Financial Mathematics 2 2011/2012
21 June 2012

1. Explain how Black and Scholes obtain the PDE

$$\begin{cases} \frac{\partial C}{\partial t} + \frac{\sigma^2}{2}x^2\frac{\partial^2 C}{\partial x^2} + rx\frac{\partial C}{\partial x} + -rC = 0 \;\text{ in } [0,T[\times[0,+\infty) \\ C(T,x) = (x-K)_+, \quad x \in [0,+\infty) \end{cases}$$

2. Consider the trinomial tree of Kamrad-Ritcken.

   (a) Compute the probabilitites p_u, p_m, p_d in order to satisfy the local consistency conditions.

   (b) Write a Matlab or Scilab code to compute European Put price in the Black Scholes model.

3. Let $Z_t = e^{-\lambda B_t}$ where $(B_t)_{t\geq 0}$ is a standard Brownian motion. Write the expression of dZ_t.

4. ParAsian options are barrier options which can be knocked in or out depending on time that the underlying asset has spent cumulatively over a barrier. In the ParAsian knock-out case the barrier option vanishes if the price of the underlying asset remain in a cumulative manner for a period longer than W over the barrier.
   Write a Matlab or Scilab code to price ParAsian option down-and-out in the Black Scholes model.
   The options parameters are $K, S_0, T, down, W$.

5. Consider the PDE

$$\begin{cases} \frac{\partial u}{\partial t}(t,x) = \frac{\sigma^2}{2}\frac{\partial^2 u}{\partial x^2}(t,x) + (r - \frac{\sigma^2}{2})\frac{\partial u}{\partial x}(t,x) - ru(t,x) \text{ in } (0,T]\times\mathbb{R}, \\ u(0,x) = \psi(x), \forall x \in \mathbb{R}, \end{cases}$$

   (a) Write the explicit finite difference scheme and the stability condition using the approximation

$$u'(x) = \frac{u(x+h) - u(x-h)}{2h} + O(h^2)$$

   (b) Write the fully implicit finite difference scheme ($\theta = 1$) in a matrix form.