<u>Primer 1:</u> Analiziramo produkcijske funkcije za podjetja industrijske dejavnosti v RS v podskupini DL 30.02 – *Proizvodnja računalnikov in druge opreme za obdelavo podatkov* na podlagi podatkov iz zaključnih računov za leto 1998 (datoteka `proizvod.dta`).

a) Opredelite izraze za izračun kritične vrednosti in *p*–vrednosti za standardizirano normalno porazdelitev, *t*–porazdelitev, *F*–porazdelitev in χ^2–porazdelitev.
b) Izračunajte 95% intervale zaupanja in natančne stopnje značilnosti (*p*–vrednosti) za regresijske koeficiente v modelu Cobb-Douglasove produkcijske funkcije.
c) Izračunajte "beta koeficiente" (standardizirane regresijske koeficiente) v modelu Cobb-Douglasove produkcijske funkcije.
d) S pomočjo *t*–testa in *F*–testa preverite predpostavko o homogenosti stopnje ena v modelu Cobb-Douglasove produkcijske funkcije.

Izpis rezultatov obdelav v programskem paketu Stata:

```
* Kriticne vrednosti in p-vrednosti porazdelitev:

display 1-normal(z0)          // p-vrednost z-statistike //
display invnormal(1-p)        // kriticna vrednost z-statistike //

display ttail(df,t0)          // p-vrednost t-statistike (rezultat je masa
                                 verjetnosti v enem repu) //
display invttail(df,p)        // kriticna vrednost t-statistike (p se nanasa na
                                 maso verjetnosti v enem repu) //

display Ftail(df1,df2,F0)     // p-vrednost F-statistike //
display invFtail(df1,df2,p)  // kriticna vrednost F-statistike //

display chi2tail(df,ch0)      // p-vrednost hi2-statistike //
display invchi2tail(df,p)    // kriticna vrednost hi2-statistike //
```

```
. * Izracun 95% intervala zaupanja za regresijske koeficiente:

. regress lq ll lk

      Source |       SS       df       MS              Number of obs =      81
-------------+------------------------------           F(  2,    78) =  190.75
       Model |  178.261263     2  89.1306313           Prob > F      =  0.0000
    Residual |    36.44752    78  .467275898           R-squared     =  0.8302
-------------+------------------------------           Adj R-squared =  0.8259
       Total |  214.708783    80  2.68385978           Root MSE      =  .68358

------------------------------------------------------------------------------
          lq |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          ll |   .9645479   .1199229     8.04   0.000     .7257997    1.203296
          lk |   .1885438   .0673358     2.80   0.006     .0544886     .322599
       _cons |   7.546026   .4617465    16.34   0.000      6.62676    8.465293
------------------------------------------------------------------------------

. display invttail(78,0.025)
1.9908471
```

```
. mata
------------------------------ mata (type end to exit) ---------------------------
: 0.9645479 - 1.9908471 * 0.1199229
  .7257997423

: 0.9645479 + 1.9908471 * 0.1199229
  1.203296058

: 0.1885438 - 1.9908471 * 0.0673358
  .0544885178

: 0.1885438 + 1.9908471 * 0.0673358
  .3225990822

: 7.546026 - 1.9908471 * 0.4617465
  6.62675932

: 7.546026 + 1.9908471 * 0.4617465
  8.46529268

: end
-------------------------------------------------------------------------------------
```

```
. * Izracun standardiziranih (beta) koeficientov CD produkcijske funkcije:

. egen lqs=std(lq)
. egen lls=std(ll)
. egen lks=std(lk)

. regress lqs lls lks

      Source |       SS       df       MS              Number of obs =      81
-------------+------------------------------           F(  2,    78) =  190.75
       Model |  66.4197375     2  33.2098687           Prob > F      =  0.0000
    Residual |  13.5802628    78  .174105933           R-squared     =  0.8302
-------------+------------------------------           Adj R-squared =  0.8259
       Total |  80.0000002    80           1           Root MSE      =  .41726

------------------------------------------------------------------------------
         lqs |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         lls |   .6971976   .0866831     8.04   0.000     .5246249    .8697703
         lks |   .2427173   .0866831     2.80   0.006     .0701445      .41529
       _cons |   3.43e-09   .0463622     0.00   1.000    -.0923001    .0923001
------------------------------------------------------------------------------

. regress lq ll lk, beta

      Source |       SS       df       MS              Number of obs =      81
-------------+------------------------------           F(  2,    78) =  190.75
       Model |  178.261263     2  89.1306313           Prob > F      =  0.0000
    Residual |    36.44752    78  .467275898           R-squared     =  0.8302
-------------+------------------------------           Adj R-squared =  0.8259
       Total |  214.708783    80  2.68385978           Root MSE      =  .68358

------------------------------------------------------------------------------
          lq |      Coef.   Std. Err.      t    P>|t|                     Beta
-------------+----------------------------------------------------------------
          ll |   .9645479   .1199229     8.04   0.000                 .6971976
          lk |   .1885438   .0673358     2.80   0.006                 .2427173
       _cons |   7.546026   .4617465    16.34   0.000                        .
------------------------------------------------------------------------------
```

```
. * Preverjanje predpostavke o homogenosti stopnje 1 za CD produkcijsko funkcijo:

. regress lq ll lk

      Source |       SS       df       MS              Number of obs =      81
-------------+------------------------------           F(  2,    78) =  190.75
       Model |  178.261263     2  89.1306313           Prob > F      =  0.0000
    Residual |    36.44752    78  .467275898           R-squared     =  0.8302
-------------+------------------------------           Adj R-squared =  0.8259
       Total |  214.708783    80  2.68385978           Root MSE      =  .68358

------------------------------------------------------------------------------
          lq |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          ll |   .9645479   .1199229     8.04   0.000     .7257997    1.203296
          lk |   .1885438   .0673358     2.80   0.006     .0544886     .322599
       _cons |   7.546026   .4617465    16.34   0.000      6.62676    8.465293
------------------------------------------------------------------------------

. vce

Covariance matrix of coefficients of regress model

        e(V) |         ll          lk       _cons
-------------+------------------------------------
          ll |  .01438151
          lk | -.00680594   .00453411
       _cons |  .03829304  -.02989139   .21320983
```

Domnevo, da je stopnja homogenosti pri Cobb-Douglasovi produkcijski funkciji enaka 1, preverimo najprej s pomočjo *t*–statistike:

H_0: $\beta_2 + \beta_3 = 1$
H_1: $\beta_2 + \beta_3 \neq 1$

$$t = \frac{(b_2+b_3)-(\beta_2+\beta_3)}{\sqrt{\mathrm{var}(b_2+b_3)}} = \frac{(b_2+b_3)-(\beta_2+\beta_3)}{\sqrt{\mathrm{var}(b_2)+\mathrm{var}(b_3)+2\,\mathrm{cov}(b_2,b_3)}} = 2{,}10214$$

$$t_{k\,(m\,=\,n-k\,\cong\,80,\ \frac{\alpha}{2}\,=\,0{,}025)} = 1{,}990$$

$|t| > t_k$, zavrnemo ničelno domnevo

```
. display 2*ttail(78, abs(2.10214))
.03876779
```

Domnevo, da je stopnja homogenosti pri Cobb-Douglasovi produkcijski funkciji enaka 1, preverimo sedaj še s pomočjo *F*–statistike. Pri tem najprej ocenimo osnovni model, nato pa vanj vključimo ničelno domnevo in dobimo model z omejitvami, ki ga ocenimo.

H_0: $\beta_2 + \beta_3 = 1 \Rightarrow \beta_2 = 1 - \beta_3$
H_1: $\beta_2 + \beta_3 \neq 1$

$\ln Q_i = \beta_1 + \beta_2 \ln L_i + \beta_3 \ln K_i + u_i$
$\ln Q_i = \beta_1 + (1 - \beta_3) \ln L_i + \beta_3 \ln K_i + u_i$
$\ln Q_i - \ln L_i = \beta_1 + \beta_3 (\ln K_i - \ln L_i) + u_i$
$\ln (Q_i / L_i) = \beta_1 + \beta_3 \ln (K_i / L_i) + u_i$

```
. gen lql=log(q/l)
. gen lkl=log(k/l)

. regress lql lkl

      Source |       SS       df       MS              Number of obs =      81
-------------+------------------------------           F(  1,    79) =   17.38
       Model |  8.47386986     1  8.47386986           Prob > F      =  0.0001
    Residual |  38.5123913    79  .487498624           R-squared     =  0.1803
-------------+------------------------------           Adj R-squared =  0.1700
       Total |  46.9862612    80  .587328265           Root MSE      =  .69821

------------------------------------------------------------------------------
         lql |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         lkl |     .25412   .0609515     4.17   0.000      .132799     .375441
       _cons |   7.303514    .456675    15.99   0.000     6.394525    8.212502
------------------------------------------------------------------------------

. mata
----------------------------- mata (type end to exit) ---------------------------
: ((38.5123913 - 36.44752) / (3 - 2)) / (36.44752 / (81 - 3))
  4.418955292

: Ftail(1,78,4.418955292)
  .0387685944

: 1 - (38.5123913 / 214.708783)
  .8206296419

: ((0.8302 - 0.8206) / (3 - 2)) / ((1 - 0.8302) / (81 - 3))
  4.409893993

: Ftail(1,78,4.409893993)
  .0389635256

: end
---------------------------------------------------------------------------------

. regress lq ll lk

      Source |       SS       df       MS              Number of obs =      81
-------------+------------------------------           F(  2,    78) =  190.75
       Model |  178.261263     2  89.1306313           Prob > F      =  0.0000
    Residual |    36.44752    78  .467275898           R-squared     =  0.8302
-------------+------------------------------           Adj R-squared =  0.8259
       Total |  214.708783    80  2.68385978           Root MSE      =  .68358

------------------------------------------------------------------------------
          lq |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          ll |   .9645479   .1199229     8.04   0.000     .7257997    1.203296
          lk |   .1885438   .0673358     2.80   0.006     .0544886     .322599
       _cons |   7.546026   .4617465    16.34   0.000      6.62676    8.465293
------------------------------------------------------------------------------

. test ll+lk=1

 ( 1)  ll + lk = 1

       F(  1,    78) =    4.42
            Prob > F =    0.0388
```

■

Primer 2: V vzorec smo izbrali 32 evropskih držav in za leto 2003 pridobili naslednje podatke (datoteka `zdravstvo.dta`):

- pričakovana življenjska doba (*PZD*; v letih);
- izdatki za zdravstvo na prebivalca (*IZDATKI*; v ameriških dolarjih);
- odstotek kadilcev med odraslimi prebivalci (*TOBAK*).

Ocenite linearni regresijski model: $PZD_i = \beta_1 + \beta_2 IZDATKI_i + \beta_3 TOBAK_i + u_i$ in s pomočjo ustreznih postopkov preverjanja domnev odgovorite na naslednja vprašanja:

a) Ali odstotek kadilcev med odraslimi prebivalci vpliva na pričakovano življenjsko dobo?
b) Ali lahko trdimo, da povečanje izdatkov za zdravstvo na prebivalca za 100 dolarjev podaljša povprečno življenjsko dobo za več kot dva meseca (ob nespremenjeni spremenljivki *TOBAK*)?
c) Preverite ničelno domnevo, da znižanje povprečne življenjske dobe zaradi povečanja odstotka kadilcev med odraslimi prebivalci za eno odstotno točko lahko »kompenziramo« s povečanjem izdatkov za zdravstvo na prebivalca za 100 dolarjev.

Izpis rezultatov obdelav v programskem paketu Stata:

```
. regress pzd izdatki tobak

      Source |       SS       df       MS              Number of obs =      32
-------------+------------------------------           F(  2,    29) =   31.97
       Model |  385.751827     2  192.875914           Prob > F      =  0.0000
    Residual |   174.97295    29  6.03354999           R-squared     =  0.6880
-------------+------------------------------           Adj R-squared =  0.6664
       Total |  560.724777    31   18.087896           Root MSE      =  2.4563

------------------------------------------------------------------------------
         pzd |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     izdatki |   .0023323   .0003709     6.29   0.000     .0015736    .0030909
       tobak |  -.2503555   .0889983    -2.81   0.009    -.4323774   -.0683335
       _cons |   79.62409   2.796632    28.47   0.000     73.90433    85.34384
------------------------------------------------------------------------------

. * Testiranje hipotez s t-testom (skalarna oblika)

. vce

Covariance matrix of coefficients of regress model

        e(V) |    izdatki        tobak        _cons 
-------------+------------------------------------
     izdatki |  1.376e-07 
       tobak |  9.960e-06     .0079207 
       _cons | -.00048708   -.24150696    7.8211512 

. display 2*ttail(29, abs(-2.81))
.00878239

. display ttail(29, 1.78)
.04277883
```

```
. display 2*ttail(29, abs(-0.16))
.87399033

. * Testiranje hipotez s t-testom (matricna oblika)

. matrix beta=(e(b))'

. matrix list beta

beta[3,1]
                  y1
izdatki    .00233227
  tobak   -.25035545
  _cons    79.624085

. matrix vcm=e(V)

. matrix list vcm

symmetric vcm[3,3]
             izdatki       tobak       _cons
izdatki    1.376e-07
  tobak    9.960e-06    .0079207
  _cons   -.00048708  -.24150696   7.8211512

. matrix c=(100\0\0)

. matrix list c

c[3,1]
     c1
r1  100
r2    0
r3    0

. matrix varcb=c'*vcm*c

. matrix list varcb

symmetric varcb[1,1]
           c1
c1  .00137602

. matrix secb=cholesky(varcb)

. matrix list secb

symmetric secb[1,1]
           c1
c1  .03709481

. matrix tb=(c' * beta - 0.167) * invsym(secb)

. matrix list tb

symmetric tb[1,1]
           c1
r1  1.7853439

. display ttail(29, 1.7853439)
.04233458
```

```
. matrix c=(100\1\0)

. matrix list c

c[3,1]
     c1
r1  100
r2    1
r3    0

. matrix varcb=c'*vcm*c

. matrix list varcb

symmetric varcb[1,1]
           c1
c1  .01128871

. matrix secb=cholesky(varcb)

. matrix list secb

symmetric secb[1,1]
           c1
c1  .10624835

. matrix tc=(c' * beta - 0) * invsym(secb)

. matrix list tc

symmetric tc[1,1]
            c1
r1  -.16121158

. display 2*ttail(29, abs(-.16121158))
.87304462

. * Testiranje hipotez z F-testom

. gen xres=izdatki-100*tobak

. regress pzd xres

      Source |       SS       df       MS              Number of obs =      32
-------------+------------------------------           F(  1,    30) =   66.05
       Model |   385.59502     1   385.59502           Prob > F      =  0.0000
    Residual |  175.129757    30  5.83765858           R-squared     =  0.6877
-------------+------------------------------           Adj R-squared =  0.6773
       Total |  560.724777    31   18.087896           Root MSE      =  2.4161

------------------------------------------------------------------------------
         pzd |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        xres |   .0023683   .0002914     8.13   0.000     .0017732    .0029634
       _cons |   79.18374   .5903236   134.14   0.000     77.97814    80.38934
------------------------------------------------------------------------------

. display Ftail(1,29,0.026)
.87301842
```

```
. regress pzd izdatki tobak

      Source |       SS       df       MS              Number of obs =      32
-------------+------------------------------           F(  2,    29) =   31.97
       Model |  385.751827     2  192.875914           Prob > F      =  0.0000
    Residual |   174.97295    29  6.03354999           R-squared     =  0.6880
-------------+------------------------------           Adj R-squared =  0.6664
       Total |  560.724777    31   18.087896           Root MSE      =  2.4563

------------------------------------------------------------------------------
         pzd |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     izdatki |   .0023323   .0003709     6.29   0.000     .0015736    .0030909
       tobak |  -.2503555   .0889983    -2.81   0.009    -.4323774   -.0683335
       _cons |   79.62409   2.796632    28.47   0.000     73.90433    85.34384
------------------------------------------------------------------------------

. test 100*izdatki+tobak=0

 ( 1)  100*izdatki + tobak = 0

       F(  1,    29) =    0.03
            Prob > F =    0.8730
```

■

Primer 3: Pri ocenjevanju funkcije povpraševanja po pivu za neko državo smo uporabili naslednje pojasnjevalne spremenljivke: ceno piva (*CP*), cene drugih alkoholnih pijač (*CDP*), cene življenjskih potrebščin (*CZP*) in razpoložljivi dohodek (*RD*).

a) Zapišite funkcijo v obliki, ki zagotavlja, da regresijski koeficienti predstavljajo različne elastičnosti povpraševanja po pivu. Kako se spremeni specifikacija funkcije, če predpostavljamo, da hkratna sprememba vseh pojasnjevalnih spremenljivk za enak odstotek ohranja odvisno spremenljivko nespremenjeno. Prikažite postopek, s katerim bi lahko ugotovili, katera izmed oblik funkcije povpraševanja je primernejša.

b) Prikažite, kako bi preverili naslednje tri domneve oziroma omejitve v okviru (ene) ničelne domneve (izhajajte iz prve funkcije v prejšnji točki):
   - funkcija povpraševanja po pivu je homogena stopnje nič;
   - razlika med elastičnostjo povpraševanja glede na cene piva in glede na cene drugih pijač je enaka nič;
   - elastičnost povpraševanja po pivu glede na razpoložljivi dohodek je enaka ena.

■

Primer 4: Na podlagi podatkov za Slovenijo (razdobje 1965-1989), ki so zbrani v datoteki `potrosnja1.dta`, ocenite naslednjo funkcijo porabe:

$$OP_t = \beta_1 + \beta_2 OD_t + \beta_3 SOC_t + \beta_4 PRO_t + \beta_5 TUJ_t + u_t$$

Pri tem smo z *OP* označili osebno porabo, *OD* je oznaka za osebne dohodke, *SOC* za socialne prejemke, *PRO* za proračunske prejemke in *TUJ* za prejemke prebivalstva iz tujine (vse spremenljivke so izražene v mio DIN po cenah iz leta 1972)

Preverite, ali bi bilo smiselno iz funkcije izločiti prejemke iz proračuna in tujine.

Izpis rezultatov obdelav v programskem paketu Stata:

```
. regress op od soc pro tuj

      Source |       SS       df       MS              Number of obs =      25
-------------+------------------------------           F(  4,    20) =  130.82
       Model |  3.69668581     4  .924171451           Prob > F      =  0.0000
    Residual |  .141293465    20  .007064673           R-squared     =  0.9632
-------------+------------------------------           Adj R-squared =  0.9558
       Total |  3.83797927    24  .159915803           Root MSE      =  .08405

------------------------------------------------------------------------------
          op |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          od |   .3981591   .1938749     2.05   0.053    -.0062569     .802575
         soc |   1.628632   .5127575     3.18   0.005      .559039    2.698226
         pro |   7.506952   6.290407     1.19   0.247    -5.614608    20.62851
         tuj |   .4450372   .5246166     0.85   0.406    -.6492939    1.539368
       _cons |   .3635363   .1152333     3.15   0.005     .1231638    .6039088
------------------------------------------------------------------------------

. * Preverjanje smiselnosti izkljucevanja pojasnjevalnih spremenljivk (F-test)

. regress op od soc

      Source |       SS       df       MS              Number of obs =      25
-------------+------------------------------           F(  2,    22) =  253.82
       Model |  3.67855587     2  1.83927794           Prob > F      =  0.0000
    Residual |   .1594234     22  .007246518           R-squared     =  0.9585
-------------+------------------------------           Adj R-squared =  0.9547
       Total |  3.83797927    24  .159915803           Root MSE      =  .08513

------------------------------------------------------------------------------
          op |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          od |   .6648524   .0872809     7.62   0.000     .4838429    .8458619
         soc |   .9483866    .283467     3.35   0.003      .360512    1.536261
       _cons |   .2653599   .0937114     2.83   0.010     .0710144    .4597054
------------------------------------------------------------------------------

. display ((.1594234-.141293465)/2) / (.141293465/(25-5))
1.2831404

. display Ftail(2,20,1.2831404)
.29901989
```

```
. * Preverjanje smiselnosti izkljucevanja pojasnjevalnih spremenljivk (LM-test)

. regress op od soc

      Source |       SS       df       MS              Number of obs =      25
-------------+------------------------------           F(  2,    22) =  253.82
       Model |  3.67855587     2  1.83927794           Prob > F      =  0.0000
    Residual |    .1594234    22  .007246518           R-squared     =  0.9585
-------------+------------------------------           Adj R-squared =  0.9547
       Total |  3.83797927    24  .159915803           Root MSE      =  .08513

------------------------------------------------------------------------------
          op |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          od |   .6648524   .0872809     7.62   0.000     .4838429    .8458619
         soc |   .9483866    .283467     3.35   0.003      .360512    1.536261
       _cons |   .2653599   .0937114     2.83   0.010     .0710144    .4597054
------------------------------------------------------------------------------

. predict test, resid

. regress test od soc pro tuj

      Source |       SS       df       MS              Number of obs =      25
-------------+------------------------------           F(  4,    20) =    0.64
       Model |  .018129935     4  .004532484           Prob > F      =  0.6391
    Residual |  .141293467    20  .007064673           R-squared     =  0.1137
-------------+------------------------------           Adj R-squared = -0.0635
       Total |  .159423401    24  .006642642           Root MSE      =  .08405

------------------------------------------------------------------------------
        test |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          od |  -.2666934   .1938749    -1.38   0.184    -.6711094    .1377226
         soc |   .6802458   .5127575     1.33   0.200    -.3893476    1.749839
         pro |   7.506952   6.290407     1.19   0.247    -5.614608    20.62851
         tuj |   .4450372   .5246166     0.85   0.406    -.6492939    1.539368
       _cons |   .0981764   .1152333     0.85   0.404    -.1421961     .338549
------------------------------------------------------------------------------

. scalar lm=25*0.1137

. display "chi2(2) = " lm
chi2(2) = 2.8425

. display "Prob > chi2 = " chi2tail(2,lm)
Prob > chi2 = .24141206
```

```
. * Preverjanje smiselnosti izkljucevanja pojasnjevalnih spremenljivk (LR-test)

. regress op od soc pro tuj

      Source |       SS       df       MS              Number of obs =      25
-------------+------------------------------           F(  4,    20) =  130.82
       Model |  3.69668581     4  .924171451           Prob > F      =  0.0000
    Residual |  .141293465    20  .007064673           R-squared     =  0.9632
-------------+------------------------------           Adj R-squared =  0.9558
       Total |  3.83797927    24  .159915803           Root MSE      =  .08405

------------------------------------------------------------------------------
          op |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          od |   .3981591   .1938749     2.05   0.053    -.0062569     .802575
         soc |   1.628632   .5127575     3.18   0.005      .559039    2.698226
         pro |   7.506952   6.290407     1.19   0.247    -5.614608    20.62851
         tuj |   .4450372   .5246166     0.85   0.406    -.6492939    1.539368
       _cons |   .3635363   .1152333     3.15   0.005     .1231638    .6039088
------------------------------------------------------------------------------

. scalar llo = e(ll)

. display llo
29.223937

. display -25/2*(ln(2*_pi)+ln(0.141293465/25)+1)
29.223937

. regress op od soc

      Source |       SS       df       MS              Number of obs =      25
-------------+------------------------------           F(  2,    22) =  253.82
       Model |  3.67855587     2  1.83927794           Prob > F      =  0.0000
    Residual |    .1594234    22  .007246518           R-squared     =  0.9585
-------------+------------------------------           Adj R-squared =  0.9547
       Total |  3.83797927    24  .159915803           Root MSE      =  .08513

------------------------------------------------------------------------------
          op |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          od |   .6648524   .0872809     7.62   0.000     .4838429    .8458619
         soc |   .9483866    .283467     3.35   0.003      .360512    1.536261
       _cons |   .2653599   .0937114     2.83   0.010     .0710144    .4597054
------------------------------------------------------------------------------

. scalar llr = e(ll)

. display llr
27.714881

. display -25/2*(ln(2*_pi)+ln(0.1594234/25)+1)
27.714881

. display "chi2(2) = " 2*(llo-llr)
chi2(2) = 3.0181129

. display "Prob > chi2 = " chi2tail(2, 2*(llo-llr))
Prob > chi2 = .22111852
```

```
. qui regress op od soc pro tuj
. estimates store mo

. qui regress op od soc
. estimates store mr

. lrtest mr mo

Likelihood-ratio test                                 LR chi2(2)  =      3.02
(Assumption: mr nested in mo)                         Prob > chi2 =    0.2211
```

■

<u>Primer 5:</u> V vzorec smo izbrali 32 evropskih držav in za leto 2003 pridobili naslednje podatke (datoteka `zdravstvo.dta`):

- pričakovana življenjska doba (*PZD*; v letih);
- izdatki za zdravstvo na prebivalca (*IZDATKI*; v ameriških dolarjih);
- odstotek kadilcev med odraslimi prebivalci (*TOBAK*);
- poraba čistega alkohola na prebivalca (*ALKO*; v litrih – upoštevane žgane pijače).

Ocenite enostavni linearni regresijski model: $PZD_i = \beta_1 + \beta_2 IZDATKI_i + u_i$, nato pa ga razširite z vključitvijo preostalih dveh pojasnjevalnih spremenljivk. Z ustreznimi testi presodite, ali bi razširjenemu modelu dali prednost pred osnovnim.

Izpis rezultatov obdelav v programskem paketu Stata:

```
. regress pzd izdatki

      Source |       SS       df       MS              Number of obs =      32
-------------+------------------------------           F(  1,    30) =   45.53
       Model |  338.007285     1  338.007285           Prob > F      =  0.0000
    Residual |  222.717492    30  7.42391639           R-squared     =  0.6028
-------------+------------------------------           Adj R-squared =  0.5896
       Total |  560.724777    31   18.087896           Root MSE      =  2.7247

------------------------------------------------------------------------------
         pzd |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     izdatki |   .0026471   .0003923     6.75   0.000     .0018459    .0034483
       _cons |   71.99059   .7502451    95.96   0.000     70.45839     73.5228
------------------------------------------------------------------------------
```

```
. * Testiranje smiselnosti vkljucevanja pojasnjevalnih spremenljivk

. regress pzd izdatki tobak alko

      Source |       SS       df       MS              Number of obs =      32
-------------+------------------------------           F(  3,    28) =   26.30
       Model |  413.850212     3  137.950071           Prob > F      =  0.0000
    Residual |  146.874565    28  5.24552017           R-squared     =  0.7381
-------------+------------------------------           Adj R-squared =  0.7100
       Total |  560.724777    31   18.087896           Root MSE      =  2.2903

------------------------------------------------------------------------------
         pzd |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     izdatki |   .0018569   .0004023     4.62   0.000     .0010329    .0026809
       tobak |  -.2238391   .0837702    -2.67   0.012    -.3954346   -.0522436
        alko |  -.6493606   .2805689    -2.31   0.028     -1.22408   -.0746412
       _cons |   81.42053   2.720683    29.93   0.000     75.84746    86.99359
------------------------------------------------------------------------------

. display ((222.717492-146.874565)/2) / (146.874565/(32-4))
7.2293047

. display Ftail(2,28,7.2293047)
.00294233


. regress pzd izdatki tobak alko

      Source |       SS       df       MS              Number of obs =      32
-------------+------------------------------           F(  3,    28) =   26.30
       Model |  413.850212     3  137.950071           Prob > F      =  0.0000
    Residual |  146.874565    28  5.24552017           R-squared     =  0.7381
-------------+------------------------------           Adj R-squared =  0.7100
       Total |  560.724777    31   18.087896           Root MSE      =  2.2903

------------------------------------------------------------------------------
         pzd |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     izdatki |   .0018569   .0004023     4.62   0.000     .0010329    .0026809
       tobak |  -.2238391   .0837702    -2.67   0.012    -.3954346   -.0522436
        alko |  -.6493606   .2805689    -2.31   0.028     -1.22408   -.0746412
       _cons |   81.42053   2.720683    29.93   0.000     75.84746    86.99359
------------------------------------------------------------------------------

. test tobak=alko=0

 ( 1)  tobak - alko = 0
 ( 2)  tobak = 0

       F(  2,    28) =    7.23
            Prob > F =    0.0029


. regress pzd izdatki

      Source |       SS       df       MS              Number of obs =      32
-------------+------------------------------           F(  1,    30) =   45.53
       Model |  338.007285     1  338.007285           Prob > F      =  0.0000
    Residual |  222.717492    30  7.42391639           R-squared     =  0.6028
-------------+------------------------------           Adj R-squared =  0.5896
       Total |  560.724777    31   18.087896           Root MSE      =  2.7247
```

```
------------------------------------------------------------------------------
         pzd |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     izdatki |   .0026471   .0003923     6.75   0.000     .0018459    .0034483
       _cons |   71.99059   .7502451    95.96   0.000     70.45839     73.5228
------------------------------------------------------------------------------

. predict test, resid

. regress test izdatki tobak alko

      Source |       SS       df       MS              Number of obs =      32
-------------+------------------------------           F(  3,    28) =    4.82
       Model |  75.8429242     3  25.2809747           Prob > F      =  0.0079
    Residual |  146.874561    28  5.24552002           R-squared     =  0.3405
-------------+------------------------------           Adj R-squared =  0.2699
       Total |  222.717485    31  7.18443499           Root MSE      =  2.2903

------------------------------------------------------------------------------
        test |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     izdatki |  -.0007902   .0004023    -1.96   0.059    -.0016142    .0000338
       tobak |  -.2238391   .0837702    -2.67   0.012    -.3954346   -.0522436
        alko |  -.6493605   .2805689    -2.31   0.028     -1.22408   -.0746412
       _cons |   9.429936   2.720682     3.47   0.002      3.85687      15.003
------------------------------------------------------------------------------


. scalar lm=32*0.3405

. display "chi2(2) = " lm
chi2(2) = 10.896

. display "Prob > chi2 = " chi2tail(2,lm)
Prob > chi2 = .00430491


. regress pzd izdatki

      Source |       SS       df       MS              Number of obs =      32
-------------+------------------------------           F(  1,    30) =   45.53
       Model |  338.007285     1  338.007285           Prob > F      =  0.0000
    Residual |  222.717492    30  7.42391639           R-squared     =  0.6028
-------------+------------------------------           Adj R-squared =  0.5896
       Total |  560.724777    31   18.087896           Root MSE      =  2.7247

------------------------------------------------------------------------------
         pzd |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     izdatki |   .0026471   .0003923     6.75   0.000     .0018459    .0034483
       _cons |   71.99059   .7502451    95.96   0.000     70.45839     73.5228
------------------------------------------------------------------------------

. scalar llo = e(ll)

. display llo
-76.448724

. display -32/2*(ln(2*_pi)+ln(222.717492/32)+1)
-76.448724
```

```
. regress pzd izdatki tobak alko

      Source |       SS       df       MS              Number of obs =      32
-------------+------------------------------           F(  3,    28) =   26.30
       Model |  413.850212     3  137.950071           Prob > F      =  0.0000
    Residual |  146.874565    28  5.24552017           R-squared     =  0.7381
-------------+------------------------------           Adj R-squared =  0.7100
       Total |  560.724777    31   18.087896           Root MSE      =  2.2903

------------------------------------------------------------------------------
         pzd |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     izdatki |   .0018569   .0004023     4.62   0.000     .0010329    .0026809
       tobak |  -.2238391   .0837702    -2.67   0.012    -.3954346   -.0522436
        alko |  -.6493606   .2805689    -2.31   0.028     -1.22408   -.0746412
       _cons |   81.42053   2.720683    29.93   0.000     75.84746    86.99359
------------------------------------------------------------------------------

. scalar lln = e(ll)

. display lln
-69.787521

. display -32/2*(ln(2*_pi)+ln(146.874565/32)+1)
-69.787521

. display "chi2(2) = " 2*(lln-llo)
chi2(2) = 13.322406

. display "Prob > chi2 = " chi2tail(2, 2*(lln-llo))
Prob > chi2 = .00127961


. qui regress pzd izdatki
. estimates store mo

. qui regress pzd izdatki tobak alko
. estimates store mn

. lrtest mo mn

Likelihood-ratio test                                  LR chi2(2)  =     13.32
(Assumption: mo nested in mn)                          Prob > chi2 =    0.0013
```

■

Primer 6: Države iz prejšnjega primera smo razdelili v dve skupini, in sicer glede na to, ali je država članica EU15 (pri teh državah ima spremenljivka *DEU* vrednost 1) ali ne (spremenljivka *DEU* ima vrednost 0). Za vsako skupino posebej ocenite regresijski model:

$$PZD_i = \beta_1 + \beta_2 IZDATKI_i + \beta_3 ALKO_i + \beta_4 TOBAK_i + u_i$$

Z ustreznim testom presodite, ali se proučevana regresijska funkcija razlikuje med omenjenima skupinama držav.

Izpis rezultatov obdelav v programskem paketu Stata:

. regress pzd izdatki alko tobak

```
      Source |       SS       df       MS              Number of obs =      32
-------------+------------------------------           F(  3,    28) =   26.30
       Model |  413.850212     3  137.950071           Prob > F      =  0.0000
    Residual |  146.874565    28  5.24552017           R-squared     =  0.7381
-------------+------------------------------           Adj R-squared =  0.7100
       Total |  560.724777    31   18.087896           Root MSE      =  2.2903

------------------------------------------------------------------------------
         pzd |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     izdatki |   .0018569   .0004023     4.62   0.000     .0010329    .0026809
        alko |  -.6493606   .2805689    -2.31   0.028     -1.22408   -.0746412
       tobak |  -.2238391   .0837702    -2.67   0.012    -.3954346   -.0522436
       _cons |   81.42053   2.720683    29.93   0.000     75.84746    86.99359
------------------------------------------------------------------------------
```

. regress pzd izdatki alko tobak if deu==0

```
      Source |       SS       df       MS              Number of obs =      17
-------------+------------------------------           F(  3,    13) =   19.58
       Model |  261.356136     3   87.118712           Prob > F      =  0.0000
    Residual |  57.8427484    13  4.44944219           R-squared     =  0.8188
-------------+------------------------------           Adj R-squared =  0.7770
       Total |  319.198884    16  19.9499303           Root MSE      =  2.1094

------------------------------------------------------------------------------
         pzd |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     izdatki |   .0015388   .0004653     3.31   0.006     .0005335    .0025441
        alko |  -.5335537   .2839051    -1.88   0.083    -1.146893     .079786
       tobak |  -.4019988   .1124047    -3.58   0.003    -.6448345   -.1591631
       _cons |   85.90512   3.793205    22.65   0.000      77.7104    94.09984
------------------------------------------------------------------------------
```

. regress pzd izdatki alko tobak if deu==1

```
      Source |       SS       df       MS              Number of obs =      15
-------------+------------------------------           F(  3,    11) =    0.07
       Model |  .252059401     3    .0840198           Prob > F      =  0.9751
    Residual |  13.3052664    11  1.20956967           R-squared     =  0.0186
-------------+------------------------------           Adj R-squared = -0.2491
       Total |  13.5573258    14  .968380413           Root MSE      =  1.0998

------------------------------------------------------------------------------
         pzd |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     izdatki |   .0000786   .0004425     0.18   0.862    -.0008953    .0010526
        alko |   .1374531   .5219541     0.26   0.797     -1.01136    1.286266
       tobak |   .0109881   .0631892     0.17   0.865    -.1280903    .1500665
       _cons |   77.97065   1.777303    43.87   0.000     74.05883    81.88247
------------------------------------------------------------------------------
```

. display Ftail(4,24,6.3861135)

```
.0011999
```

■