7. Domača naloga - Vektorski prostori (2. del)
Algebra 1, finančna matematika

1. V prostoru $\mathbb{R}^4$ sta dana podprostora U in V. Prostor U ima bazo

$$\mathcal{B}_1 = \left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} \right\}$$

prostor V pa bazo

$$\mathcal{B}_2 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -1 \\ 1 \end{bmatrix} \right\}.$$

Poišči bazi podprostorov $U \cap V$ in $U + V$.

2. Dana sta vektorska podprostora

$$U = \{p \in \mathbb{R}_3[x] \colon p(-1) = p(1) = 0\} \quad \text{in}$$

$$V = \{p \in \mathbb{R}_3[x] \colon p'''(0) = p'(0) = 0\}$$

v prostoru $\mathbb{R}_3[x]$ polinomov stopnje največ 3. Poišči baze prostorov $U + V$ in $U \cap V$.

3. Dana sta vektorska podprostora

$$U = \{p \in \mathbb{R}_3[x], p(0) = p'(0) = 0\}$$

in

$$V = Lin\{x^3 - x + 1, x^3 - x^2, x^2 - x + 1\}$$

v prostoru $\mathbb{R}_3[x]$ polinomov stopnje največ 3. Poišči baze prostorov U, V, $U + V$ in $U \cap V$.

4. V prostoru $\mathbb{R}_3[x]$ polinomov stopnje največ 3 sta dana podprostora

$$U = Lin\{x^3 - x - 1, x^2 + x + 1, x^3 - x^2 - 2x - 2\}$$

in

$$V = \{p(x) = ax^3 + bx^2 + bx - a; a, b \in \mathbb{R}\}.$$

Poišči baze podprostorov $U, V, U + V$ in $U \cap V$.

5. V prostoru $\mathbb{R}_3[x]$ polinomov stonje največ 3 sta dana podprostora

$$U = \{p \in \mathbb{R}_3[x], p(1) = p(-1), p''(0) = 2p(1)\}$$

in

$$V = Lin\{x^2, 1\}.$$

Poišči bazi prostorov $U + V$ in $U \cap V$.

6. V prostoru $\mathbb{R}^4$ je dana baza

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}.$$

Poišči vektor koeficientov razvoja vektorja

$$x = \begin{bmatrix} 2 \\ 0 \\ -1 \\ 3 \end{bmatrix}$$

po tej bazi.

7. Podprostor U v $\mathbb{R}^4$ ima bazo

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -1 \\ 1 \end{bmatrix} \right\}.$$

Pokaži, da je vektor

$$x = \begin{bmatrix} 1 \\ -1 \\ -3 \\ -1 \end{bmatrix}$$

element prostora U in poišči njegov vektor koeficientov razvoja po bazi $\mathcal{B}$.

8. Podprostor U v prostoru $\mathbb{R}_3[x]$ ima bazo

$$\mathcal{B} = \{x^3 - x - 1, x^2 + x + 1, x^3 - x^2 - 2x\}.$$

Pokaži, da je polinom

$$p(x) = 2$$

element prostora U in poišči njegov vektor koeficientov razvoja po bazi $\mathcal{B}$.

9. V prostoru $\mathbb{R}^4$ je dana baza

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}.$$

Poišči prehodno matriko iz baze $\mathcal{B}$ v standardno bazo prostora $\mathbb{R}^4$ in prehodno matriko iz standardne baze v bazo $\mathcal{B}$.

10. Vektorski prostor U ima bazi

$$\mathcal{B}_1 = \left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \\ 1 \end{bmatrix} \right\}$$

in

$$\mathcal{B}_2 = \left\{ \begin{bmatrix} 2 \\ -1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -2 \\ 2 \end{bmatrix} \right\}.$$

Poišči prehodno matriko iz baze $\mathcal{B}_1$ v bazo $\mathcal{B}_2$ in prehodno matriko iz baze $\mathcal{B}_2$ v bazo $\mathcal{B}_1$.

11. Prostor $\mathbb{R}_2[x]$ ima baze

$$\mathcal{S} = \{x^2, x, 1\},$$

$$\mathcal{B}_1 = \{x^2 - x - 1, x + 1, x^2 + 1\}$$

in

$$\mathcal{B}_2 = \{x^2 + x + 1, 2x^2 + x, x^2 - 2\}.$$

Poišči prehodne matrike $P_{\mathcal{B}_1\mathcal{S}}$, $P_{\mathcal{B}_2\mathcal{S}}$, $P_{\mathcal{S}\mathcal{B}_1}$, $P_{\mathcal{S}\mathcal{B}_2}$, $P_{\mathcal{B}_1\mathcal{B}_2}$ in $P_{\mathcal{B}_2\mathcal{B}_1}$.

Rešitve:

1. baza $U + V = \left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ 2 \end{bmatrix} \right\}$, baza $U \cap V = \left\{ \begin{bmatrix} 2 \\ 0 \\ -1 \\ 1 \end{bmatrix} \right\}$

2. baza $U + V = \{x^3 - x, x^2 - 1, x^2\}$, baza $U \cap V = \{x^2 - 1\}$

3. baza $U = \{x^3, x^2\}$, baza $V = \{x^3 - x + 1, x^3 - x^2\}$,
   baza $U + V = \{x^3, x^2, x^3 - x + 1\}$, baza $U \cap V = \{x^3 - x^2\}$

4. baza $U = \{x^3 - x - 1, x^2 + x + 1\}$, baza $V = \{x^3 - 1, x^2 + x\}$,
   baza $U + V = \{x^3 - x - 1, x^2 + x + 1, x^3 - 1, x^2 + x\}$, baza $U \cap V = \{\}$

5. baza $U + V = \{x^3 - x, x^2, 1\}$, baza $U \cap V = \{x^2\}$

6. $x_{\mathcal{B}} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

7. $x_{\mathcal{B}} = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$

8. $p_{\mathcal{B}} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$

9. $P_{\mathcal{B}\mathcal{S}} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ -1 & 0 & -1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$, $P_{\mathcal{S}\mathcal{B}} = \frac{1}{2}\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ -1 & -1 & -1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

10. $P_{\mathcal{B}_1\mathcal{B}_2} = \frac{1}{3}\begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix}$, $P_{\mathcal{B}_2\mathcal{B}_1} = \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix}$

11. $P_{\mathcal{B}_1\mathcal{S}} = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix}$, $P_{\mathcal{B}_2\mathcal{S}} = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & -2 \end{bmatrix}$, $P_{\mathcal{S}\mathcal{B}_1} = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$,
$P_{\mathcal{S}\mathcal{B}_2} = \begin{bmatrix} -2 & 4 & -1 \\ 2 & -3 & 1 \\ -1 & 2 & -1 \end{bmatrix}$, $P_{\mathcal{B}_1\mathcal{B}_2} = \begin{bmatrix} -5 & 3 & -3 \\ 4 & -2 & 3 \\ -2 & 1 & -2 \end{bmatrix}$, $P_{\mathcal{B}_2\mathcal{B}_1} = \begin{bmatrix} 1 & 3 & 3 \\ 2 & 4 & 3 \\ 0 & -1 & -2 \end{bmatrix}$