Časovna vrednost denarja

$$N = P \cdot A(0,T)$$
$$P = N \cdot D(0,T)$$

Navadno obrestovanje

$$A(0,T) = 1 + L \cdot T; \quad T \in \mathbb{R}_0^+$$
$$D(0,T) = (1 + L \cdot T)^{-1}; \quad T \in \mathbb{R}_0^+$$

Diskretno obrestno obrestovanje (k-krat na leto)

$$A_k(0, \tfrac{h}{k}) = (1 + \tfrac{R_N}{k})^h; \quad h \in \mathbb{N}_0$$
$$D_k(0, \tfrac{h}{k}) = (1 + \tfrac{R_N}{k})^{-h}; \quad h \in \mathbb{N}_0$$
$$(1 + R_E) = (1 + \tfrac{R_N}{k})^k$$

Zvezno obrestno obrestovanje

$$A(0,T) = e^{Y \cdot T} = (1 + R_E)^T; \quad T \in \mathbb{R}_0^+$$
$$D(0,T) = e^{-Y \cdot T} = (1 + R_E)^{-T}; \quad T \in \mathbb{R}_0^+$$
$$(1 + R_E) = e^Y$$

Krediti

$$a = G \cdot \tfrac{r(1+r)^n}{(1+r)^n - 1}$$
$$R_j = G \cdot \tfrac{(1+r)^n - (1+r)^j}{(1+r)^n - 1}$$
$$R_{j+1} = R_j(1+r) - a$$

Terminske obrestne mere

$$A(t,U) = A(t,T) \cdot A(t,T,U)$$
$$L(t,T,U) = \tfrac{1}{U-T} \cdot \left(\tfrac{1+(U-t)L(t,U)}{1+(T-t)L(t,T)} - 1 \right)$$
$$Y(t,T,U) = \tfrac{1}{U-T} \cdot \big((U-t)Y(t,U) - (T-t)Y(t,T) \big)$$

Obveznice

Brezkuponska obveznica

$$P_t^{\mathrm{ZCB}} = N \cdot D(t,T)$$

Kuponska obveznica

$$t_i - t_{i-1} = \Delta$$
$$P_t^{\mathrm{CB}} = \textstyle\sum_{t_i > t} C_i \cdot D(t,t_i) + N \cdot D(t,t_n)$$
$$D_t = \tfrac{1}{P_t^{\mathrm{CB}}} \big(\textstyle\sum_{t_i > t} C_i \cdot t_i \cdot D(t,t_i) + N \cdot t_n \cdot D(t,t_n) \big)$$

Obveznica s spremenljivimi kuponi

$$t_i - t_{i-1} = \Delta$$
$$F_i = N \Delta L(t_{i-1}, t_i)$$
$$P_{t_i}^{\mathrm{FL}} = N$$
$$P_t^{\mathrm{FL}} = \tfrac{1}{1+(t_i - t)L(t,t_i)} \cdot (N + F_i) \text{ za } t \in (t_{i-1}, t_i)$$

Terminski posli

$$K = (S_0 - I(0,T)) \cdot A(0,T)$$
$$F_t = (S_t - I(t,T)) \cdot A(t,T)$$
$$V_T^{\mathrm{FW}} = S_T - K$$
$$V_t^{\mathrm{FW}} = (F_t - K) \cdot D(t,T)$$

Valutni terminski posli (na enoto tuje valute)

$$1f = S_t \; d$$
$$K = S_0 \cdot \tfrac{D^f(0,T)}{D^d(0,T)} = S_0 \cdot D^f(0,T) \cdot A^d(0,T)$$
$$V_T^{\mathrm{FW}} = S_T - K$$
$$V_t^{\mathrm{FW}} = S_t \cdot D^f(t,T) - K \cdot D^d(t,T)$$

Dogovor o terminski obrestni meri

$$L_{\mathrm{FRA}} = L(0,T,U)$$
$$V_T^{\mathrm{FRA}} = N(U-T)(L(T,U) - L_{\mathrm{FRA}})D(T,U)$$
$$V_t^{\mathrm{FRA}} = N(U-T)(L(t,T,U) - L_{\mathrm{FRA}})D(t,U)$$

Zamenjave obrestnih mer

$$t_i - t_{i-1} = \Delta$$
$$C_i = N \Delta L_{\mathrm{SWAP}}$$
$$F_i = N \Delta L(t_{i-1}, t_i)$$

Zamenjava kot portfelj FRA-jev

$$L_{\mathrm{SWAP}} = \frac{\sum_{i=1}^n L(0, t_{i-1}, t_i) D(0,t_i)}{\sum_{i=1}^n D(0,t_i)}$$
$$V_t^{\mathrm{SWAP}} = N \Delta \textstyle\sum_{t_i > t} (L(t, t_{i-1}, t_i) - L_{\mathrm{SWAP}}) D(t,t_i)$$

Zamenjava kot portfelj obveznic

$$L_{\mathrm{SWAP}} = \frac{1 - D(0,t_n)}{\Delta \sum_{i=1}^n D(0,t_i)}$$
$$V_t^{\mathrm{SWAP}} = P_t^{\mathrm{FL}} - P_t^{\mathrm{CB}}$$

Opcije

Evropske opcije

$$c_T^{\mathrm{E}} = \max\{S_T - K, 0\}$$
$$p_T^{\mathrm{E}} = \max\{K - S_T, 0\}$$
$$\max\{S_t - KD(t,T) - I(t,T), 0\} \leq c_t^{\mathrm{E}} \leq S_t - I(t,T)$$
$$\max\{KD(t,T) + I(t,T) - S_t, 0\} \leq p_t^{\mathrm{E}} \leq KD(t,T)$$

Pariteta evropske nakupne in prodajne opcije

$$p_t^{\mathrm{E}} + S_t - I(t,T) = c_t^{\mathrm{E}} + KD(t,T)$$

Ameriške opcije

$$c_T^{\mathrm{A}} = \max\{S_T - K, 0\}$$
$$p_T^{\mathrm{A}} = \max\{K - S_T, 0\}$$
$$c_t^{\mathrm{E}} \leq c_t^{\mathrm{A}} \quad \text{in} \quad p_t^{\mathrm{E}} \leq p_t^{\mathrm{A}}$$

Če je $I(t,T) = 0$, velja $c_t^{\mathrm{E}} = c_t^{\mathrm{A}}$.

$$\max\{S_t - KD(t,T) - I(t,T), S_t - K, 0\} \leq c_t^{\mathrm{A}} \leq S_t$$
$$\max\{KD(t,T) + I(t,T) - S_t, K - S_t, 0\} \leq p_t^{\mathrm{A}} \leq K$$

Neenakost za ameriško nakupno in prodajno opcijo

$$c_t^{\mathrm{A}} + KD(t,T) \leq p_t^{\mathrm{A}} + S_t \leq c_t^{\mathrm{A}} + K + I(t,T)$$

Diskretni model finančnega trga

Časovna množica $\mathcal{T} = \{0, 1, \ldots, T\}$
Vrednostni papirji (instrumenti) $S^1, S^2, \ldots, S^N$
S^i_t cena/vrednost papirja S^i v trenutku t
$\Omega = \{\omega_1, \omega_2, \ldots, \omega_K\}$ množica stanj v času T

Enoobdobni model $\mathcal{T} = \{0, 1\}$

Vektor cen $c \in \mathbb{R}^N$

$$c = \begin{bmatrix} S^1_0 \\ \vdots \\ S^N_0 \end{bmatrix}$$

Matrika izplačil $M \in \mathbb{R}^{K \times N}$

$$M = \begin{bmatrix} S^1_1(\omega_1) & S^2_1(\omega_1) & \cdots & S^N_1(\omega_1) \\ \vdots & \vdots & & \vdots \\ S^1_1(\omega_K) & S^2_1(\omega_K) & \cdots & S^N_1(\omega_K) \end{bmatrix}$$

S^1 netvegan, če $\forall j : S^1_1(\omega_j) = S^1_0(1+R)$
R netvegana obdobna obrestna mera
Portfelj vrednostnih papirjev $\theta \in \mathbb{R}^N$

$$\theta = \begin{bmatrix} \theta_1 \\ \vdots \\ \theta_N \end{bmatrix}$$

Pogojna terjatev $X : \Omega \to \mathbb{R}$

$$X = \begin{bmatrix} X(\omega_1) \\ \vdots \\ X(\omega_K) \end{bmatrix}$$

X nenegativna, če $\forall i : X(\omega_i) \geq 0$
X pozitivna, če $(\forall i : X(\omega_i) \geq 0) \wedge (\exists j : X(\omega_j) > 0)$
X strogo pozitivna, če $\forall i : X(\omega_i) > 0$
$V_0(\theta) \in \mathbb{R}$ vrednost/cena portfelja θ v času 0

$$V_0(\theta) = \langle \theta, c \rangle$$

$V_1(\theta) : \Omega \to \mathbb{R}$ vrednost/izplačila portfelja θ v času 1

$$V_1(\theta) = M \cdot \theta$$

θ arbitražni, če $V_0(\theta) \leq 0$ in $V_1(\theta)$ pozitivna
θ arbitražni, če $V_0(\theta) < 0$ in $V_1(\theta)$ nenegativna
Trg brez arbitraže $\Leftrightarrow \big(V_1(\theta)$ pozitivna $\Rightarrow V_0(\theta) > 0\big)$

θ izvedbeni portfelj terjatve X, če $V_1(\theta) = X$
Terjatev dosegljiva, če obstaja njen izvedbeni portfelj
$\mathcal{M} \leq \mathbb{R}^K$ množica dosegljivih pogojnih terjatev
Trg poln $\Leftrightarrow \operatorname{rang}(M) = K$

Zakon ene cene: $V_1(\theta) = V_1(\phi) \Rightarrow V_0(\theta) = V_0(\phi)$
Trg brez arbitraže $\Rightarrow$ zakon ene cene

Cenovni funkcional $\pi_0 : \mathcal{M} \to \mathbb{R}$

$$\pi_0(X) = V_0(\theta), \text{ kjer } V_1(\theta) = X$$

π_0 krepko pozitiven, če $\forall X : \big(X$ pozitivna $\Rightarrow \pi_0(X) > 0\big)$
Trg brez arbitraže $\Leftrightarrow \pi_0$ krepko pozitiven

$\widehat{\pi}_0 : \mathbb{R}^K \to \mathbb{R}$ krepko pozitivne razširitve π_0

$$\widehat{\pi}_0(X) = \langle X, \psi \rangle$$
$$M^T \cdot \psi = c \text{ in } \psi \text{ strogo pozitiven}$$

$\psi \in \mathbb{R}^K$ *vektor cen stanj* (Rieszov izrek)

Prvi izrek FM: Trg brez arbitraže $\Leftrightarrow \exists\ \widehat{\pi}_0$
Drugi izrek FM: Trg brez arbitraže poln $\Leftrightarrow \exists!\ \widehat{\pi}_0$
X dosegljiva $\Leftrightarrow \mathrm{m}\big(\widehat{\pi}_0(X)\big) = 1$.
θ^* najcenejši superzaščitni portfelj terjatve $X \notin \mathcal{M}$

$$V_0(\theta^*) = \sup\big(\widehat{\pi}_0(X)\big)$$

Ekvivalentna martingalska verjetnost

$p_i = P(\omega_i)$ naravna verjetnost stanja ω_i
$q_i = Q(\omega_i)$ ekvivalentna martingalska verjetnost ω_i
S^1 *numerar*
Če S^1 netvegan, Q do tveganja nevtralna verjetnost

Vektor diskontiranih cen $\widetilde{c} = \frac{1}{S^1_0} c \in \mathbb{R}^N$
Matrika diskontiranih izplačil $\widetilde{M} \in \mathbb{R}^{K \times N}$

$$\widetilde{M} = \begin{bmatrix} 1 & \frac{S^2_1(\omega_1)}{S^1_1(\omega_1)} & \cdots & \frac{S^N_1(\omega_1)}{S^1_1(\omega_1)} \\ \vdots & \vdots & & \vdots \\ 1 & \frac{S^2_1(\omega_K)}{S^1_1(\omega_K)} & \cdots & \frac{S^N_1(\omega_K)}{S^1_1(\omega_K)} \end{bmatrix}$$

Določanje ekvivalentne matringalske verjetnosti

$$E_Q(\widetilde{S}^i_1) = \widetilde{S}^i_0 \text{ in } q_j > 0 \quad \forall i, j$$

Q ekvivalentna matringalska $\Rightarrow \forall \theta : E_Q(\widetilde{V}_1(\theta)) = \widetilde{V}_0(\theta)$
Diskontirana pogojna terjatev $\widetilde{X} : \Omega \to \mathbb{R}$

$$\widetilde{X} = \begin{bmatrix} \frac{X(\omega_1)}{S^1_1(\omega_1)} \\ \vdots \\ \frac{X(\omega_K)}{S^1_1(\omega_K)} \end{bmatrix}$$
$$\pi^Q_0(X) = S^1_0 \cdot E_Q(\widetilde{X}) = \widehat{\pi}_0(X)$$

Prvi izrek FM: Trg brez arbitraže $\Leftrightarrow \exists\ Q$
Drugi izrek FM: Trg brez arbitraže poln $\Leftrightarrow \exists!\ Q$

$r_\theta : \Omega \to \mathbb{R}$ *enostavni donos* portfelja θ

$$r_\theta = \frac{V_1(\theta) - V_0(\theta)}{V_0(\theta)}$$

S^1 netvegan $\Rightarrow \pi^Q_0(X) = \frac{1}{1+R} E_Q(X)$ in $\forall \theta : E_Q(r_\theta) = R$

Enoobdobni binomski model

$$c = \begin{bmatrix} B_0 \\ S_0 \end{bmatrix} \qquad M = \begin{bmatrix} B_0(1+R) & S_0 u \\ B_0(1+R) & S_0 d \end{bmatrix}$$

$(-1 < R) \wedge (0 \le d < u) \Rightarrow$ trg poln, zakon ene cene

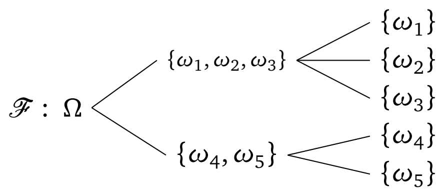
$$\psi_1 = \frac{1+R-d}{(u-d)(1+R)} \qquad q_1 = \frac{1+R-d}{u-d}$$

$$\psi_2 = \frac{u-(1+R)}{(u-d)(1+R)} \qquad q_2 = \frac{u-(1+R)}{u-d}$$

Trg brez arbitraže $\Leftrightarrow d < 1+R < u$

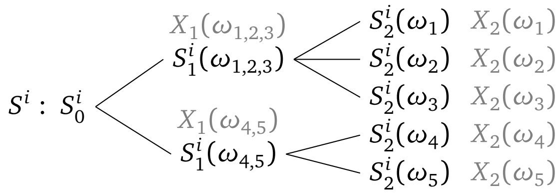
Večobdobni model $\mathscr{T} = \{0, 1, \ldots, T\}$

Informacijska struktura



Prilagojen *cenovni proces* $S^i_t : \Omega \to \mathbb{R}$ instrumenta S^i

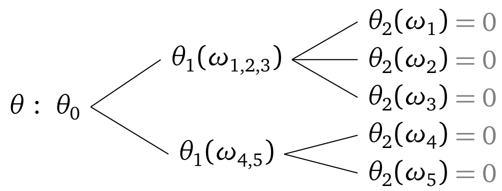
$X_1, X_2 : \Omega \to \mathbb{R}$ pogojni terjatvi z zapadlostima 1, 2



Trgovalna strategija θ

$\theta_t : \Omega \to \mathbb{R}^N$ portfelj, ustvarjen v trenutku t (!)

$\theta^i_t : \Omega \to \mathbb{R}$ količina instrumenta S^i v θ_t



Nabavna vrednost $V^A_t(\theta) = \sum_{i=1}^N \theta^i_t S^i_t$

Prodajna vrednost, $t > 0$, $V^L_t(\theta) = \sum_{i=1}^N \theta^i_{t-1} S^i_t$

θ str. samofinanciranja, če $\forall t : V^A_t(\theta) = V^L_t(\theta) \overset{\text{def}}{=} V_t(\theta)$

θ str. samofin. $\Rightarrow \big(V_T(\theta) = 0 \Rightarrow \forall t : V_t(\theta) = 0\big)$

θ arbitražna, če θ str. samofin., $V_0(\theta) = 0$, $V_T(\theta) \neq 0$ in $V_t(\theta)$ nenegativna za $1 \le t \le T$

θ arbitražna, če θ str. samofin., $V_0(\theta) < 0$ in $V_t(\theta)$ nenegativna za $1 \le t \le T$

θ izvedbena str. X_T, če θ str. samofin. in $V_T(\theta) = X_T$

Trjatev dosegljiva, če obstaja njena izvedbena strategija

Zakon ene cene: $V_T(\theta) = V_T(\phi) \Rightarrow V_0(\theta) = V_0(\phi)$

Zakon ene cene $\Leftrightarrow \big(X_T = 0 \Rightarrow \forall \theta$ izv. str.: $V_0(\theta) = 0\big)$

Trg brez arbitraže $\Rightarrow$ zakon ene cene

Cenovni funkcional $\pi_0(X_T) = V_0(\theta)$, kjer θ izv. str. X_T

Martingali

X_t martingal, če $E(X_t | \mathscr{F}_s) = X_s$ za $0 \le s \le t \le T$

X_t martingal $\Leftrightarrow E(X_{t+1} | \mathscr{F}_t) = X_t$ za $0 \le t < T$

X_t martingal $\Rightarrow E(X_t) = X_0$

$(X_t, Y_t$ martingala, $X_T = Y_T) \Rightarrow X_t = Y_t$ za $0 \le t < T$

X_t nad/pod martingal, če $E(X_t | \mathscr{F}_s) \le X_s / E(X_t | \mathscr{F}_s) \ge X_s$

Ekvivalentna martingalska verjetnost

S^1 numerar; $S^1_t > 0$ za vse t

Če S^1 bančni račun, Q do tveganja nevtralna verjetnost

Če S^1 ZCB, Q do prihodnosti nevtralna verjetnost

$\widetilde{S}^i_t = \frac{S^i_t}{S^1_t}$ *diskontirani cenovni proces S^i*

$\widetilde{V}_t(\theta) = \frac{V_t(\theta)}{S^1_t}$ *diskontirani vrednostni proces θ*

Določanje ekvivalentne martingalske verjetnosti

$$E_Q(\widetilde{S}^i_{t+1} | \mathscr{F}_t) = \widetilde{S}^i_t \text{ in } q_j > 0 \quad \forall i, j$$

Naj bo θ strategija samofinanciranja:

Q e.m.v. $\Leftrightarrow \forall j : q_j > 0$ in $E_Q(\widetilde{V}_{t+1}(\theta) | \mathscr{F}_t) = \widetilde{V}_t(\theta)$

Q e.m.v. $\Leftrightarrow \forall j : q_j > 0$ in $E_Q(\widetilde{V}_T(\theta)) = \widetilde{V}_0(\theta)$

$\widetilde{X}_T = \frac{X_T}{S^1_T}$ diskontirana pogojna terjatev z zapadlostjo T

$$\pi^Q_0(X_T) = S^1_0 E_Q(\widetilde{X}_T)$$

Prvi izrek FM: Trg brez arbitraže $\Leftrightarrow \exists\, Q$

Drugi izrek FM: Trg brez arbitraže poln $\Leftrightarrow \exists!\, Q$

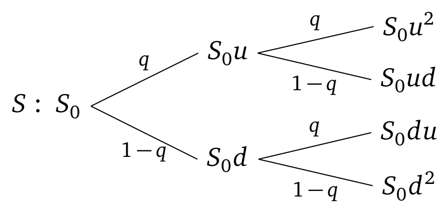
X_T dosegljiva $\Leftrightarrow m\big(\pi^Q_0(X_T)\big) = 1$

Y^*_T najcenejša dosegljiva dominantna za $X_T \notin \mathscr{M}$

$$\pi^Q_0(Y^*_T) = \sup\big(\pi^Q_0(X_T)\big)$$

Večobdobni binomski model

$B :\ 1$ —— $1+R$ —— $(1+R)^2$



Trg brez arbitraže $\Leftrightarrow d < 1+R < u$

$$q = \frac{1+R-d}{u-d}$$

$$B_t = (1+R)^t$$

$$S_t \overset{Q}{\sim} \begin{pmatrix} S_0 d^t & \cdots & S_0 u^i d^{t-i} & \cdots & S_0 u^t \\ (1-q)^t & \cdots & \binom{t}{i} q^i (1-q)^{t-i} & \cdots & q^t \end{pmatrix}$$

X_T pogojna terjatev z zapadlostjo T, $X_T = f(S_T)$

$$\pi^Q_t(X_T) = \frac{1}{(1+R)^{T-t}} E_Q(X_T | \mathscr{F}_t)$$

$$\pi^Q_0(X_T) = \frac{1}{(1+R)^T} E_Q(X_T)$$

$$\pi^Q_0(X_T) = \frac{1}{(1+R)^T} \sum_{i=0}^T \binom{T}{i} q^i (1-q)^{T-i} f(S_0 u^i d^{T-i})$$

Evropska nakupna in prodajna opcija

$$m = \left\lceil \frac{\ln K - \ln(S_0 d^T)}{\ln u - \ln d} \right\rceil \qquad n = \left\lfloor \frac{\ln K - \ln(S_0 d^T)}{\ln u - \ln d} \right\rfloor$$
$$c_T^{\mathrm{E}} = \max\{S_T - K, 0\}$$
$$c_0^{\mathrm{E}} = \frac{1}{(1+R)^T} \sum_{i=m}^{T} \binom{T}{i} q^i (1-q)^{T-i} (S_0 u^i d^{T-i} - K)$$
$$p_T^{\mathrm{E}} = \max\{K - S_T, 0\}$$
$$p_0^{\mathrm{E}} = \frac{1}{(1+R)^T} \sum_{i=0}^{n} \binom{T}{i} q^i (1-q)^{T-i} (K - S_0 u^i d^{T-i})$$

Digitalna nakupna in prodajna opcija

$$m = \left\lfloor \frac{\ln K - \ln(S_0 d^T)}{\ln u - \ln d} \right\rfloor + 1 \qquad n = \left\lceil \frac{\ln K - \ln(S_0 d^T)}{\ln u - \ln d} \right\rceil - 1$$
$$c_T^{\mathrm{D}} = 1_{\{S_T > K\}} \qquad c_0^{\mathrm{D}} = \frac{1}{(1+R)^T} \sum_{i=m}^{T} \binom{T}{i} q^i (1-q)^{T-i}$$
$$p_T^{\mathrm{D}} = 1_{\{S_T < K\}} \qquad p_0^{\mathrm{D}} = \frac{1}{(1+R)^T} \sum_{i=0}^{n} \binom{T}{i} q^i (1-q)^{T-i}$$

Konstrukcija izvedbene strategije

- čas $t = T-1$, stanje ω, portfelj $\theta_\omega = (\alpha_\omega, \beta_\omega)^T$

$$\alpha_\omega B_{\omega u} + \beta_\omega S_{\omega u} = X_{\omega u}$$
$$\alpha_\omega B_{\omega d} + \beta_\omega S_{\omega d} = X_{\omega d}$$
$$V_t(\theta_\omega) = \alpha_\omega B_\omega + \beta_\omega S_\omega$$

- čas $t = T-2, T-3, \ldots, 0$

$$\alpha_\omega B_{\omega u} + \beta_\omega S_{\omega u} = V_{t+1}(\theta_{\omega u}) + X_{\omega u}$$
$$\alpha_\omega B_{\omega d} + \beta_\omega S_{\omega d} = V_{t+1}(\theta_{\omega d}) + X_{\omega d}$$
$$V_t(\theta_\omega) = \alpha_\omega B_\omega + \beta_\omega S_\omega$$

Black-Scholesova formula

$$\ln \frac{S_T}{S_0} \overset{Q}{\sim} \mathcal{N}\Big((r - \tfrac{\sigma^2}{2})T, \sigma^2 T\Big)$$
$$d_1 = \frac{\ln \frac{S_0}{K} + Y \cdot T + \frac{\sigma^2}{2} T}{\sigma \sqrt{T}} \qquad d_2 = d_1 - \sigma\sqrt{T}$$
$$c_0^{\mathrm{E}} = S_0 \Phi(d_1) - K e^{-Y \cdot T} \Phi(d_2)$$
$$p_0^{\mathrm{E}} = K e^{-Y \cdot T} \Phi(-d_2) - S_0 \Phi(-d_1)$$

Konvergenca binomskega modela z n obdobji

$$u = e^{\sigma \sqrt{T/n}} \qquad p = \frac{1}{2} + \frac{\mu}{2\sigma}\sqrt{\frac{T}{n}}$$
$$d = e^{-\sigma \sqrt{T/n}} \qquad \mu = \frac{1}{T} E_P(\ln \frac{S_n}{S_0})$$
$$R = e^{Y \cdot T/n} - 1 \qquad \sigma^2 = \frac{1}{T} \mathrm{var}_P(\ln \frac{S_n}{S_0})$$

Časi ustavljanja in optimalno ustavljanje

$\tau : \Omega \to \mathcal{T} = \{0, 1, \ldots, T\}$ *slučajni čas*

$\tau : \Omega \to \mathcal{T}$ *čas ustavljanja*, če $\forall t : \{\tau = t\} \in \mathcal{F}_t$

$\tau : \Omega \to \mathcal{T}$ čas ustavljanja $\Leftrightarrow \forall t : \{\tau \leq t\} \in \mathcal{F}_t$

$\mathcal{S}_{s,t} = \{\tau : \tau$ čas ustavljanja in $s \leq \tau \leq t\}$

X_t slučajni proces, X_t^τ *ustavljeni proces*

$$X_t^\tau = X_t \cdot 1_{\{\tau \geq t\}} + \sum_{s=0}^{t-1} X_s \cdot 1_{\{\tau = s\}} = \begin{cases} X_t; & t \leq \tau \\ X_\tau; & t > \tau \end{cases}$$

$X_\tau = X_T^\tau$ *končna vrednost* procesa X_t

X_t martingal $\Rightarrow X_t^\tau$ martingal in $E(X_\tau) = X_0$

X_t nadmartingal $\Rightarrow X_t^\tau$ nadmartingal in $E(X_\tau) \leq X_0$

X_t podmartingal $\Rightarrow X_t^\tau$ podmartingal in $E(X_\tau) \geq X_0$

τ^* *optimalni čas ustavljanja*, če $E(X_{\tau^*}) = \max_{\tau \in \mathcal{S}_{0,T}} E(X_\tau)$

τ^* optimalni v času t, če $E(X_{\tau^*} | \mathcal{F}_t) = \max_{\tau \in \mathcal{S}_{t,T}} E(X_\tau | \mathcal{F}_t)$

U_t *Snellova ovojnica* procesa X_t

$$U_T = X_T$$
$$U_t \overset{\text{def}}{=} \max_{\tau \in \mathcal{S}_{t,T}} E(X_\tau | \mathcal{F}_t) = \max\{X_t, E(U_{t+1} | \mathcal{F}_t)\}$$

$\tau^* \in \mathcal{S}_{0,T}$ optimalen $\Leftrightarrow U_t^{\tau^*}$ martingal in $U_{\tau^*} = X_{\tau^*}$

$\tau^* \in \mathcal{S}_{0,T}$ optimalen $\Rightarrow E(X_{\tau^*}) = U_0$

$\tau^*_{\min} \in \mathcal{S}_{0,T}$ najmanjši optimalni čas ustavljanja

$$\tau^*_{\min} = \min\{t : X_t = U_t\}$$

$U_t = M_t + A_t$ Doobov razcep

$$M_0 = U_0 \qquad M_{t+1} = M_t + U_{t+1} - E_Q(U_{t+1} | \mathcal{F}_t)$$
$$A_0 = 0 \qquad A_{t+1} = A_t - U_t + E_Q(U_{t+1} | \mathcal{F}_t)$$

$\tau^*_{\max} \in \mathcal{S}_{0,T}$ največji optimalni čas ustavljanja

$$\tau^*_{\max} = \begin{cases} \min\{t : A_{t+1} < 0\}; & \exists t : A_{t+1} < 0 \\ T; & \text{sicer} \end{cases}$$

Ameriške pogojne terjatve

τ strategija izvršitve = čas ustavljanja

Z_t *notranja vrednost* pogojne terjatve (če izvršimo)

U_t *vrednostni proces* pogojne terjatve

$$U_T = Z_T$$
$$U_t = \max\{Z_t, E_Q(U_{t+1} \cdot \frac{S_t^1}{S_{t+1}^1} | \mathcal{F}_t)\}$$

$\widetilde{Z}_t = \frac{Z_t}{S_t^1}$ in $\widetilde{U}_t = \frac{U_t}{S_t^1}$ diskontirana procesa

$$\widetilde{U}_T = \widetilde{Z}_T$$
$$\widetilde{U}_t = \max\{\widetilde{Z}_t, E_Q(\widetilde{U}_{t+1} | \mathcal{F}_t)\}$$

$\widetilde{U}_t$ je Snellova ovojnica $\widetilde{Z}_t$

τ^* optimalna strategija = optimalni čas ustavljanja

$$\pi_t^Q(X) = U_t = S_t^1 E_Q(\widetilde{Z}_{\tau^*} | \mathcal{F}_t)$$
$$\pi_0^Q(X) = U_0 = S_0^1 E_Q(\widetilde{Z}_{\tau^*})$$
$$\tau^*_{\min} = \min\{t : U_t = Z_t\}$$

Vrednotenje am. opcije $Z_t = f(S_t)$ v binomskem modelu

$$U_T = f(S_T)$$
$$U_t = \max\{f(S_t), \frac{1}{1+R} E_Q(U_{t+1} | \mathcal{F}_t)\}$$

f zvezna $\Rightarrow U_t$ zvezna

f padajoča/konveksna $\Rightarrow U_t$ padajoča/konveksna