

Rešitve enajste domače naloge

1. (a) Velja

$$P_4(x) = f(a) + (x-a)f'(a) + \frac{1}{2}(x-a)^2 f''(a) + \frac{1}{6}f^{(3)}(a) + (x-a)^3 + \frac{1}{24}f^{(4)}(a)(x-a)^4.$$

Za $f(x) = e^{x/2}$ dobimo $1 + \frac{x}{2} + \frac{x^2}{8} + \frac{x^3}{48} + \frac{x^4}{384}$.

- (b) Velja $f(x) = P_4(x) + R_4(x)$, kjer je $R_4(x) = \frac{f^{(5)}(b)}{5!}x^5$ za nek $b \in (1, 0)$. Sledi

$$|R_4(1)| \leq \frac{e^{b/2}}{5! \cdot 2^5} \leq \frac{e^{1/2}}{5! \cdot 2^5} = 4,3 \cdot 10^{-4}.$$

- (c) Iščemo k , da velja $R_k(1) = \frac{f^{(k+1)}(b)}{(k+1)!} \leq 10^{-11}$. Velja

$$R_k(1) \leq \frac{e^{1/2}}{(k+1)! \cdot 2^{k+1}}.$$

Izračunamo $R_{10}(1) = 2.01679 \cdot 10^{-11}$, $R_{11}(1) = 8.4 \cdot 10^{-13}$. Potrebujemo 11 členov.

- 2.

$$\ln(3) + \sum_{k=1}^{10} \frac{(-1)^k (x-1)^k}{k \cdot 3^k}.$$

3. $4 + (x-2)^2 - (x-2)^3 + (x-2)^4$.