Rešitve devete domače naloge

1. (a) $y' = \dfrac{(2x\sin x + x^2\cos x)(1+\tan x) - x^2 \sin x \frac{1}{\cos^2 x}}{(1+\tan x)^2}$

   (b) $y' = \dfrac{(\frac{2}{\sqrt{1-4x^2}} + \frac{1}{\arccos x}\frac{-1}{\sqrt{1-x^2}})(x^e + e^x) - (\arcsin(2x) + \ln(\arccos x))(ex^{e-1} + e^x)}{(x^e+e^x)^2}$

   (c) $y' = 3^3 x^{3^3-1} + 3^{x^3}\ln 3 \cdot 3x^2 + 3^{3^x}\ln 3 \cdot 3^x \ln 3$

   (d) $y' = (1+x)^{\frac{1}{x}} \dfrac{\frac{x}{1+x} - \ln(1+x)}{x^2}$

   (e) $y' = e^x \ln^2(\cos^3(\ln x)) + e^x 2\, \ln(\cos^3(\ln x)) \dfrac{1}{\cos^3(\ln x)} 3\cos^2(\ln(x)(-\sin(\ln x)) \dfrac{1}{x}$

2. (a)

$$\begin{aligned}\lim_{x\uparrow 0} \frac{\arcsin \frac{1-x^2}{1+x^2} - \frac{\pi}{2}}{x} &= \lim_{x\uparrow 0} \frac{1}{\sqrt{1-(\frac{1-x^2}{1+x^2})^2}} \frac{-2x(1+x^2)-(1-x^2)2x}{(1+x^2)^2} = \\ &= \lim_{x\uparrow 0} \frac{-4x(1+x^2)}{\sqrt{(1+x^2)^2-(1-x^2)^2}} = \lim_{x\uparrow 0}\frac{-4x}{\sqrt{4x^2}} = 2.\end{aligned}$$

   Levi odvod je 2, desni odvod je -2.

   (b) Levi odvod je -1, desni odvod je 1.

3.

$$\begin{aligned}\lim_{x\uparrow -1}\frac{f(x)-f(-1)}{x-(-1)} &= \lim_{x\uparrow -1}\frac{\frac{\pi}{2^{x+1}-2} - \frac{\pi}{-1}}{x+1} = \lim_{x\uparrow -1} \pi \frac{\frac{1}{2}\frac{1}{2^x-1}+1}{x+1} = \\ &= \lim_{x\uparrow -1} \pi \frac{1}{2}\frac{1}{(2^x-1)^2} 2^x \ln 2 = \pi \frac{1}{2}\frac{1}{(-\frac{1}{2})^2}\frac{1}{2}\ln 2 = \pi \ln 2.\end{aligned}$$

$$\lim_{x\downarrow -1}\frac{f(x)-f(-1)}{x-(-1)} = \lim_{x\downarrow -1}\frac{\pi(\sqrt{x+1}-1) - \frac{\pi}{-1}}{x+1} = \lim_{x\downarrow -1}\pi\frac{1}{2}\frac{1}{\sqrt{x+1}} = \infty$$

   Analogno si pogledamo obnašanje odvoda funkcije v točki $x = 0$. Funkcija je odvedljiva na $\mathbb{R} \setminus \{-1\}$.

4. $a = 2,\ b = -1,\ c = \pi - 3$

5. Označimo z $g(x)$ enačbo normale. Velja

$$\begin{aligned} f'(x) &= \frac{1}{2}\frac{1}{\sqrt{\ln x}}\frac{1}{x} \ \Rightarrow\ -\frac{1}{f'(e)} = -2e \ \Rightarrow \\ g(x) &= -\frac{1}{f'(e)}x + c \Rightarrow g(e) = 1 = -2e^2 + c \ \Rightarrow c = 1 + 2e^2.\end{aligned}$$

   Enačba normale: $g(x) = -2ex + (1+2e^2)$.

6. Poiščeš točko x_0 na grafu, kjer velja $f'(x_0) = -3$. Nato izračunaš tangento na f v x_0.

   Enačba tangente: $g(x) = -3(x-2)$.

7. Levi odvod je 1, desni odvod je -1, torej je $\phi = 90°$.

8. Enačbe tangent v točki $x = x_0$:

$$y_1 = f'(x_0)x + (f(x_0) - f'(x_0)x_0),$$
$$y_2 = g'(x_0)x + (g(x_0) - g'(x_0)x_0),$$
$$y_3 = \frac{1}{2}(f+g)'(x_0)x + (\frac{1}{2}(f+g)(x_0) - \frac{1}{2}(f+g)'(x_0)x_0) =: k(x).$$

   Naj se tangenti funkcij f in g v toǩi $x = x_0$ sekata v točki (a, b). Poračunamo samo še, da je $k(a) = b$.

$$\begin{aligned} k(a) &= \frac{1}{2}(f+g)'(x_0)a + (\frac{1}{2}(f+g)(x_0) - \frac{1}{2}(f+g)'(x_0)x_0) = \\ &= \frac{1}{2}(f'(x_0)a + (\frac{1}{2}f(x_0) - \frac{1}{2}f'(x_0)x_0) + g'(x_0)a + (\frac{1}{2}g(x_0) - \frac{1}{2}g'(x_0)x_0)) = \\ &= \frac{1}{2}(b+b) = b. \end{aligned}$$