[3]{n}}{\sqrt[3]{n^4}-2}$,

   (d) $\lim\limits_{n\to\infty} \dfrac{\sqrt{n^3-2n}+\sqrt[3]{n^4-5n^2}}{\sqrt[4]{n^6+n^5}}$,

   (e) $\lim\limits_{n\to\infty} \dfrac{(2n-1)(3-n)(7+n^2)}{(2n+1)(2+n)(5n+3)(n+4)}$,

   (f) $\lim\limits_{n\to\infty} (\sqrt{n+3}-\sqrt{n})$,

   (g) $\lim\limits_{n\to\infty} n^{\frac{2}{3}}(\sqrt[3]{n+3}-\sqrt[3]{n})$,

   (h) $\lim\limits_{n\to\infty} \dfrac{\sqrt[3]{1+2n^3+8n^6}+\sqrt{n^4+3n+2}}{n^2-\sqrt{n^2-n+2}}$,

   (i) $\lim\limits_{n\to\infty} \dfrac{1}{(\sqrt{n^2+3}-n)\cdot n}$,

   (j) $\lim\limits_{n\to\infty} (n-\sqrt{n^2+n})$,

   (k) $\lim\limits_{n\to\infty} n^{\frac{3}{2}}(\sqrt{n^3+1}-\sqrt{n^3-1})$,

   (l) $\lim\limits_{n\to\infty} \dfrac{5^n+2^n}{5^{n+1}+2^{n+1}}$,

   (m) $\lim\limits_{n\to\infty} \dfrac{3\cdot 2^n+2\cdot 3^n}{3^{n-1}+1}$.

4. Od katerega člena dalje se vsi členi zaporedja $a_n = \frac{n+1}{2-3n}$ ločijo od limite za manj kot 10^{-3}?

5. Izračunaj vsote geometrijskih vrst:

(a) $1 - \dfrac{1}{2} + \dfrac{1}{4} - \dfrac{1}{8} + \ldots$,

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{3}{4^{n-1}}$,

(c) $1 + \cos^2 x + \cos^4 x + \ldots$,

(d) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{x-1}{2}\right)^n$.

6. Določi vsa stekališča naslednjih zaporedij:

(a) $a_n = 1 - \frac{1}{n}$,

(b) $a_{2n-1} = \frac{1}{n+1}$, $a_{2n} = \frac{n}{n+1}$,

(c) $a_n = 1 + (-1)^n \cdot 2^{-n}$,

(d) $a_n = \sin\left(\frac{n\pi}{2}\right) + \frac{1}{n}$.

7. Izračunaj limite:

(a) $\displaystyle\lim_{n\to\infty} \left(1 + \frac{2}{n}\right)^n$,

(b) $\displaystyle\lim_{n\to\infty} \left(1 + \frac{2}{n}\right)^{3n+4}$,

(c) $\displaystyle\lim_{n\to\infty} \left(\frac{2n-4}{2n+3}\right)^{5n}$,

(d) $\displaystyle\lim_{n\to\infty} \left(\frac{n^2+2}{n^2-1}\right)^{n^2+n}$,

(e) $\displaystyle\lim_{n\to\infty} \left(\frac{n^2-n}{n^2+2n+1}\right)^{n+2}$,

(f) $\displaystyle\lim_{n\to\infty} \frac{\sqrt[3]{1+2\sqrt{n}}}{1+\sqrt[6]{2n}}$,

(g) $\displaystyle\lim_{n\to\infty} \frac{e^{-n}+n}{e^n+n}$,

(h) $\displaystyle\lim_{n\to\infty} \left(\frac{n+2}{n+1}\right)^{n+3}$,

(i) $\displaystyle\lim_{n\to\infty} \left(\frac{n^2+5}{n^2-4}\right)^{n^2}$,

(j) $\displaystyle\lim_{n\to\infty} \left(\frac{n^2+2n}{n^2+1}\right)^{n}$,

(k) $\displaystyle\lim_{n\to\infty} n(\ln n - \ln(n+1))$,

(l) $\displaystyle\lim_{n\to\infty} \left(\frac{n^2+4n}{n^2-n+1}\right)^{n}$.

8. Seštej vrste:

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$,

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(2n-1)(2n+1)} = \frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} + \ldots$,

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+2)} = \frac{1}{1\cdot 3} + \frac{1}{2\cdot 4} + \frac{1}{3\cdot 5} + \frac{1}{4\cdot 6} + \ldots$.

9. Izračunaj vsote vrst:

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{9n^2+3n-2}$,

(b) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^2-1}$,

(c) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{x}{x+1}\right)^n$,

(d) $\displaystyle\sum_{n=4}^{\infty} \frac{3^n}{2^{2n-1}}$,

(e) $\displaystyle\sum_{n=0}^{\infty} \frac{2\cdot 3^n + 3\cdot 2^n}{6^n}$,

(f) $\displaystyle\sum_{n=1}^{\infty} \frac{1-2^n}{5^{n+1}}$.

10. Od katerega člena dalje se vsi členi zaporedja $\frac{1}{n^\alpha}$ od limite razlikujejo za manj kot $\frac{1}{1000}$, če je $\alpha = \frac{1}{3}$?

11. Dokaži, da za $x > 0$ velja $\lim_{n\to\infty} \sqrt[n]{x} = 1$.

12. Z izračunom delnih vsot ugotovi, ali vrsta $\displaystyle\sum_{n=1}^{\infty} \log\left(1+\frac{1}{n}\right)$ konvergira.

13. S kvocientnim kriterijem ugotovi konvergenco vrst:

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$,

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^4}$,

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{c^n}{n!}$,

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n\cdot n!}{n^n}$,

(e) $\displaystyle\sum_{n=1}^{\infty} n\cdot c^{n-1}$, $c > 0$.

14. S pomočjo primerjalnega kriterija določi konvergenco vrst

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$,

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n(n+1)}}$,

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{(4n-3)(4n-1)}}$,

(d) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^2-n}$,

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{n+3}{n^2+2n-1}$,

(f) $\displaystyle\sum_{n=2}^{\infty} \frac{n\alpha^n}{(n-1)(n+3)}$, $\alpha > 0$,

(g) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$, $x > 0$.

15. Ali naslednje vrste konvergirajo?

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n(n+1)}$,

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^\alpha}$ za $\alpha > 0$,

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{x^n}$ za $x \in \mathbb{R}$,

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n^4+1}}$,

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n+\sqrt{n}}$.

16. S korenskim kriterijem določi konvergenco vrste

$$\sum_{n=1}^{\infty} \frac{n^2}{2^n}$$

in vrste

$$\sum_{n=1}^{\infty} \left(1+\frac{1}{\sqrt{n}}\right)^{-n^{\frac{3}{2}}}.$$

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DOMAČA NALOGA – rezultati

Zaporedja. Osnovni pojmi. Limita. Računanje z zaporedji. Število e. Konvergenca številske vrste, geometrijska vrsta.

1. Naredili na vajah.
2. (a) Vaje.
   (b) Vaje.
   (c) Naraščajoče.
3. (a) $\lim\limits_{n\to\infty} \dfrac{2n+5}{3-n} = 2$,
   (b) $\lim\limits_{n\to\infty} \dfrac{5n^2-7n+4}{n^3-4n} = 0$,
   (c) $\lim\limits_{n\to\infty} \dfrac{n+\sqrt[3]{n}}{\sqrt[3]{n^4}-2} = 0$,
   (d) $\lim\limits_{n\to\infty} \dfrac{\sqrt{n^3-2n}+\sqrt[3]{n^4-5n^2}}{\sqrt[4]{n^6+n^5}} = 1$,
   (e) $\lim\limits_{n\to\infty} \dfrac{(2n-1)(3-n)(7+n^2)}{(2n+1)(2+n)(5n+3)(n+4)} = -\dfrac{1}{5}$,
   (f) $\lim\limits_{n\to\infty} (\sqrt{n+3}-\sqrt{n}) = 0$,
   (g) $\lim\limits_{n\to\infty} n^{\frac{2}{3}}(\sqrt[3]{n+3}-\sqrt[3]{n}) = 1$,
   (h) $\lim\limits_{n\to\infty} \dfrac{\sqrt[3]{1+2n^3+8n^6}+\sqrt{n^4+3n+2}}{n^2-\sqrt{n^2-n+2}} = 3$,
   (i) $\lim\limits_{n\to\infty} \dfrac{1}{(\sqrt{n^2+3}-n)\cdot n}$,
   (j) $\lim\limits_{n\to\infty} (n-\sqrt{n^2+n}) = -\dfrac{1}{2}$,
   (k) $\lim\limits_{n\to\infty} n^{\frac{3}{2}}(\sqrt{n^3+1}-\sqrt{n^3-1}) = \dfrac{2}{3}$,
   (l) $\lim\limits_{n\to\infty} \dfrac{5^n+2^n}{5^{n+1}+2^{n+1}} = \dfrac{1}{5}$,
   (m) $\lim\limits_{n\to\infty} \dfrac{3\cdot 2^n+2\cdot 3^n}{3^{n-1}+1} = 6$.
4. Limita je $-1/3$, manj od 10^{-3} se ločijo členi od 557.-ega naprej.
5. (a) $1-\dfrac{1}{2}+\dfrac{1}{4}-\dfrac{1}{8}+\ldots = \dfrac{2}{3}$,

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{3}{4^{n-1}} = 4,$

(c) $1 + \cos^2 x + \cos^4 x + \ldots = \dfrac{1}{\sin^2 x} \quad$ za $x \neq k\pi,$

(d) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{x-1}{2}\right)^n = \frac{x-1}{3-x}$ za $-1 < x < 3$, če pa je $x \geq 3$ ali $x \leq -1$, vrsta divergira.

6. (a) Edino stekališče je 1.
   (b) Stekališči sta 0 in 1.
   (c) Stekališče je 1.
   (d) Stekališča so -1, 0 in 1.

7. (a) $\displaystyle\lim_{n\to\infty} \left(1 + \frac{2}{n}\right)^n = e^2,$

   (b) $\displaystyle\lim_{n\to\infty} \left(1 + \frac{2}{n}\right)^{3n+4} = e^6,$

   (c) $\displaystyle\lim_{n\to\infty} \left(\frac{2n-4}{2n+3}\right)^{5n} = e^{-\frac{35}{2}},$

   (d) $\displaystyle\lim_{n\to\infty} \left(\frac{n^2+2}{n^2-1}\right)^{n^2+n} = e^3,$

   (e) $\displaystyle\lim_{n\to\infty} \left(\frac{n^2-n}{n^2+2n+1}\right)^{n+2} = e^{-3},$

   (f) $\displaystyle\lim_{n\to\infty} \frac{\sqrt[3]{1+2\sqrt{n}}}{1+\sqrt[6]{2n}} = \sqrt[6]{2},$

   (g) $\displaystyle\lim_{n\to\infty} \frac{e^{-n}+n}{e^n+n} = 0,$

   (h) $\displaystyle\lim_{n\to\infty} \left(\frac{n+2}{n+1}\right)^{n+3} = e,$

   (i) $\displaystyle\lim_{n\to\infty} \left(\frac{n^2+5}{n^2-4}\right)^{n^2} = e^9,$

   (j) $\displaystyle\lim_{n\to\infty} \left(\frac{n^2+2n}{n^2+1}\right)^{n} = e^2,$

   (k) $\displaystyle\lim_{n\to\infty} n(\ln n - \ln(n+1)) = 1,$

   (l) $\displaystyle\lim_{n\to\infty} \left(\frac{n^2+4n}{n^2-n+1}\right)^{n} = e^5.$

8. (a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = 1,$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(2n-1)(2n+1)} = \frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} + \ldots = \frac{1}{2},$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+2)} = \frac{1}{1\cdot 3} + \frac{1}{2\cdot 4} + \frac{1}{3\cdot 5} + \frac{1}{4\cdot 6} + \ldots = \frac{3}{4}.$

9. (a) $\sum_{n=0}^{\infty} \frac{1}{9n^2+3n-2} = -\frac{1}{3}$,

   (b) $\sum_{n=2}^{\infty} \frac{1}{n^2-1} = \frac{3}{4}$,

   (c) $\sum_{n=1}^{\infty} \left(\frac{x}{x+1}\right)^n = x+1$ za $x > -1$, sicer ne konvergira,

   (d) $\sum_{n=4}^{\infty} \frac{3^n}{2^{2n-1}} = 8 \cdot \left(\frac{3}{4}\right)^4$,

   (e) $\sum_{n=0}^{\infty} \frac{2 \cdot 3^n + 3 \cdot 2^n}{6^n} = \frac{11}{2}$,

   (f) $\sum_{n=1}^{\infty} \frac{1-2^n}{5^{n+1}} = -\frac{1}{24}$.

10. Od člena $10^9 + 1$.

11. Ločimo primere $x > 1$, $x = 1$, $x < 1$.

12. Divergira.

13. (a) Konvergira,

    (b) konvergira,

    (c) konvergira,

    (d) konvergira,

    (e) konvergira za $c < 1$ in divergira za $c \geq 1$.

14. (a) Primerjamo z $\sum_{n=2}^{\infty} \frac{1}{n(n-1)}$, ki konvergira,

    (b) primerjamo z $\sum_{n=1}^{\infty} \frac{1}{n+1}$, ki divergira,

    (c) primerjamo z $\sum_{n=1}^{\infty} \frac{1}{4n}$, ki divergira,

    (d) primerjamo z $\sum_{n=2}^{\infty} \frac{1}{(n-1)^2}$, ki konvergira,

    (e) primerjamo z $\sum_{n=1}^{\infty} \frac{1}{n}$, ki divergira,

    (f) uporabimo kvocientni kriterij in ugotovimo, da za $\alpha < 1$ konvergira, $\alpha > 1$ divergira, za $\alpha = 1$ pa primerjamo z vrsto $\sum \frac{1}{n}$ in zato divergira,

    (g) uporabimo kvocientni kriterij in ugotovimo, da za $x < 1$ konvergira, $x > 1$ divergira, za $x = 1$ pa primerjamo z vrsto $\sum \frac{1}{n}$ in zato divergira.

15. (a) Konvergira,

    (b) konvergira (za $\alpha > 1$ celo absolutno),

    (c) konvergira za $x > 1$, divergira za $0 < x \leq 1$, konvergira za $x = 0$, divergira za $-1 \leq x < 0$ in konvergira za $x < -1$,

    (d) divergira za $x > 1$, konvergira za $-1 \leq x \leq 1$ in divergira za $x < -1$,

    (e) divergira za $x \geq 1$, konvergira za $-1 \leq x < 1$ in divergira za $x < -1$.

16. Obe vrsti konvergirata.