Rešitve druge domače naloge

1. (a) Elipsa z enačbo $\frac{3}{4}y^2 + \frac{3}{4}(x-6)^2 = 3$.

   (b) Hiperbola z enačbo $(x-3)^2 - y^2 = 8$.

   (c) Za $z := x + iy$ in $x < 1$ so rešitev točke 'znotraj' parabole $y^2 = -4(x-1)$. Za $z := x + iy$ in $x > 1$ so rešitev točke 'znotraj' parabole $y^2 = 4(x-1)$.

   (d) $\{z = x + iy \in \mathbb{C} \mid x = y, x \neq 0\}$

   (e) Rešitev je elipsa z enačbo $8(x - \frac{5}{2})^2 + 9y^2 = 18$. Ideja:

   - $|z-2| = 3 - |z-3|$
   - $\Leftrightarrow |z-2|^2 = 9 - 2|z-3| + |z-3|^2$, če $3 - |z-3| \geq 0$
   - Vstavimo $z := x + iy$ in dobimo $14 - 2x = 6\sqrt{(x-3)^2 + y^2}$.
   - $\Leftrightarrow (14-2x)^2 = 16\left[(x-3)^2 + y^2\right]$, če je $14 - 2x \geq 0$, tj. $x \leq 7$.
   - Dobimo $8(x - \frac{5}{2})^2 + 9y^2 = 18$.
   - Preverimo še, da vse rešitve zadoščajo pogojema pridobljenima med izpeljavo. Npr. $8(x - \frac{5}{2})^2 \leq 18 \Rightarrow x \in [1, 4]$ (x res manjši od 7) in $9y^2 \leq 18 \Rightarrow y^2 \leq 2$. $3 - |z-3| \geq 0$ velja natanko tedaj, ko $y^2 \leq 9 - (x-3)^2$. Za $x \in [1, 4]$ bo pogoju vedno zadoščeno, če je $y^2 \leq 5$. Ker pa vemo $y^2 \leq 2$, je tudi temu pogoju zadoščeno.

2. $i\frac{3^3}{2^9}$

3. (a) $z \in \left\{\sqrt{5} \cdot (\cos(\phi) + i\sin(\phi)) \mid \phi \in \left\{\frac{\pi}{10}, \frac{\pi}{2}, \frac{9\pi}{10}, \frac{13\pi}{10}, \frac{17\pi}{10}\right\}\right\}$

   (b) $z \in \left\{\sqrt[6]{2} \cdot (\cos(\phi) + i\sin(\phi)) \mid \phi \in \left\{\frac{\pi}{4}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{21\pi}{12}\right\}\right\}$

   (c) $z \in \left\{\frac{1}{2}, \frac{1}{2} + \frac{1}{2}i, \frac{1}{2} - \frac{1}{2}i\right\}$

   (d) $z \in \left\{0, i\sqrt[7]{2}, -i\sqrt[7]{2}\right\}$

4. $(1 - \sqrt{2 - \sqrt{3}}) + i(\sqrt{2 + \sqrt{3}} - 1)$.

   Zarotirati $3+i$ okoli $1-i$ za kot $\pi/3$ pomeni zarotirati $(3+i)-(1-i) = 2 + 2i$ za $\pi/3$ okoli izhodišča in translirati nazaj za vektor $1 - i$.

- $$\begin{aligned} 2+2i &= 2\left(\cos\left(\frac{\pi}{4}\right)+i\sin\left(\frac{\pi}{4}\right)\right) \to 2\left(\cos\left(\frac{7\pi}{12}\right)+i\sin\left(7\frac{\pi}{12}\right)\right) \\ \cos\left(\frac{7\pi}{12}\right) &= -\cos\left(\frac{5\pi}{12}\right) = -\sin\left(\frac{\pi}{12}\right) = -\frac{\sqrt{2}}{2}\sqrt{1-\cos\left(\pi/6\right)} = \\ &= -\frac{\sqrt{2-\sqrt{3}}}{2} \\ \sin\left(\frac{7\pi}{12}\right) &= \frac{2+\sqrt{3}}{2} \end{aligned}$$

$$2\left(\cos\left(\frac{7\pi}{12}\right)+i\sin\left(\frac{7\pi}{12}\right)\right) = -\sqrt{2-\sqrt{3}}+i\sqrt{2+\sqrt{3}}$$

- $$2\left(\cos\left(\frac{7\pi}{12}\right)+i\sin\left(\frac{7\pi}{12}\right)\right) \to 2\left(\cos\left(\frac{7\pi}{12}\right)+i\sin\left(\frac{7\pi}{12}\right)\right)+(1-i) =$$

$$= \left(1-\sqrt{2-\sqrt{3}}\right)+i\left(\sqrt{2+\sqrt{3}}-1\right)$$

5. $\frac{1}{2}z^2+\frac{1}{2}\bar{z}^2-z\bar{z}+z+\bar{z}+2(z-\bar{z})i-4=0$. $(x=\frac{z+\bar{z}}{2},\ y=\frac{z-\bar{z}}{2i})$