

**6. Domača naloga – Determinante**  
**Algebra 1, finančna matematika**

1. Izračunaj determinanto

$$\begin{vmatrix} 2 & 2 & 1 & 1 & 1 \\ 1 & 2 & 2 & 1 & 1 \\ 1 & 1 & 2 & 2 & 1 \\ 1 & 1 & 1 & 2 & 2 \\ 1 & 1 & 1 & 1 & 2 \end{vmatrix}.$$

2. Izračunaj determinanto

$$\begin{vmatrix} 2 & 1 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{vmatrix}.$$

3. Poišči vsa taka realna števila  $x$ , za katera je determinanta

$$\begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & x & 1 & 2 & 1 \\ 1 & 1 & x & 1 & 1 \\ 1 & 2 & 1 & x & 1 \\ 1 & 1 & 1 & 1 & 2 \end{vmatrix}$$

enaka 0.

4. Določi vsa taka realna števila  $x$ , da bo matrika  $A$  obrnljiva.

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2x & 3 & 4 \\ 2 & 3 & 4x & 5 \\ 3 & 4 & 5 & 6x \end{bmatrix}.$$

5. Izračunaj determinanto

$$\begin{vmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{vmatrix}.$$

6. Reši enačbo, v kateri nastopa determinanta velikosti  $n \times n$

$$\begin{vmatrix} 1 & 1 & 1 & \cdots & 1 & 1 \\ 1 & 1-x & 1 & \cdots & 1 & 1 \\ 1 & 1 & 2-x & \ddots & 1 & 1 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 1 & 1 & 1 & \ddots & n-2-x & 1 \\ 1 & 1 & 1 & \cdots & 1 & n-1-x \end{vmatrix} = 0$$

7. Izračunaj determinanto velikosti  $n \times n$

$$\begin{vmatrix} -1 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & -1 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & -1 & 1 & \cdots & 0 & 0 \\ 0 & 0 & 0 & -1 & \ddots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \ddots & -1 & 1 \\ 1 & 1 & 1 & 1 & \cdots & 1 & 1 \end{vmatrix}$$

8. Izračunaj determinanto velikosti  $n \times n$

$$\begin{vmatrix} 0 & 1 & 1 & \cdots & 1 & 1 \\ 1 & 0 & 1 & \cdots & 1 & 1 \\ 1 & 1 & 0 & \ddots & 1 & 1 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 1 & 1 & 1 & \ddots & 0 & 1 \\ 1 & 1 & 1 & \cdots & 1 & 0 \end{vmatrix}$$

9. Izračunaj determinanto velikosti  $2n \times 2n$

$$\begin{vmatrix} 1 & 0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 2 & \cdots & 0 & 2 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & n-1 & n-1 & 0 & 0 & \cdots & 0 \\ 1 & 2 & \cdots & n-1 & n & n+1 & n+2 & \cdots & 2n \\ 0 & 0 & \cdots & 0 & n+1 & n+1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & n+2 & 0 & n+2 & \cdots & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 2n & 0 & 0 & \cdots & 2n \end{vmatrix}.$$

10. Dana je matrika  $A = [a_{ij}]_{i,j=1}^n \in \mathbb{R}^{n,n}$ . Naj bo

$$p(x) = \begin{vmatrix} a_{11} + x & a_{12} + x & \cdots & a_{1n} + x \\ a_{21} + x & a_{22} + x & \cdots & a_{2n} + x \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} + x & a_{n2} + x & \cdots & a_{nn} + x \end{vmatrix}.$$

- (a) Dokaži, da je  $p$  linearen polinom v spremenljivki  $x$ .  
(b) Dokaži:  $p(x) = (1-x) \det A + xp(1)$ .

*Rešitve:*

1. 4
2. 6
3.  $x = 0, x = 1, x = 2$
4.  $x \neq 1$  in  $x \neq -1/2$
5. 1
6.  $x = 0, x = 1, \dots, x = n - 2$
7.  $(-1)^{n-1}(n-1)$
8.  $(-1)^{n-1}n$
9.  $-(2n)!(2n-1)$