MATEMATIKA I

/rd/	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
/°/	0	30	45	60	90	180	270
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	–1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	–1	0
tg	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞	0	∞
ctg	∞	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	∞	0

NEDOLOČENI IZRAZI:

$$\frac{\infty}{\infty}, \frac{0}{0}, \infty \cdot 0, \infty - \infty, 1^{\infty}$$

$$ax^2 + bx + c = 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$|x| = \begin{cases} x, x \geq 0 \\ -x, x < 0 \end{cases}$$

$$|x \cdot y| = |x| \cdot |y|$$

$$|x + y| \leq |x| + |y|$$

$$\sqrt{x^2} = |x|$$

$$\operatorname{sgn} x = \begin{cases} -1; x < 0 \\ 0; x = 0 \\ 1; x > 0 \end{cases}$$

$$\sin(-\alpha) = -\sin\alpha$$

$$\cos(-\alpha) = \cos\alpha$$

$$\operatorname{tg}(-\alpha) = -\operatorname{tg}\alpha$$

$$\operatorname{ctg}(-\alpha) = -\operatorname{ctg}\alpha$$

$$\sin(\pi \pm \alpha) = \mp\sin\alpha$$

$$\cos(\pi \pm \alpha) = -\cos\alpha$$

$$\operatorname{tg}(k\pi \pm \alpha) = \pm\operatorname{tg}\alpha$$

$$\operatorname{ctg}(k\pi \pm \alpha) = \pm\operatorname{ctg}\alpha$$

$$\sin(k2\pi \pm \alpha) = \pm\sin\alpha$$

$$\cos(k2\pi \pm \alpha) = \cos\alpha$$

$$\sin\left(\frac{\pi}{2} \pm \alpha\right) = \cos\alpha$$

$$\cos\left(\frac{\pi}{2} \pm \alpha\right) = \pm\sin\alpha$$

$$\operatorname{tg}\left(\frac{\pi}{2} \pm \alpha\right) = \mp\operatorname{ctg}\alpha$$

$$\operatorname{ctg}\left(\frac{\pi}{2} \pm \alpha\right) = \mp\operatorname{tg}\alpha$$

$$\sin\alpha \pm \sin\beta = 2\sin\frac{\alpha \pm \beta}{2}\cos\frac{\alpha \mp \beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha + \beta}{2}\cos\frac{\alpha - \beta}{2}$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha + \beta}{2}\sin\frac{\alpha - \beta}{2}$$

$$\operatorname{tg}\alpha \pm \operatorname{tg}\beta = \frac{\sin(\alpha \pm \beta)}{\cos\alpha\cos\beta}$$

$$\operatorname{ctg}\alpha \pm \operatorname{ctg}\beta = \frac{\sin(\alpha \pm \beta)}{\sin\alpha\sin\beta}$$

$$\sin\alpha\sin\beta = -\frac{1}{2}(\cos(\alpha + \beta) - \cos(\alpha - \beta))$$

$$\cos\alpha\cos\beta = \frac{1}{2}(\cos(\alpha + \beta) + \cos(\alpha - \beta))$$

$$\sin\alpha\cos\beta = \frac{1}{2}(\sin(\alpha + \beta) + \sin(\alpha - \beta))$$

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\operatorname{tg}(\alpha \pm \beta) = \frac{\operatorname{tg}\alpha \pm \operatorname{tg}\beta}{1 \mp \operatorname{tg}\alpha\operatorname{tg}\beta}$$

$$\operatorname{ctg}(\alpha \pm \beta) = \frac{\operatorname{ctg}\alpha\operatorname{ctg}\beta \mp 1}{\operatorname{ctg}\beta \pm \operatorname{ctg}\alpha}$$

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$

$$\operatorname{tg} 2\alpha = \frac{2\operatorname{tg}\alpha}{1 - \operatorname{tg}^2\alpha}$$

$$\operatorname{ctg} 2\alpha = \frac{\operatorname{ctg}^2\alpha - 1}{2\operatorname{ctg}\alpha}$$

$$\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$$

$$\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$$

$$\operatorname{tg} 3\alpha = \frac{3\operatorname{tg}\alpha - \operatorname{tg}^3\alpha}{1 - 3\operatorname{tg}^2\alpha}$$

$$\operatorname{ctg} 3\alpha = \frac{\operatorname{ctg}^3\alpha - 3\operatorname{ctg}\alpha}{3\operatorname{tg}^2\alpha - 1}$$

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1 - \cos\alpha}{2}}$$

$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1 + \cos\alpha}{2}}$$

$$\operatorname{tg}\frac{\alpha}{2} = \frac{1 - \cos\alpha}{\sin\alpha} = \pm\sqrt{\frac{1 - \cos\alpha}{1 + \cos\alpha}}$$

$$\operatorname{ctg}\frac{\alpha}{2} = \frac{1 + \cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1 - \cos\alpha} = \pm\sqrt{\frac{1 + \cos\alpha}{1 - \cos\alpha}}$$

$$\sin^2\alpha + \cos^2\alpha = 1$$

$$1 + \operatorname{tg}^2\alpha = \frac{1}{\cos^2\alpha}$$

$$1 + \operatorname{ctg}^2\alpha = \frac{1}{\sin^2\alpha}$$

$$\arcsin x + \arccos x = \frac{\pi}{2}$$

$$\operatorname{arctg} x + \operatorname{arcctg} x = \frac{\pi}{2}$$

EULERJEVE FORMULE:

$e^{ix} = \cos x + i \sin x \qquad e^{k2\pi i} = 1; k \in Z$

$\sin x = \frac{e^{ix} - e^{-ix}}{2} \qquad \sin x = -\operatorname{sh}(ix)$

$\cos x = \frac{e^{ix} + e^{-ix}}{2} \qquad \cos x = \operatorname{ch}(ix)$

$$\operatorname{sh} x = \frac{e^{x} - e^{-x}}{2}$$

$$\operatorname{ch} x = \frac{e^{x} + e^{-x}}{2}$$

$$\operatorname{th} x = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}} = \frac{\operatorname{sh} x}{\operatorname{ch} x} = \frac{1}{\operatorname{cth} x}$$

$$\operatorname{cth} x = \frac{e^{x} + e^{-x}}{e^{x} - e^{-x}} = \frac{\operatorname{ch} x}{\operatorname{sh} x} = \frac{1}{\operatorname{th} x}$$

$\operatorname{ch}^2 x - \operatorname{sh}^2 x = 1$

$\operatorname{sh}(-x) = -\operatorname{sh} x$

$\operatorname{ch}(-x) = \operatorname{ch} x$

$\operatorname{th}(-x) = -\operatorname{th} x$

$\operatorname{cth}(-x) = -\operatorname{cth} x$

$$\operatorname{arsh} x = \ln\left(x + \sqrt{1 + x^2}\right)$$

$$\operatorname{arth} x = \frac{1}{2} \ln \frac{1 + x}{1 - x}$$

$z = a + bi$

$\bar{z} = a - bi$

$|z| = \sqrt{z\bar{z}} = \sqrt{a^2 + b^2}$

$|z \cdot w| = |z| \cdot |w|$

$\overline{z \cdot w} = \bar{z} \cdot \bar{w}$

$|z + w| \le |z| + |w|$

$|z - w| \ge ||z| - |w||$

$|z + w| \ge |z| - |w|$

$z = a + bi$

$z = |z| \cdot (\cos\varphi + i \cdot \sin\varphi)$

$|z| = \sqrt{a^2 + b^2}$

$$\varphi = \operatorname{arctg}\frac{b}{a} + k\pi, k = 0,1,2$$

$$a = 0, b > 0 \Rightarrow \varphi = \frac{\pi}{2}$$

$$a = 0, b < 0 \Rightarrow \varphi = \frac{3\pi}{2}$$

$a > 0, b = 0 \Rightarrow \varphi = 0$

$a < 0, b = 0 \Rightarrow \varphi = \pi$

$z \cdot w = |z| \cdot |w| \cdot (\cos(\varphi_z + \varphi_w) + i \cdot \sin(\varphi_z + \varphi_w))$

$$\frac{z}{w} = \frac{|z|}{|w|} \cdot (\cos(\varphi_z - \varphi_w) + i \cdot \sin(\varphi_z - \varphi_w))$$

$z^{-1} = |z|^{-1} \cdot (\cos\varphi - i \cdot \sin\varphi)$

MOIVREOVA FORMULA:

$z^n = |z|^n \cdot (\cos\varphi + i \cdot \sin\varphi)^n = |z|^n \cdot (\cos(n\varphi) + i \cdot \sin(n\varphi))$

$x^n - z = 0$

$$x_{k+1} = \sqrt[n]{|z|} \cdot \left(\cos\frac{\varphi + k2\pi}{n} + i \cdot \sin\frac{\varphi + k2\pi}{n}\right) = x_1 \cdot \omega^k$$

$(k = 0,1,2,\ldots,n-1)$

$$x_1 = \sqrt[n]{|z|} \cdot \left(\cos\frac{\varphi}{n} + i \cdot \sin\frac{\varphi}{n}\right) \quad \text{primitivni koren enote}$$

$$\omega = \cos\frac{2\pi}{n} + i \cdot \sin\frac{2\pi}{n}$$

$$(a + b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

$$\binom{n}{k} = \frac{n!}{k! \cdot (k - r)!} \qquad n \in N$$

$$\binom{r}{k} = \frac{r \cdot (r-1)(r-2)\ldots(r-k+1)}{k!} \qquad r \in R$$

$$\binom{n}{k} = \binom{n}{n-k}$$

$$\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}$$

$$\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n = e$$

$$\lim_{n\to\infty} \sqrt[n]{n} = 1$$

$$\lim_{n\to\infty} \sqrt[n]{C} = 1 \quad C > 1$$

$$\lim_{n\to\infty} \frac{\sin x}{x} = 1$$

$$\lim_{n\to\infty} \frac{\cos x}{x} = 1$$

$$\lim_{n\to\infty} \frac{\operatorname{tg} x}{x} = 1$$

$$\lim_{n\to 0} \left(x \cdot \sin\frac{1}{x}\right) = 0$$

DIVERGENTNE VRSTE:

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$$

KONVERGENTNE VRSTE:

$\sum_{n=1}^{\infty}\frac{1}{n^2}$

$\sum_{n=1}^{\infty}\frac{1}{n^k}\quad k>1$

$\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$

$\sum_{n=1}^{\infty}\frac{(-1)^n}{n^k}\quad k>0$

$\sum_{n=1}^{\infty}a\,q^n\quad |q|<1, a\neq 0$

$\sum_{n=1}^{\infty}\frac{1}{n(n+1)}$

$\sum_{n=2}^{\infty}\frac{1}{n(n-1)}$

$\sum_{n=1}^{\infty}\frac{1}{n!}$

TAYLORJEVE VRSTE:

$$e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}\quad \forall x$$

$$\sin x=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}\quad \forall x$$

$$\cos x=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}\quad \forall x$$

$$\operatorname{sh}x=\sum_{n=0}^{\infty}\frac{x^{2n+1}}{(2n+1)!}$$

$$\operatorname{ch}x=\sum_{n=0}^{\infty}\frac{x^{2n}}{(2n)!}$$

$$\ln(1+x)=\sum_{n=0}^{\infty}(-1)^n\frac{x^{n+1}}{n+1}\quad x\in(-1,1]$$

$$(1+x)^r=\sum_{n=0}^{\infty}\binom{r}{n}\cdot x^n\quad x\in(-1,1), r\in R$$

$$(a+b)^r=\sum_{n=0}^{\infty}\binom{r}{n}\cdot a^{r-n}\cdot b^n\quad |a|>|b|$$

$$\frac{1}{1-x}=\sum_{n=0}^{\infty}x^n\quad x\in(-1,1)$$

GEOMETRIJSKA VRSTA:

$$\sum_{n=0}^{\infty}a\cdot q^n\qquad s_n=\begin{cases}a\cdot\frac{1-q^n}{1-q}, |q|>1, a\neq 0\\ n\cdot a, q=1, a\neq 0\end{cases}$$

$$s=\frac{a}{1-q}, |q|<1, a\neq 0$$

ODVODI:

$(kx+n)'=k$

$(C)'=0$

$(C\cdot f)'(x)=C\cdot f'(x)$

$\left(\sum_{i=1}^{n} f_i\right)'(x)=\sum_{i=1}^{n} f_i'(x)$

$(f_1\cdot\ldots\cdot f_n)'=f_1'\cdot f_2\cdot\ldots\cdot f_n+$
$\quad f_1\cdot f_2'\cdot f_3\cdot\ldots\cdot f_n+\ldots+f_1\cdot f_2\cdot\ldots\cdot f_n'$

$(x^n)'=n\cdot x^{n-1};n\in R$

$(a^x)'=a^x\cdot \ln a;a>0$

$(a^x)^{(n)}=a^x\cdot \ln^n a;a>0$

$(e^x)'=e^x$

$x=e^{\ln x}$

$(\ln x)'=\frac{1}{x}$

$(\ln x)^{(n)}=\frac{(n-1)!}{x^n}$

$(\sin x)'=\cos x$

$(\cos x)'=-\sin x$

$(tgx)'=\frac{1}{\cos^2 x}$

$(ctgx)'=-\frac{1}{\sin^2 x}$

$(\arcsin x)'=\frac{1}{\sqrt{1-x^2}}$

$(\arccos x)'=-\frac{1}{\sqrt{1-x^2}}$

$(arctg)'=\frac{1}{1+x^2}$

$(arcctg)'=-\frac{1}{1+x^2}$

$(shx)'=chx$

$(chx)'=shx$

$(thx)'=\frac{1}{ch^2x}$

INTEGRALI:

$\int(f\pm g)dx=\int fdx\pm\int gdx$

$\int C\cdot fdx=C\cdot\int fdx$

$\int udv=uv-\int vdu$ PERPARTES

$\int\frac{dx}{x}=\ln|x|+C$

$\int\frac{f'}{f}dx=\ln|f|+C$

$\int x^n dx=\frac{x^{n+1}}{n+1}+C;n\neq 1$

$\int a^x dx=\frac{a^x}{\ln a}+C$

$\int e^x dx=e^x+C$

$\int a^{kx}dx=\frac{a^x}{k\cdot\ln a}+C$

$\int e^{kx}dx=\frac{1}{k}e^x+C$

$\int\sin x dx=-\cos x+C$

$\int\cos x dx=\sin x+C$

$\int shxdx=chx+C$

$\int chxdx=shx+C$

$\int\frac{dx}{\sin^2 x}=-ctgx+C$

$\int\frac{dx}{\cos^2 x}=tgx+C$

$\int\frac{dx}{\sqrt{1-x^2}}=\arcsin x+C$

$\int\frac{dx}{1+x^2}=arctgx+C$

$\int\frac{dx}{\sqrt{a^2-x^2}}=\arcsin\frac{x}{a}+C$

$\int\frac{dx}{a^2+x^2}=\frac{1}{a}arctg\frac{x}{a}+C$

$\int\frac{dx}{\sqrt{x^2+k}}=\ln\left|x+\sqrt{x^2+k}\right|+C$

$$\int \frac{p^{(n)}(x)}{\sqrt{ax^2+bx+c}}dx = q^{(n-1)}(x)\sqrt{ax^2+bx+c} + A\int \frac{dx}{\sqrt{ax^2+bx+c}}$$

$$\int \frac{Ax+B}{(x^2+px+q)^n}dx = \frac{T^{(2n-3)}(x)}{(x^2+px+q)^{n11}} + \int \frac{Cx+D}{x^2+px+q}dx$$

$$\int \frac{S^{(m)}(x)}{(x-k)^n\sqrt{ax^2+bx+c}}, m < n : x-k = \frac{1}{t}$$

$$\int_a^b \left(f(x) \pm g(x)\right)dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$$

$$\int_a^b C \cdot f(x)dx = C \cdot \int_a^b f(x)dx$$

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx \quad a < c < b$$

$$\int_a^b f(x)dx = -\int_b^a f(x)dx$$

$$\int_a^a f(x)dx = 0$$