Vaje iz fizikalnih merjenj 2

Klemen Bučar, F2 IJS

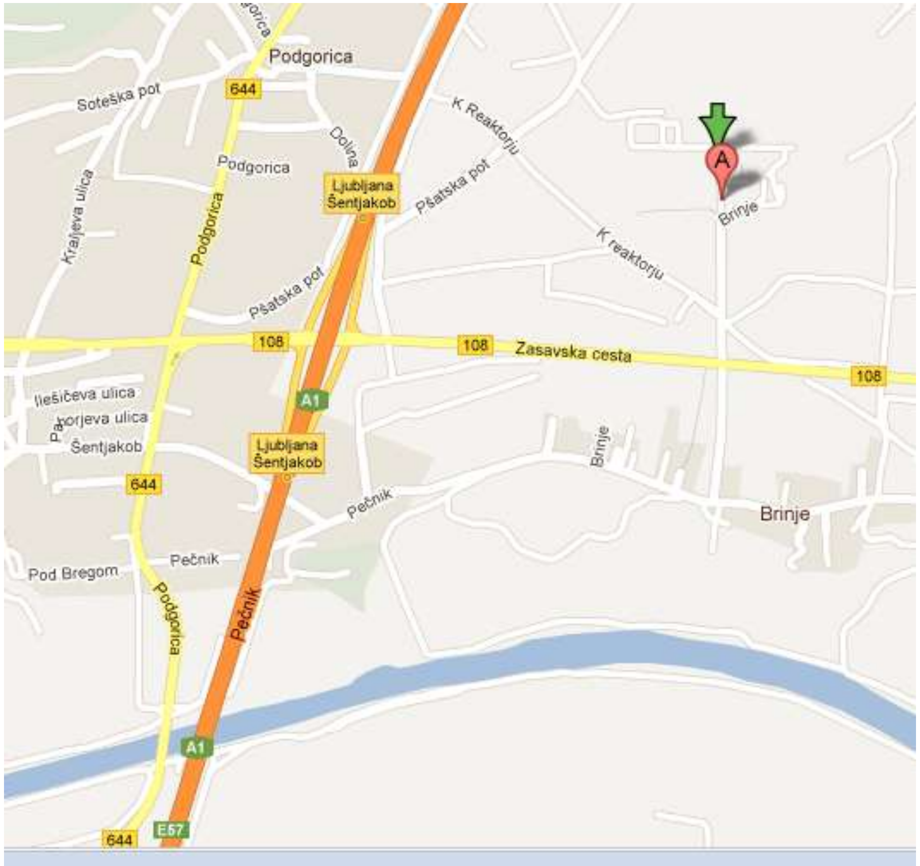
Kontakt

klemen.bucar@ijs.si

Telefon: 01-5885275

Pisarna: RCP, Brinje,

Pospeševalnik



O predmetu

- 2 kolokvija + pisni izpit iz vaj

Literatura:

1. Leo, *Techniques for Nuclear and Particle Physics Experiments*, Springer
2. Cvelbar*, Merjenje ionizirajočega sevanja*
3. Wilson, *An Introduction to Particle Accelerators*, Oxford
4. Lee, *Accelerator Physics*

(5. Ž. Šmit, *Spektroskopske metode s pospešenimi ioni)*

Energije reakcij

- Atomska in mol. fizika: 0.1ev .. 100 keV
- Fizika jedra: 0,1 MeV .. 100 MeV
- F. osnovnih delcev GeV, TeV, PeV,...

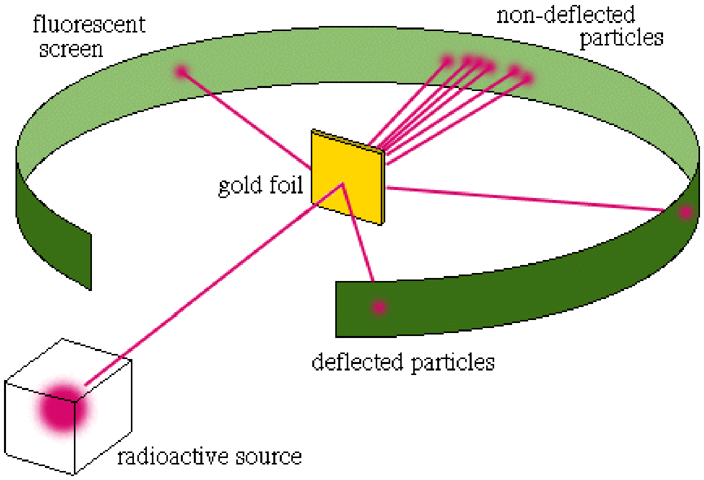
Orodje: POSPEŠEVALNIKI

Ideja pospeševalnika

1. Izvir
2. Pospeševanje
3. Transport žarka
4. Tarča
5. Spektrometri

Naravni pospeševalniki

- α delci (Ra, Th), ≈5 MeV, 1911 Rutherford



- kozmični žarki („galaktični posp.“) do 10^{20} eV

Pospeševalniki

Enosmerni (elektrostatični)

- katodna cev (CRT)
- rentgenska cev
- van de Graaffov posp.
- tandentron

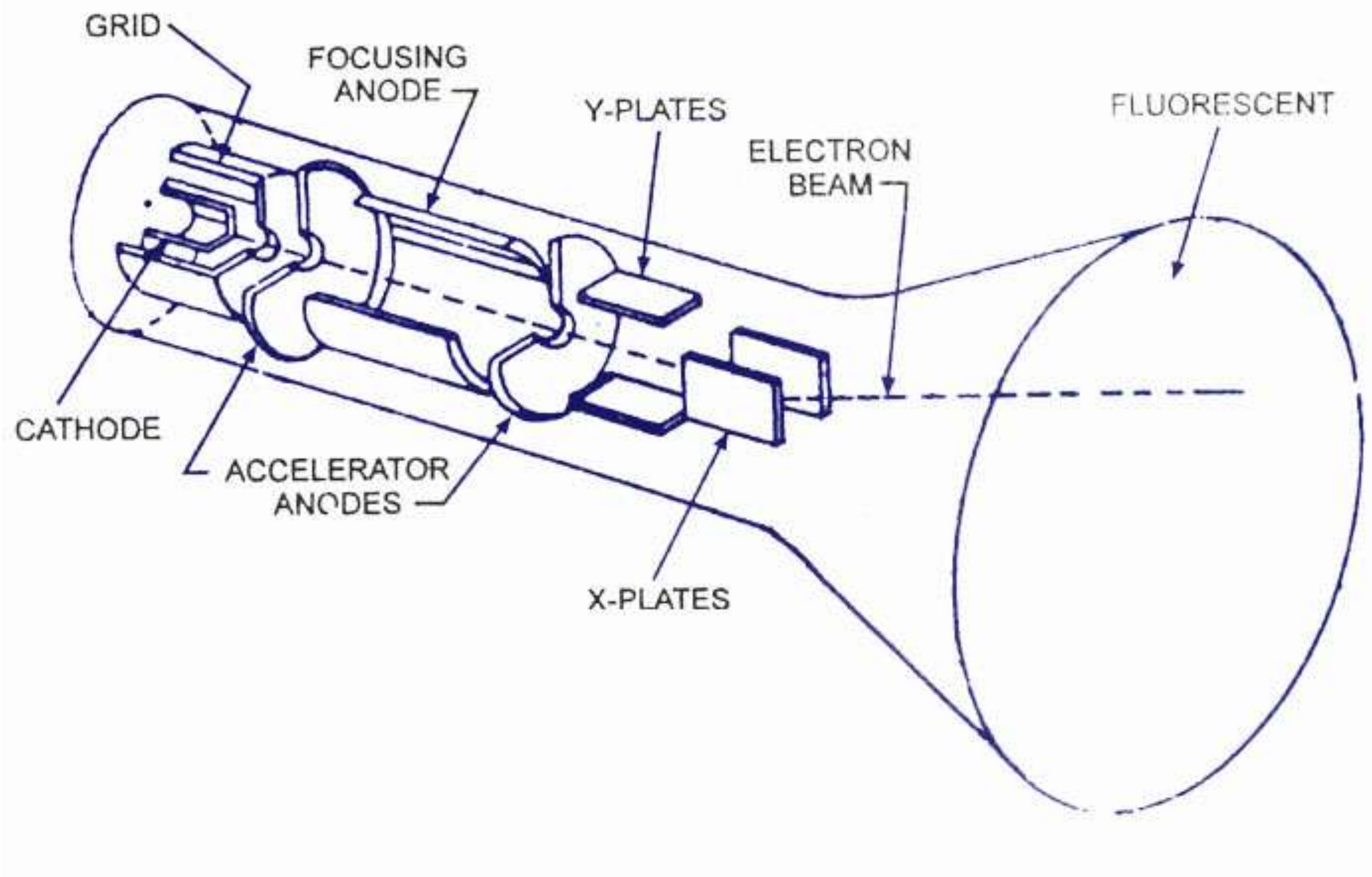
- stalen žarek

Izmenični

- Linac
- ciklotron
- sinhrotron
- ...

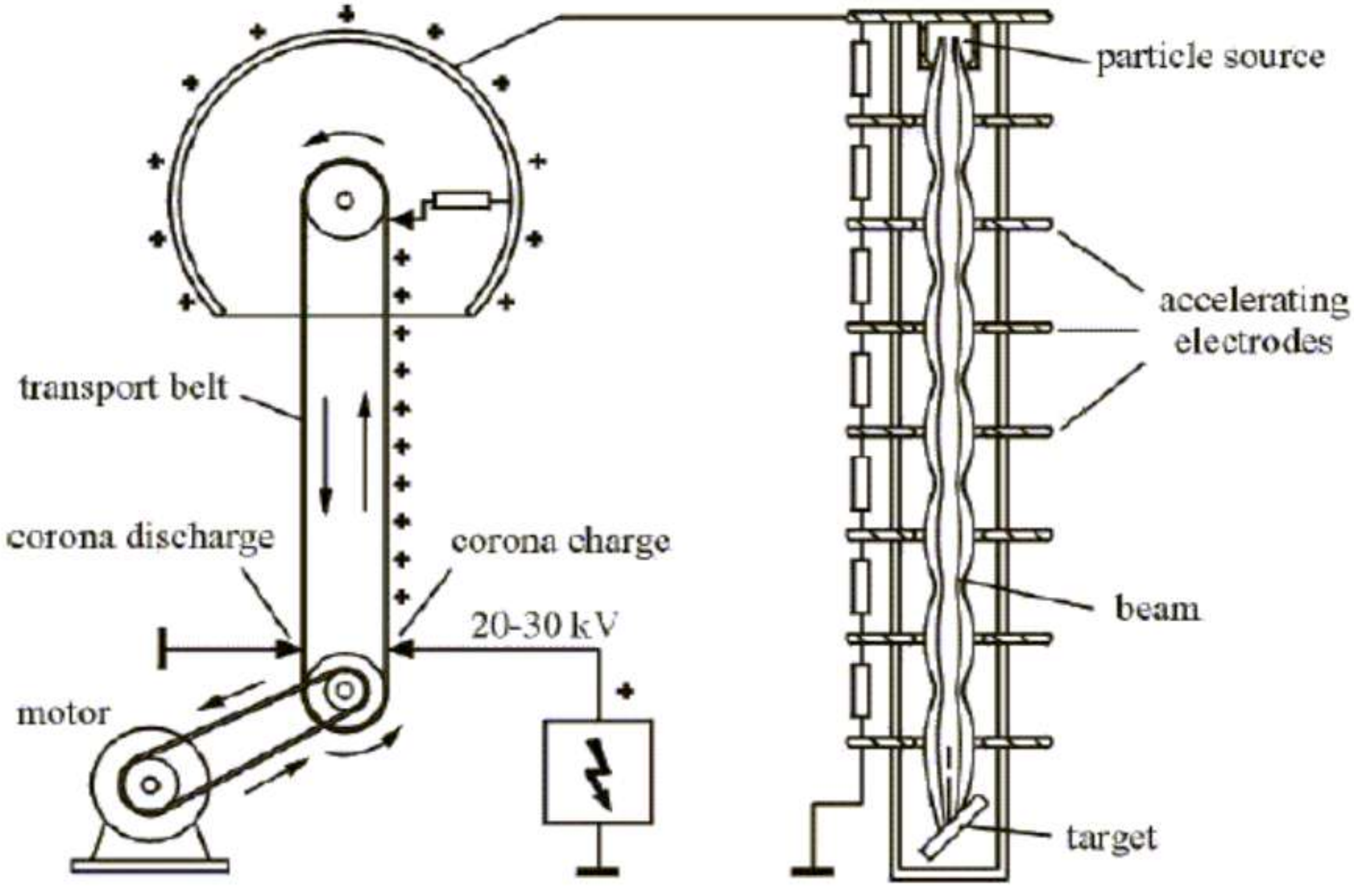
- gruče

CRT



Cathode Ray Tube

van de Graaff



Naloga za ogrevanje

van de Graaffov pospeševalnik

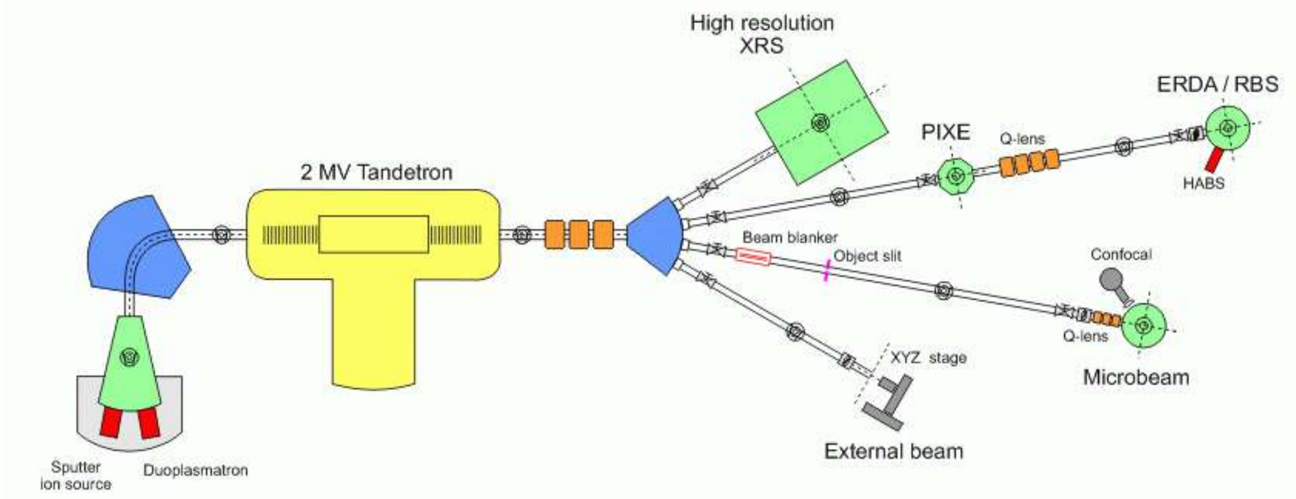
- Največja napetost: 15 MV.
- Elektrode po 50 kV.
- R_1=500 MΩ.

Kolišen tok teče po uporih? Tok ionov je tipično okoli 10 μA.

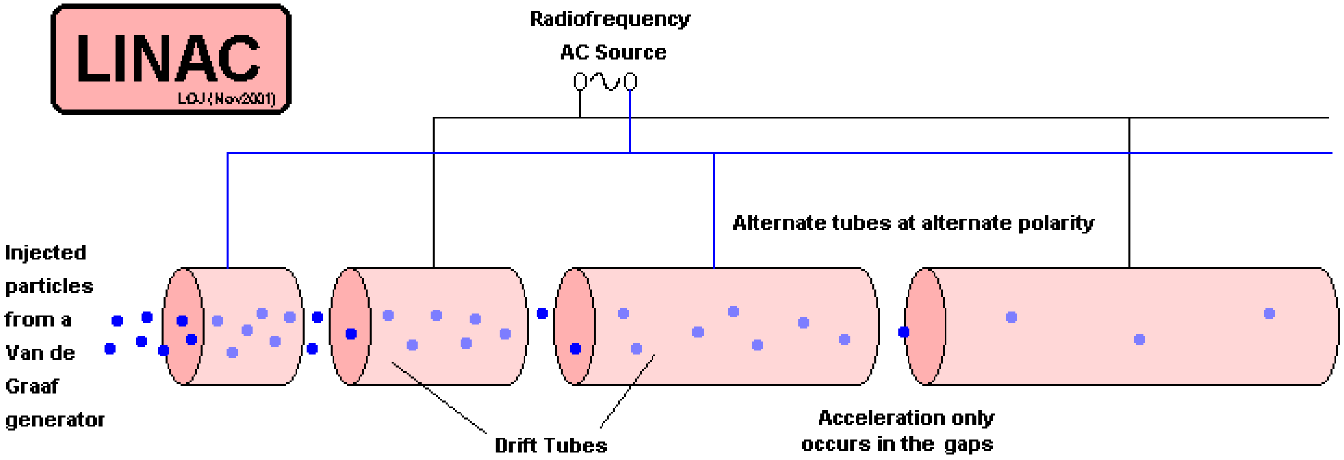
Tandetron

IJS – ionski pospeševalnik

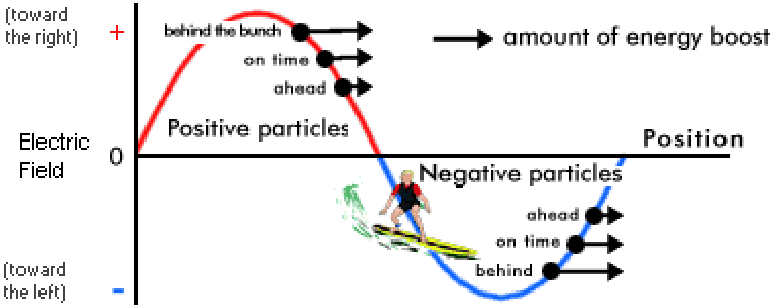
- 2 MV
- 4 eksperimentalne linije
- predviden ogled http://www.rcp.ijs.si/mic/



Linearni pospeševalnik

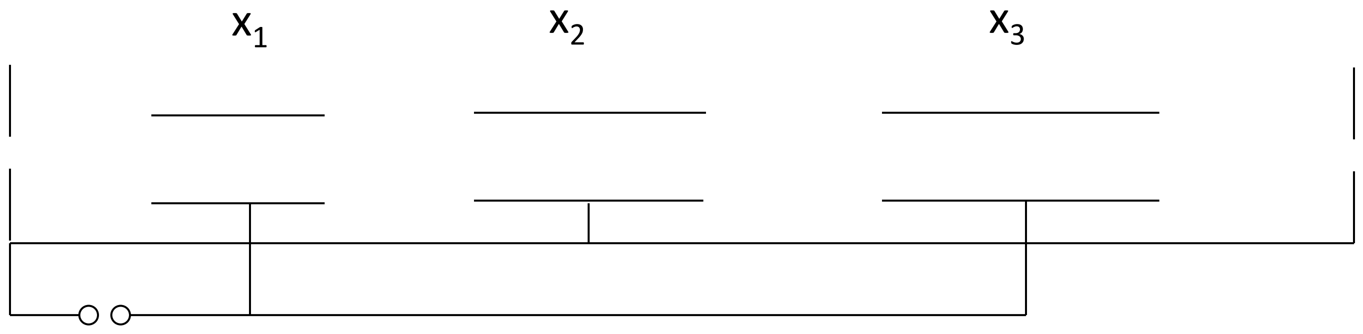


SLAC @ Stanford

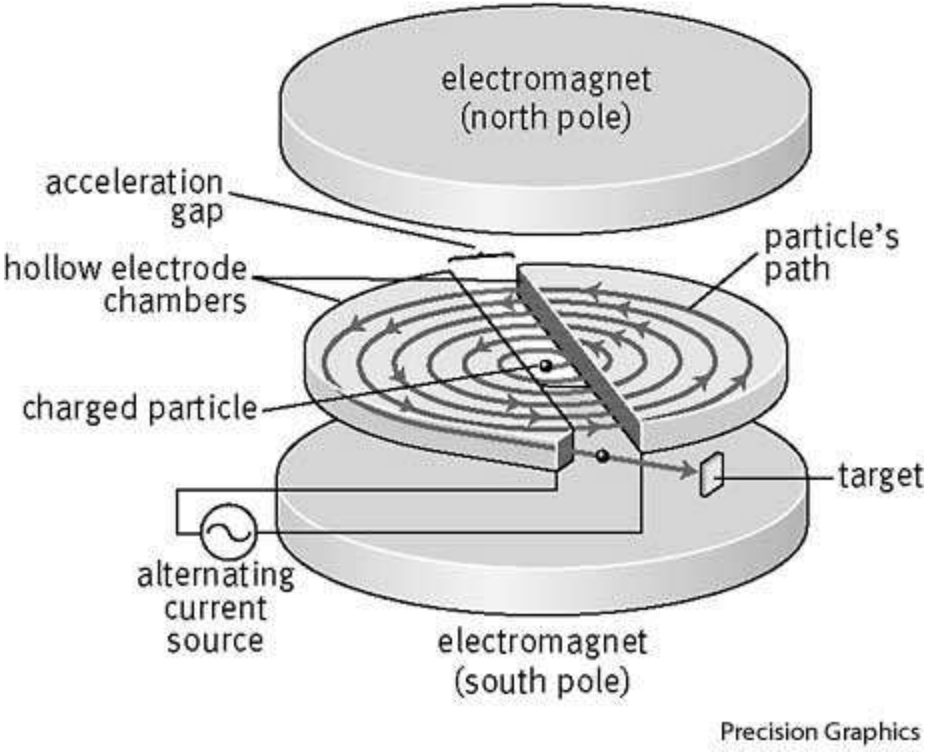


Naloga

Z linearnim pospeševalnikom pri frekvenci 0.5 Ghz pospešujemo elektrone od 100 keV do 1.3 MeV. Pospeševalnik ima tri votline. Kolikšne naj bodo njihove dolžine?



CIKLOTRON



Precision Graphics

- Ciklotronska frekvenca
- Gibanje v osi z?

Šibko fokusiranje

Magnetno polje dobi majhno radialno komponento,

ki lahko fokusira v smeri osi z in radija.

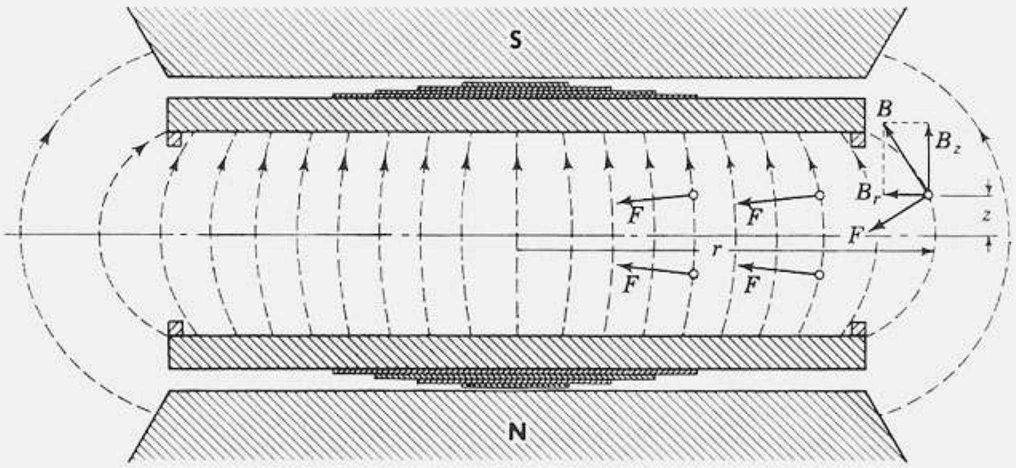


Fig. 6-7. Radially decreasing magnetic field between poles of a cyclotron magnet, showing shims for field correction.

Lorentzova sila

$$\frac{d}{dt}\, m\gamma\vec{v} = e(\vec{E} + \vec{v} \times \vec{B})$$

Kartezične:

X: $$\frac{d}{dt}\, m\gamma\dot{x} = e(E_x + \dot{y}B_z - \dot{z}B_y)$$

Y: $$\frac{d}{dt}\, m\gamma\dot{y} = e(E_y + \dot{z}B_x - \dot{x}B_z)$$

Z: $$\frac{d}{dt}\, m\gamma\dot{z} = e(E_z + \dot{x}B_y - \dot{y}B_x)$$

Lorentzova sila

$$\frac{d}{dt}\, m\gamma\vec{v} = e(\vec{E} + \vec{v} \times \vec{B})$$

Cilindrične:

r: $\frac{d}{dt}(m\gamma\dot{r}) - m\gamma\dot{\varphi}^2 r = e(E_r + r\dot{\varphi}B_z - \dot{z}B_\varphi)$

φ: $\frac{1}{r}\frac{d}{dt}(m\gamma r^2\dot{\varphi}) = e(E_\varphi + \dot{z}B_r - \dot{r}B_z)$

z: $\frac{d}{dt}(m\gamma\dot{z}) = e(E_z + \dot{r}B_\varphi - r\dot{\varphi}B_r)$

Šibko fokusiranje v smeri z

- Polje z majhno radialno komponento:

$$B_r = \frac{\partial B_r}{\partial z}\, z$$

- Enačba gibanja:

$$\ddot{z} + n\, {\omega_c}^2\, z = 0$$

Indeks polja: $n = -\frac{r_0}{B_z}\frac{\partial B_r}{\partial z},$ stabilno pri $n > 0$

Določanje tira

z(t) → z(s), $\dot{z} = z'v, \ddot{z} = z''v^2$

Tir reši: $z'' + \frac{n}{{r_0}^2} z = 0,\ \text{harmonično gibanje}$

Matrični zapis:

$$\begin{bmatrix} z \\ z' \end{bmatrix} = \begin{bmatrix} \cos(s/s_z) & s_z \sin(s/s_z) \\ -1/s_z \sin(s/s_z) & \cos(s/s_z) \end{bmatrix} \begin{bmatrix} z_0 \\ {z_0}' \end{bmatrix}$$

$$s_z = r_0/\sqrt{n}$$

„sektorski magnet“

Podobno velja za radialno gibanje

- polje: $B_z = B_0 + \frac{\partial B_z}{\partial r}\,(\,r - r_0) = B_0(1 - n\frac{\rho}{r_0})$
- gibalna enačba: $\ddot{\rho} + (1 - n)\frac{v^2}{{r_0}^2}\rho = 0$
- harmonično gibanje, če $n < 1$.
- matrični zapis:

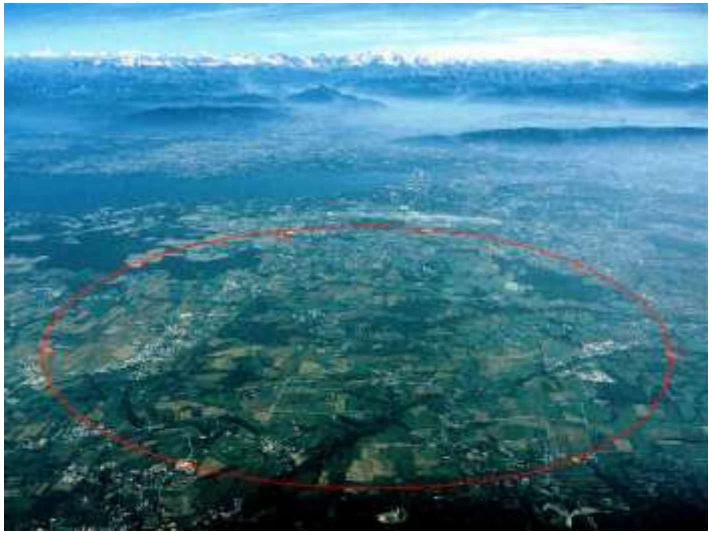
$$\begin{bmatrix} z \\ z' \end{bmatrix} = \begin{bmatrix} \cos(s/s_r) & s_r\sin(s/s_r) \\ -1/s_r\sin(s/s_r) & \cos(s/s_r) \end{bmatrix}\begin{bmatrix} z_o \\ {z_0}' \end{bmatrix}$$

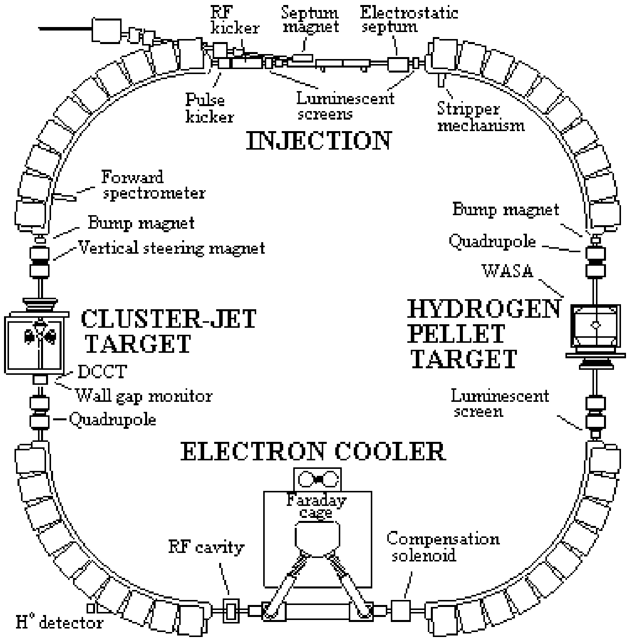
$$s_r = r_0/\sqrt{1-n}$$

SINHROTRONI

- stalen radij kroženja
- frekvenca in jakost EM polja spreminjajoči
- pospeševanje, fokusiranje, zavoj, potovanje

CERN, Ženeva





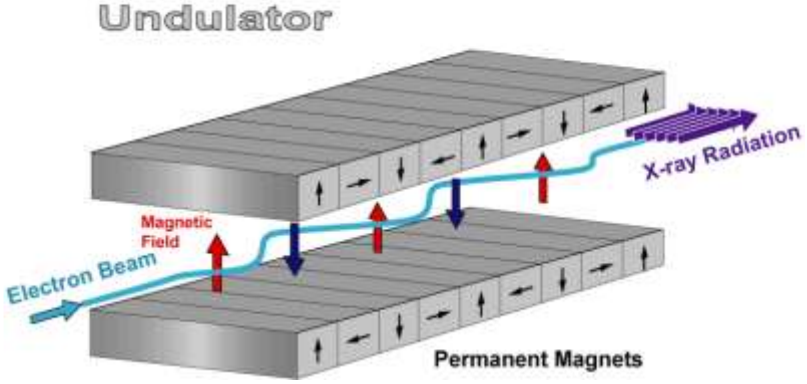
Uppsala

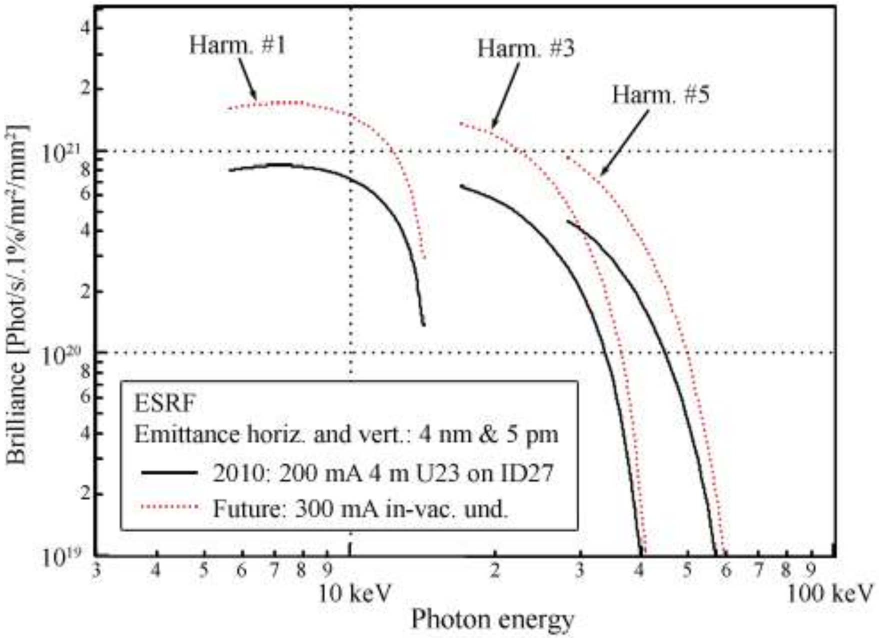
Sinhrotroni kot vir fotonov

- Izkoriščamo sevanje pospešenih delcev
- Magneti, wigglerji, undulatorji

Rezultat:

- koheretna polarizirana svetloba
- zvezen spekter, IR – žarki X (50keV)
- veliko aplikacij

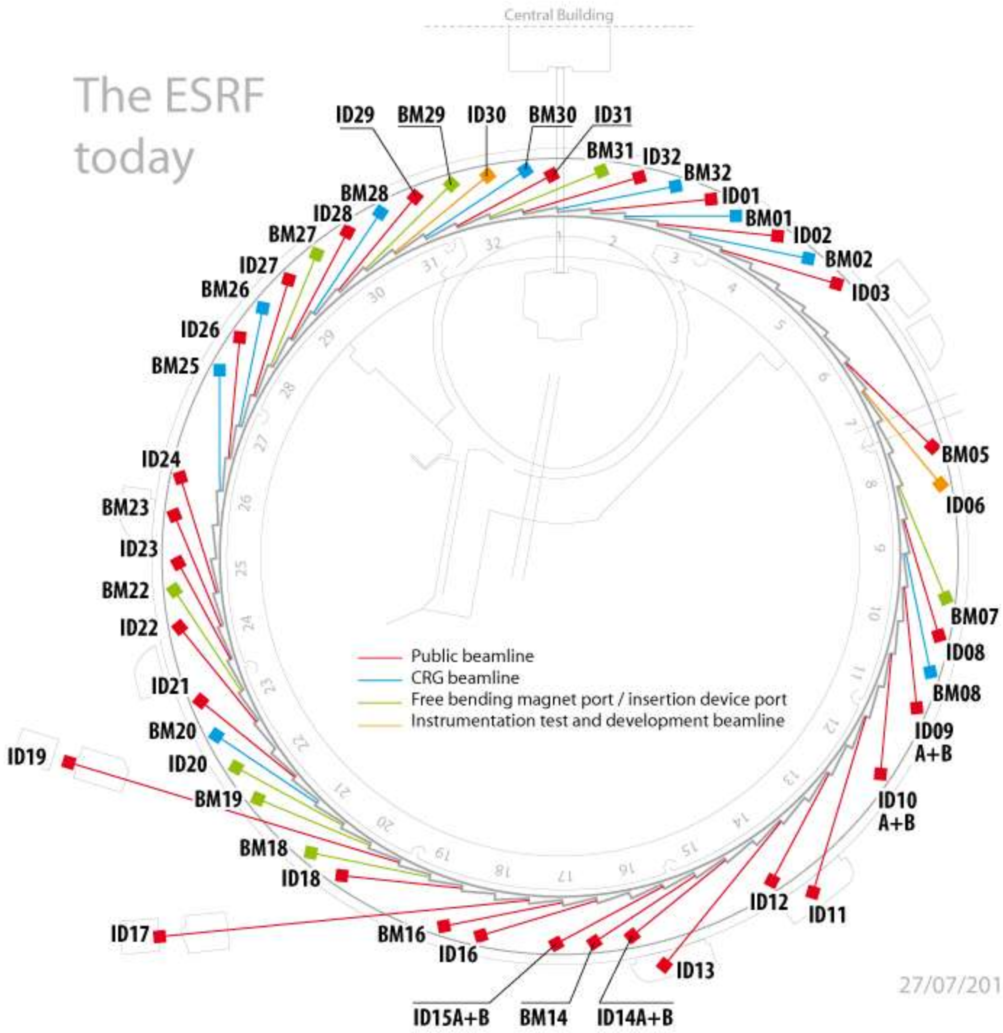




ELETTRA

- 2,4 GeV
- UV, X žarki
- 24 linij
- FEL

The ESRF today



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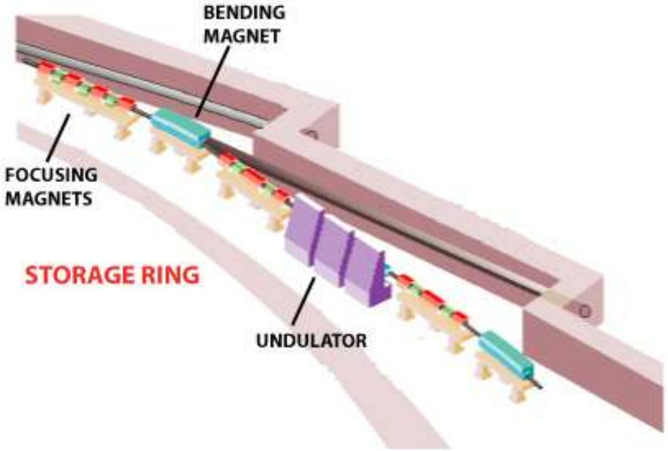
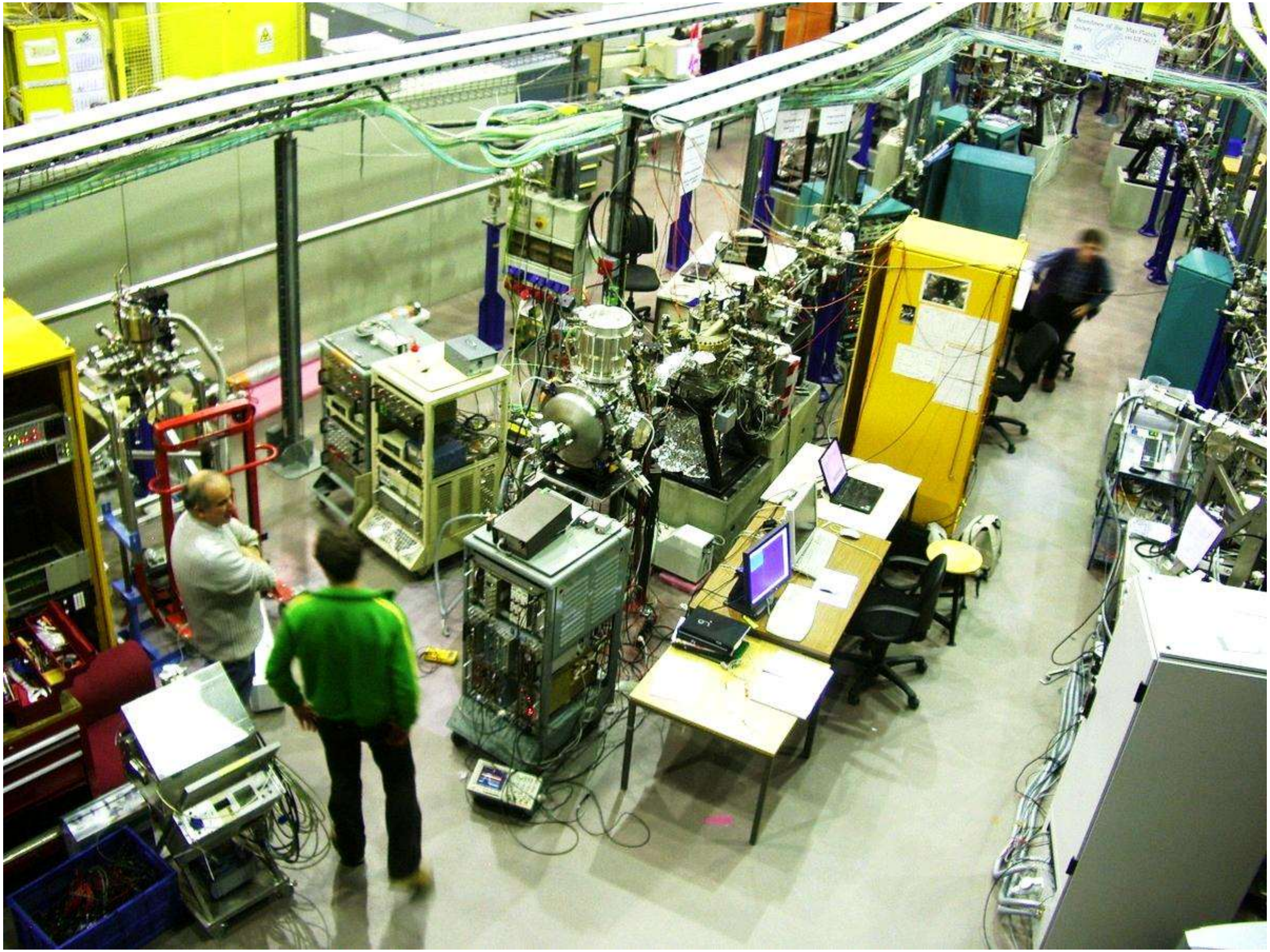
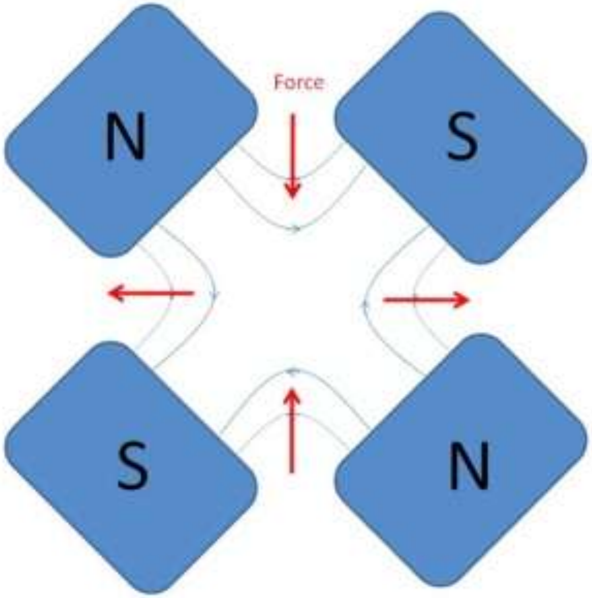
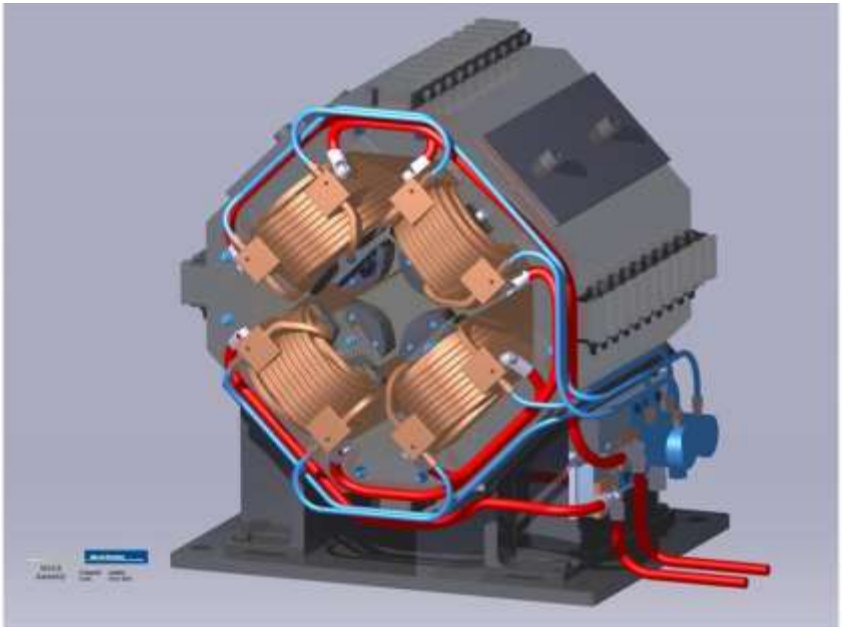



Beamlines of the Max Planck Society

Močno fokusiranje (kvadrupolni magnet)





- V eni smeri fokusira, v drugi defokusira
- V parih dosežemo fokusiranje

Kvadrupolno polje

- $B_x = \frac{\partial B_x}{\partial y} y, \; B_y = \frac{\partial B_y}{\partial x} x$
- Iz $\mathrm{rot}\left(\vec{B}\right) = 0$ dobimo $\frac{\partial B_x}{\partial y} = \frac{\partial B_y}{\partial x} = g$
- enačbi gibanja: $m\gamma\ddot{x} = -e\dot{z}B_y, m\gamma\ddot{y} = e\dot{z}B_x$
- tir: $x'' + k^2 x = 0, \; y'' - k^2 y = 0$

$k = \sqrt{eg/p}, p = m\gamma v$

Matrika kvadrupolnega magneta

Smer X: $\begin{bmatrix} \cos(kz) & 1/k\,\sin(kz) \\ -k\,\sin(kz) & \cos(kz) \end{bmatrix}$

Smer Y: $\begin{bmatrix} \cosh(kz) & 1/k\,\sinh(kz) \\ k\,\sinh(kz) & \cosh(kz) \end{bmatrix}$

Tanek kvadrupolni magnet, kz<<1, debelina z→0:

$$\begin{bmatrix} 1 & 0 \\ \pm k^2 \mathrm{z} & 1 \end{bmatrix}$$

Tanka leča

$$\begin{bmatrix} 1 & 0 \\ \pm 1/f & 1 \end{bmatrix}, \quad -\frac{1}{f}\text{ zbiralna}, \quad \frac{1}{f}\text{ razpršilna}$$

Prazen prostor dolžine l

$$\begin{bmatrix} 1 & l \\ 0 & 1 \end{bmatrix}$$

Dva kvadrupolna magneta

$f_2, f_1,$ razdalja d, ($f > 0$, zbiralna leča)

$$\begin{bmatrix} 1 - d/f_2 & d \\ -\frac{1}{f_2} - \frac{1}{f_1} + \frac{d}{f_1 f_2} & 1 - d/f_1 \end{bmatrix}$$

1. $f_2 < 0, f_1 > 0, f_1 = f_2 = f$
2. $f_2 > 0, f_1 < 0, f_1 = f_2 = f$

Neto fokusiranje v obeh smereh, gorišči pa NE sovpadata.

Kje je gorišče za vzporeden žarek?

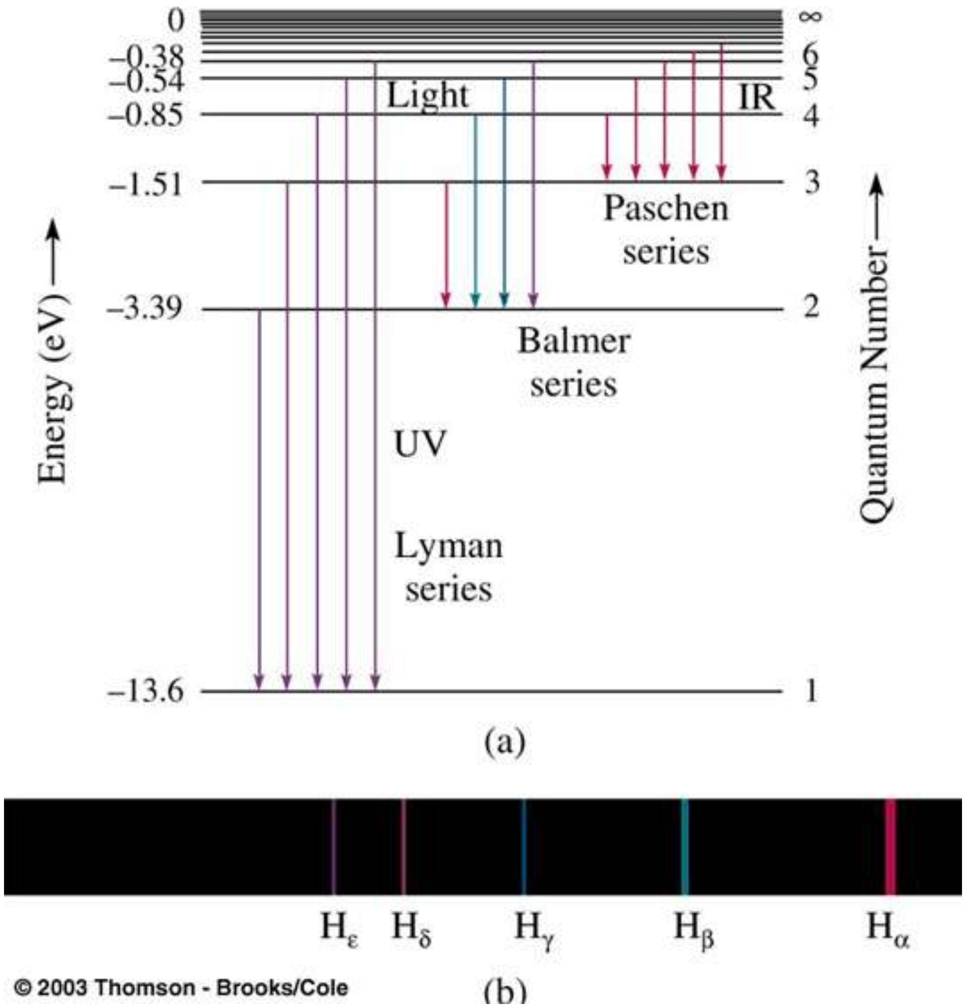
- lečo obdamo s praznim prostorom a in b
- preslikamo točko $\begin{bmatrix} y \\ 0 \end{bmatrix}$ in zahtevamo, da žarek seka os x za vsak y

1. $-bd + f(f-d) = 0 \rightarrow b = \frac{f(f-d)}{d}$
2. $-bd + f(f+d) = 0 \rightarrow b = \frac{f(f+d)}{d}$

Gorišči ne sovpadata!

Atomska fizika

Vodikov atom



© 2003 Thomson - Brooks/Cole

$$\boxed{E_n = -\frac{E_0}{n^2}}$$

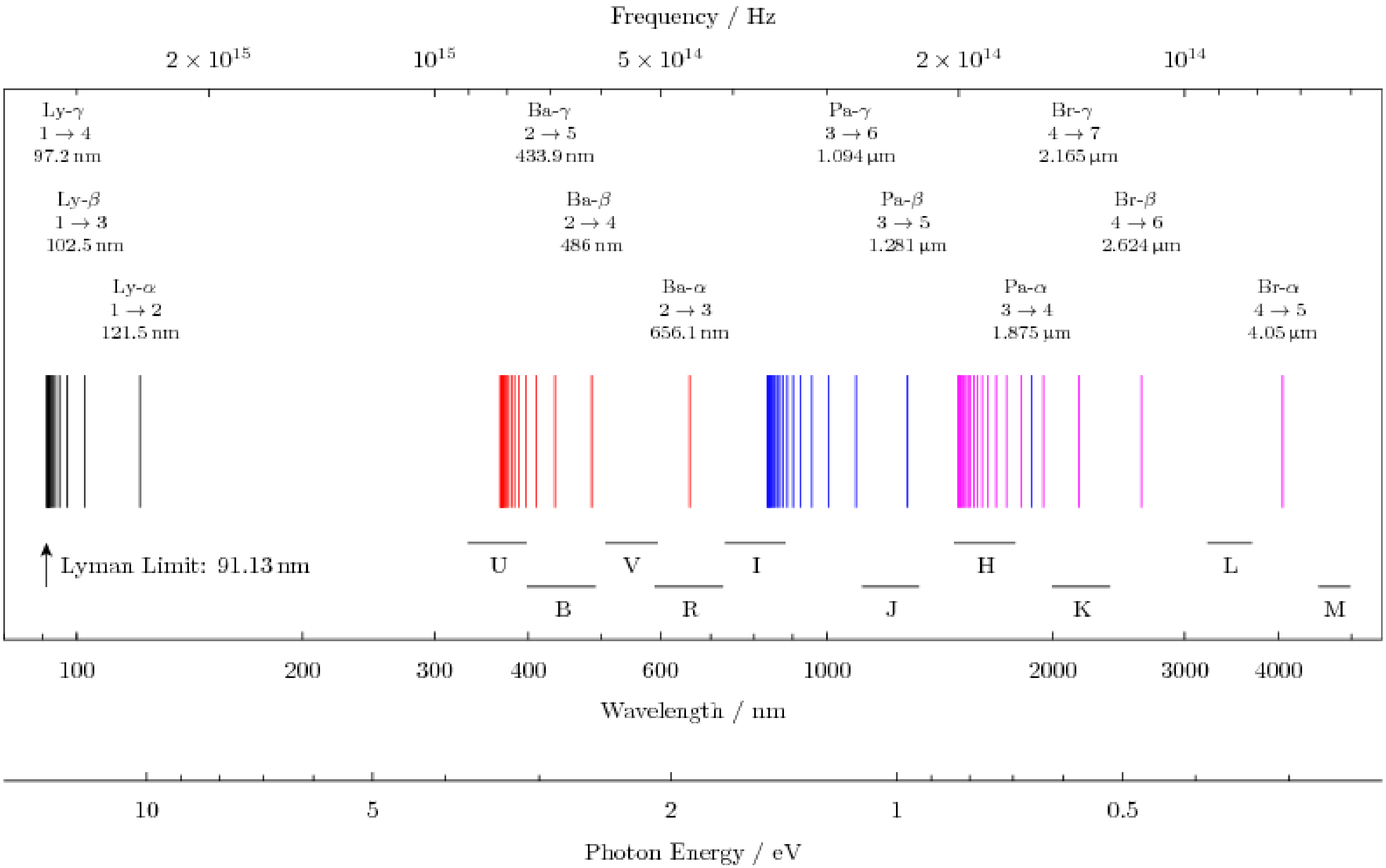
$$E_0 = 13{,}6058\ \text{eV}$$

$$a_0 = 53\ \text{pm}$$

$$a_n = a_0 n^2$$

$$E_n = -E_0\,\frac{Z^2}{n^2}$$

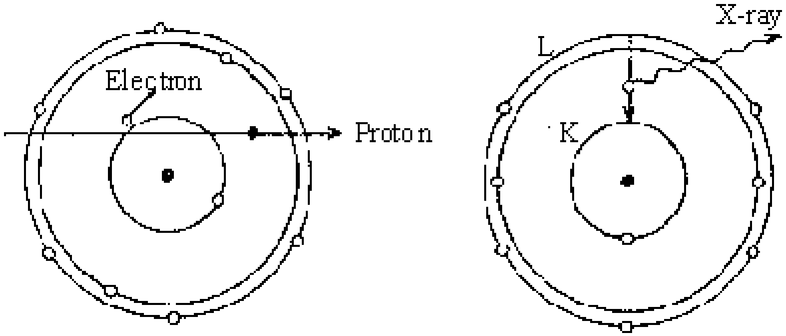
Prehodi med stanji

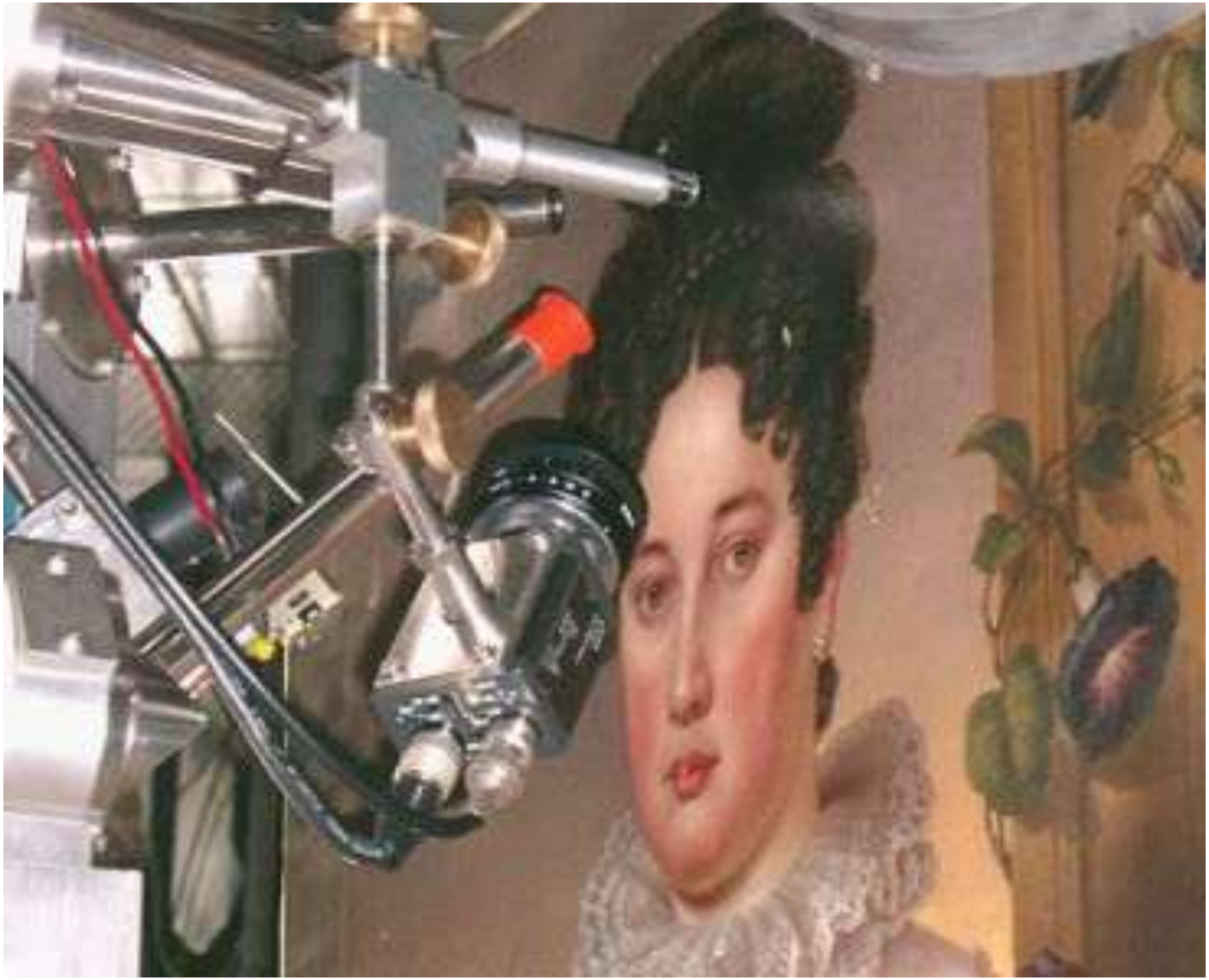


Uporaba: rentgenska spektroskopija

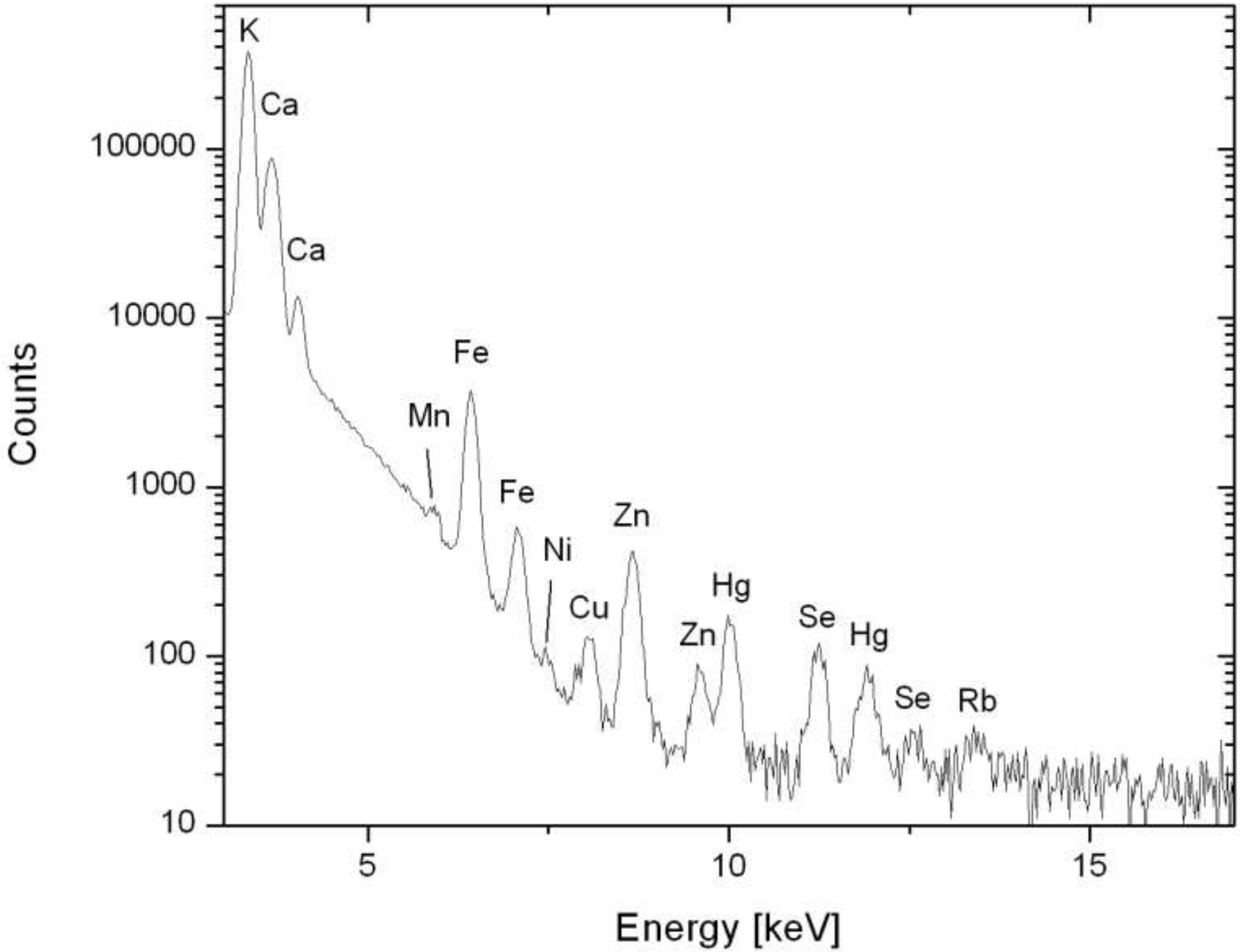
Analizni tehniki:

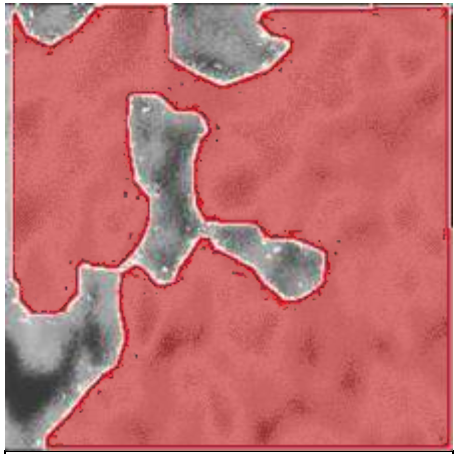
1. absorpcijska,
2. emisijska, npr. PIXE.





Germanium (iGe) X-ray spectrum (324003, whole)

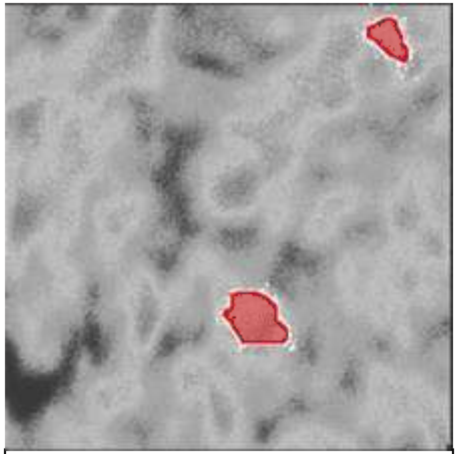




Celota

Analiza koncentracij celotnega vzorca

Analiza koncentracij Hg gnezd



Hg gnezda 1-2

Elementi	Konc [ppm]	Stat.err. %	lod [ppm]
K	9675.8	0.06	4.8
Ca	1552.7	0.84	22.5
Mn	8.5	9.12	1.4
Fe	118.8	0.84	1.2
Ni	0.2	280.12	0.9
Cu	28.6	2.47	0.8
Zn	67.6	1.56	0.9
Se	53.6	3.93	2.7
Rb	17.0	19.26	5.8
Hg	138.8	2.93	4.7

Hipofiza

File nr.
324014

Elementi	Konc [ppm]	Stat.err. %	lod [ppm]
K	10501.0	0.3	20.5
Ca	1647.1	1.69	47.1
Mn	14.1	34.51	8.7
Fe	156.5	4.47	6.7
Ni	3.2	116.65	6.5
Cu	48.3	11.82	7.5
Zn	74.6	10.3	9.8
Se	201.4	11.67	31.3
Rb	58.6	46.55	35.6
Hg	606.6	8.03	37.1

Aktivnost

- Aktivnost: $A = \frac{N}{\tau}$ [Bq]
- Primer: aktivnost K-40 v 1kg KCl

M(K) = 39,1 kg/kmol

M(Cl) = 35,45 kg/kmol

w(K-40)=0,0117%

t ½ (K-40)= 1,25 10^9 let

A=1,66 10^4 Bq

Razpadi

- Po koliko razpolovnih časih dosežemo 97% ravnovesne aktivnosti, če nuklid v reaktorju nastaja s konstantno hitrostjo? (5)

3 nivojski sistem

1 → 2 → 3

$$N_1 = N_0 \exp(-\lambda_1 t)$$

$$N_2 = N_0 \frac{\lambda_1}{\lambda_1 - \lambda_2} (\exp(-\lambda_2 t) - \exp(-\lambda_1 t))$$

$$N_3 = N_0 \left(1 + \frac{\lambda_2 \exp(-\lambda_1 t) - \lambda_1 \exp(-\lambda_2 t)}{\lambda_1 - \lambda_2}\right)$$

Kdaj nivo 2 doseže max?

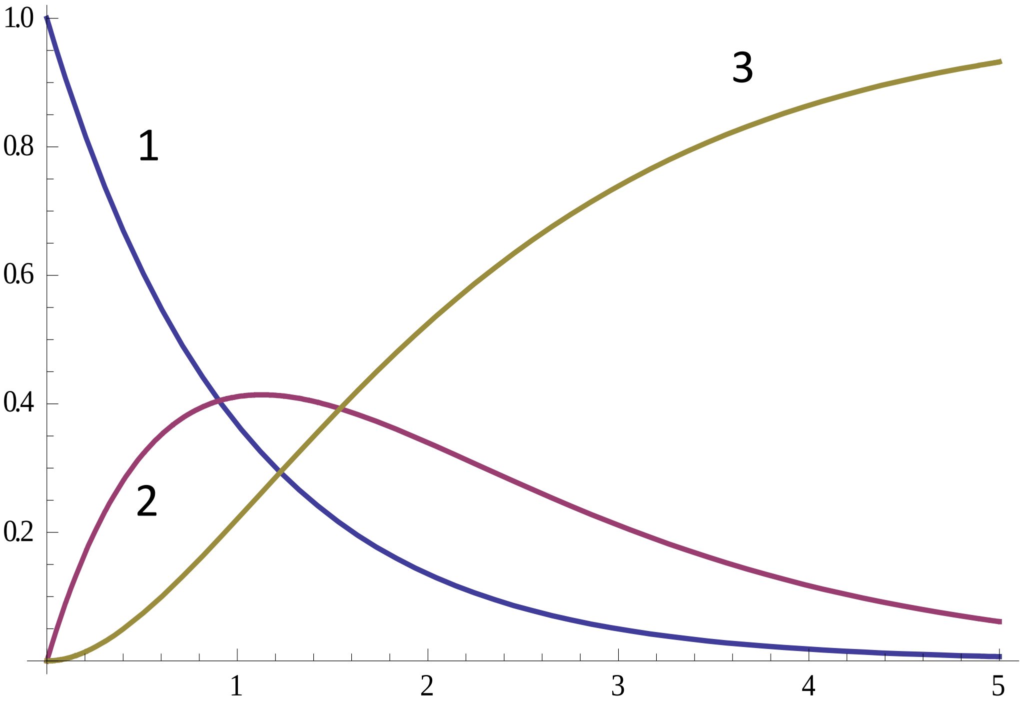
$$t_{max} = \frac{\ln(\lambda_1/\lambda_2)}{\lambda_1 - \lambda_2}$$

$A_2(t_{max}) = ?$

$A_1(t_{max}) = ?$

$$\frac{A_2}{A_1} = ?$$

$$A = \frac{N}{\tau} \neq \frac{dN}{dt}$$



$$\frac{A_2}{A_1} = \frac{\lambda_2}{\lambda_2 - \lambda_1}\left(1 - e^{-(\lambda_2 - \lambda_1)t}\right)$$

1. $\lambda_2 < \lambda_1$: razmerje se s časom povečuje
2. $\lambda_2 > \lambda_1$: razmerje se sčasoma stabilizira
3. $\lambda_2 \gg \lambda_1$: razmerje gre hitro proti 1

Primer, razpad β- pri Sr-90:

$$\mathrm{Sr-90} \xrightarrow{28y} \mathrm{Y-90} \xrightarrow{64.8h} \mathrm{Zr-90}$$

Sodelovanje sevanja s snovjo

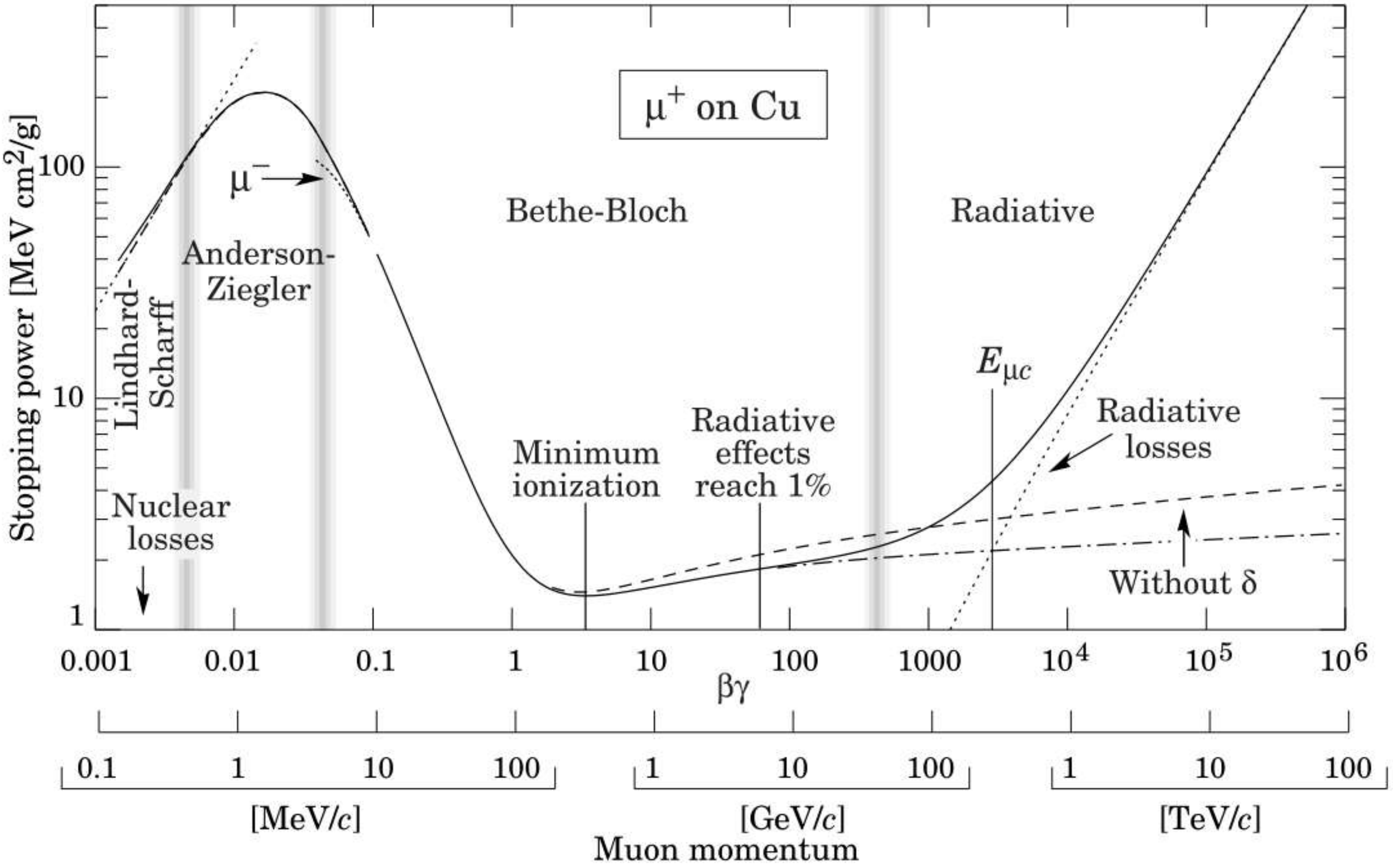
Bethe-Blochova formula

$$-\frac{dE}{\rho dx} = K\frac{Z}{A}\frac{{z_1}^2}{\beta^2}\left[\ln\left(\frac{2\,m_e c^2}{I}\,\frac{\beta^2}{1-\beta^2}\right) - \beta^2\right]$$

$$K = 4\pi\, N_a r_e^2 m_e c^2 = 0{,}307\ \mathrm{MeV}\frac{\mathrm{cm}^2}{\mathrm{g}}$$

$$I \approx Z \cdot 13{,}5\ \mathrm{eV}$$

Velja za $\frac{v}{c} = \beta > 0.1$ in $z_1 < 13,\ \mathrm{M} \gg m_e$


μ$^+$ on Cu
Stopping power [MeV cm^2/g]
100
10
1
Lindhard-Scharff
Anderson-Ziegler
μ$^-$
Bethe-Bloch
Radiative
$E_{\mu c}$
Radiative losses
Minimum ionization
Radiative effects reach 1%
Nuclear losses
Without δ
0.001
0.01
0.1
1
10
100
1000
10^4
10^5
10^6
βγ
0.1
1
10
100
[MeV/c]
1
10
100
[GeV/c]
1
10
100
[TeV/c]
Muon momentum

Naloga: Kakšno gibalno količino in kinetično energijo imajo minimalno ionizirajoči delci e^-, μ, π, p in α?

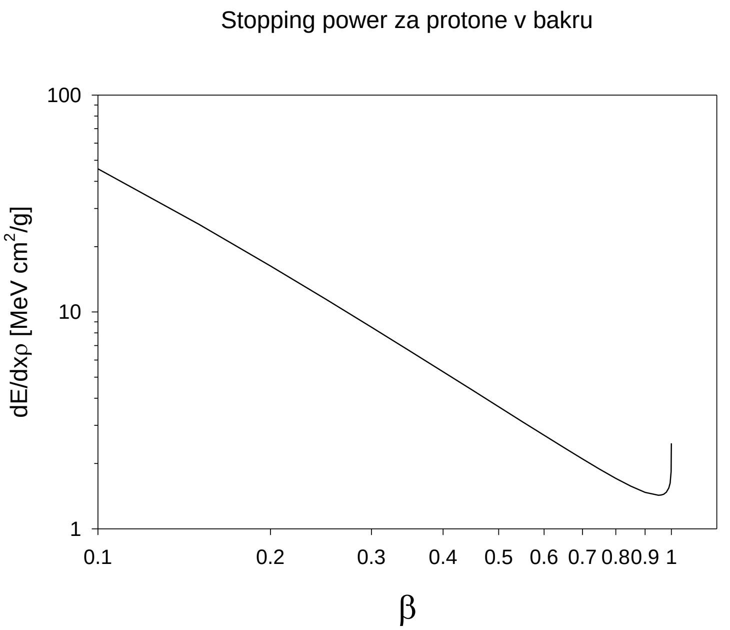
$$\Downarrow$$

$$\beta = 0{,}96$$

$$T = ?\,, p = ?$$

$$pc = 3{,}43 E_0, \qquad T = \left(\sqrt{1 + 3{,}43^2} - 1\right) E_0$$

delec	mc^2[MeV]	pc[MeV]	T[MeV]
e	0,511	1,75	1,31
μ	107	367	275
π	140	480	360
p	938	3200	2410
α	3800	13000	9800


Stopping power za protone v bakru

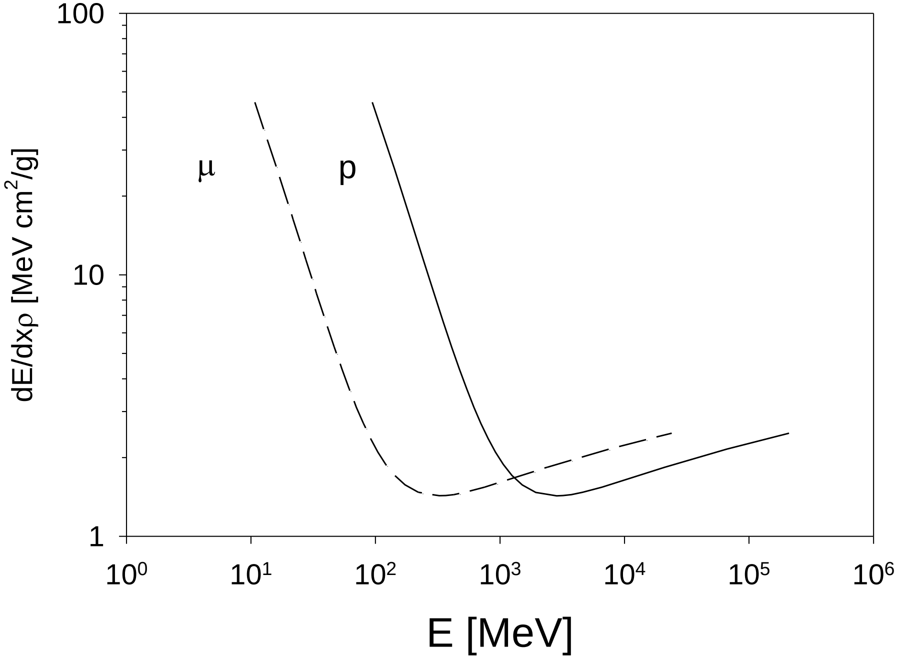
Baker: Z=29, A=63.55, ρ=9.0 g/cm^3

Naloga: Kozmične mione merimo z 2 cm debelim plastičnim scintilatorjem z gostoto 1.03 g/cm^3. Kakšna je povprečna energija, ki ostane v detektorju?

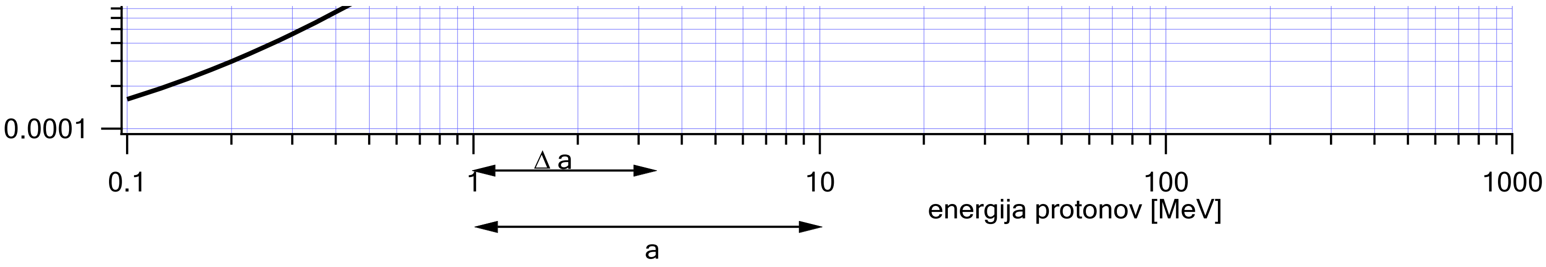
$$-\frac{dE}{\rho\, dx} \approx 1{,}5\ \mathrm{MeV\ cm^2/g}$$

$$\Delta E \approx 3\ \mathrm{MeV}$$

Stopping power za mione in protone v bakru



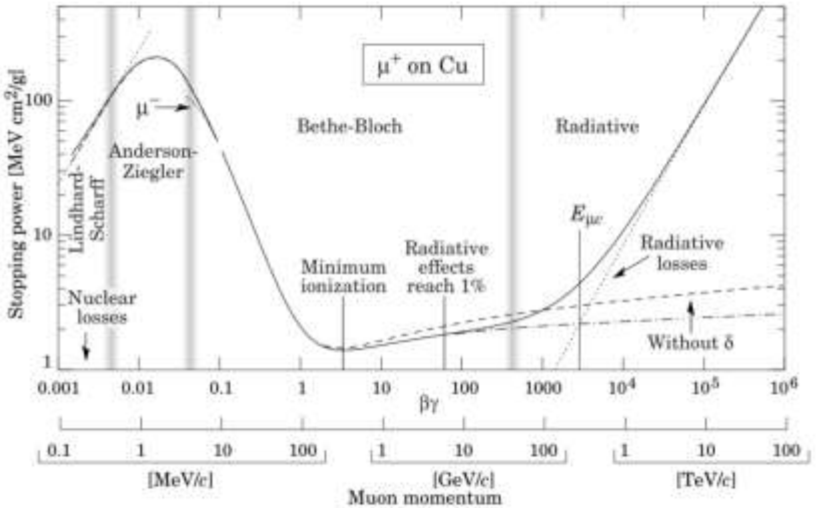
Odčitavanje z logaritemskega grafa

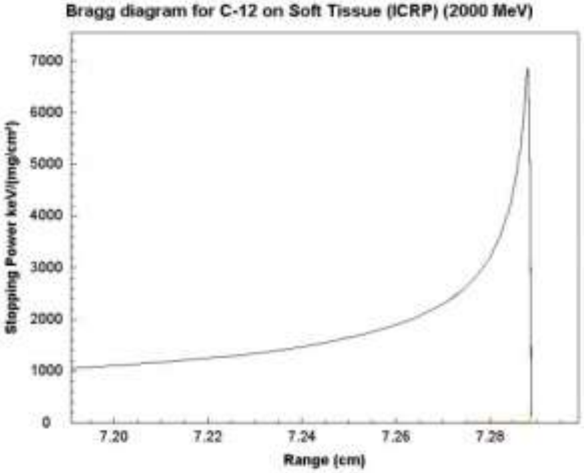


$$E = E_1\,10^{\frac{\Delta a}{a}\log\frac{E_2}{E_1}}$$

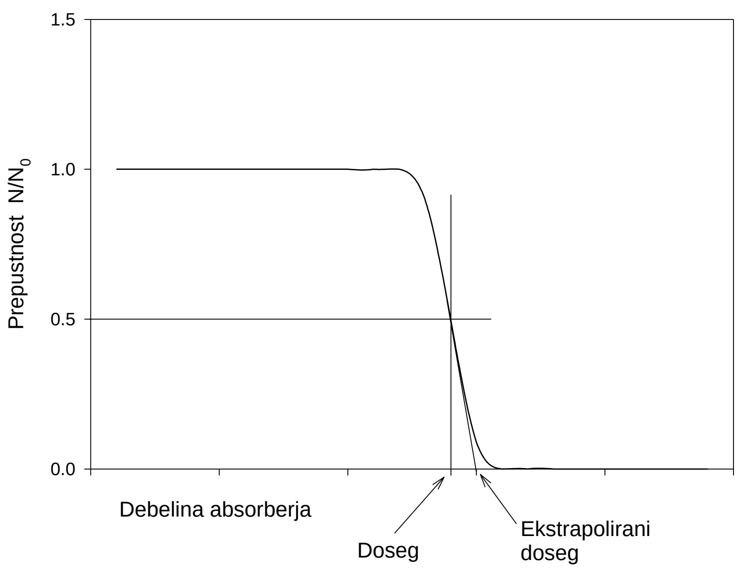
$\Delta a/a$	E
0	1
0.30	2.00
0.50	3.16
0.70	5.00
1	10

Doseg delcev





Delež prepuščenih delcev v odvisnosti od razdalje



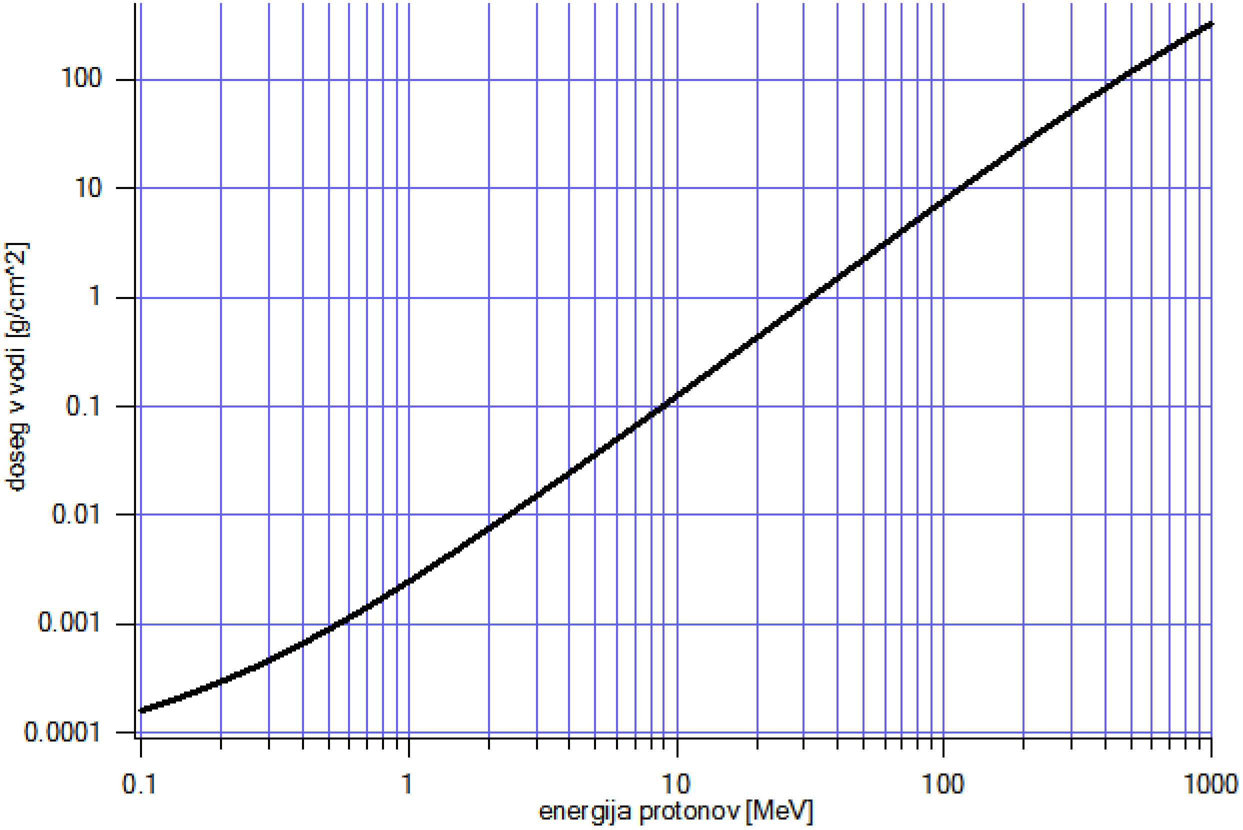
Doseg delcev

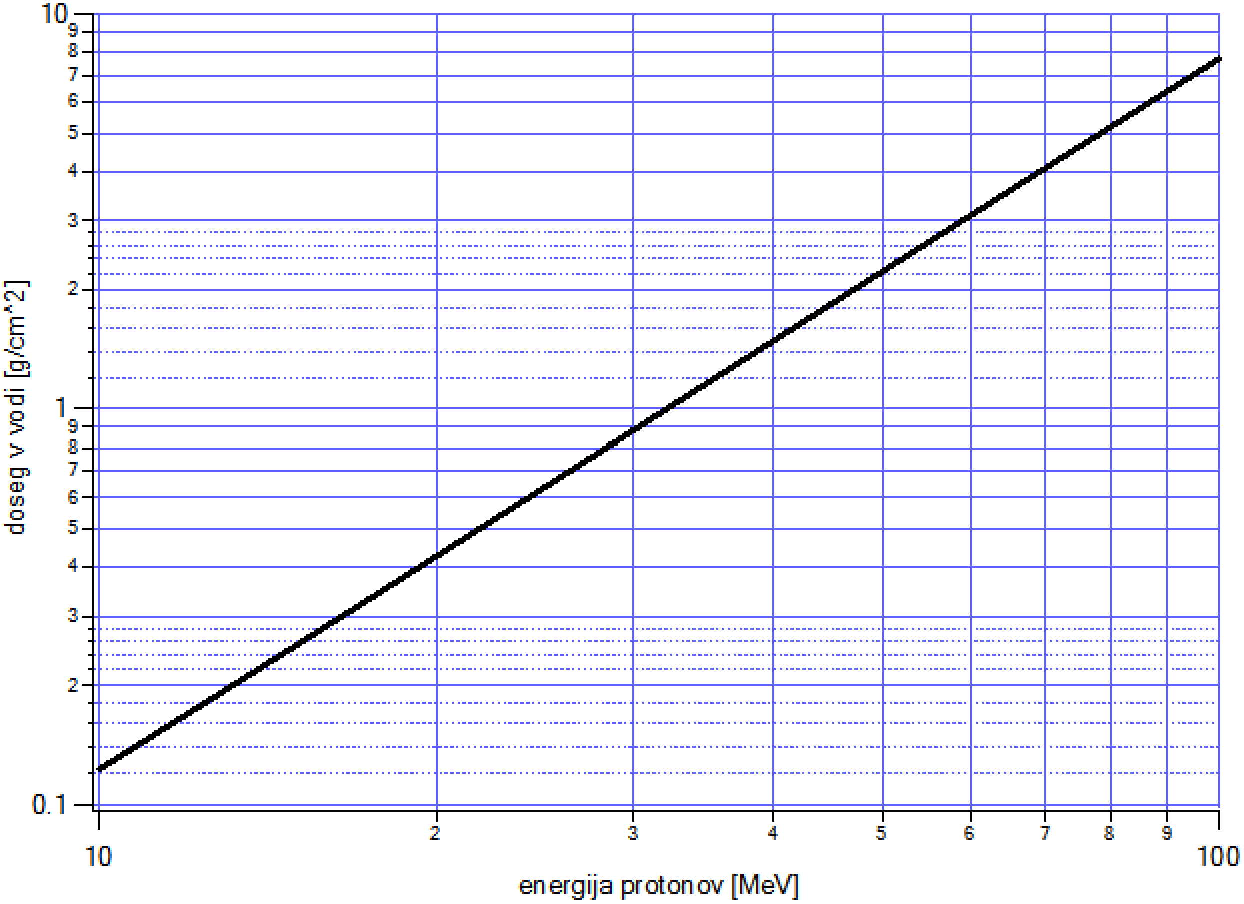
Uporabno:

- Določanje energije delcev
- Velikost detektorjev
- Debeline ščitov
- Terapija

Naloga: Kakšna mora biti energija curka protonov, da z njimi uničimo tumor v možganih na globini 1 cm?

Podatki za doseg in izgube energij:
http://physics.nist.gov/PhysRefData/Star/Text/contents.html





$E \approx 32\ \mathrm{MeV}$

Preseki = verjetnost za reakcije

$$\frac{N_r}{t} = \frac{N_0 \sigma_T}{t\ S}$$

Skupni presek za reakcijo: $\sigma_T = N_T \sigma$

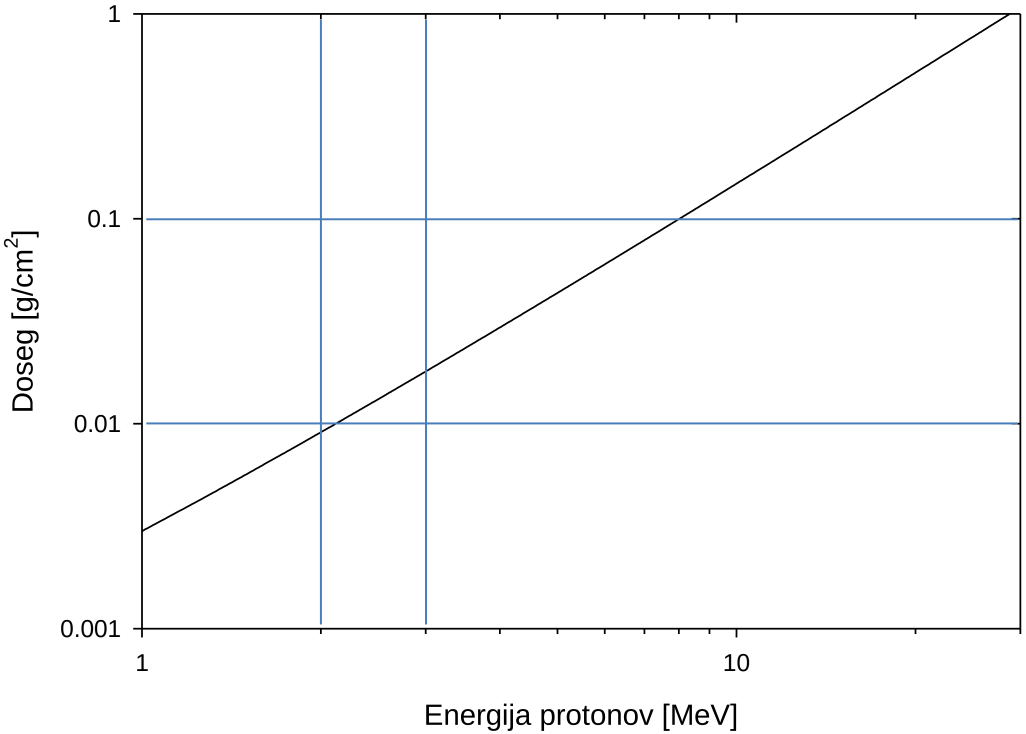
Velja za tanko tarčo, S = min(tarča, žarek)

S protoni iz tandem van de Graafovega pospeševalnika pri terminalni napetosti 1.5 MV obstreljujemo tarčo iz litija. Presek za reakcijo ^{7}Li(p,n)^{7}Be je 345 mb, konstanten do praga pri 2 MeV, pri nižjih energijah protonskega žarka pa hitro pade na nič. Koliko reakcij bo nastalo, če na tarči izmerimo naboj 100 μAs?

$$1\ \mathrm{b} = 10^{-24}\ \mathrm{cm}^2$$

$$N_R = 1{,}6\ \times 10^{11}$$

Doseg protonov v litiju



Uporaba B-B

- Oceni, kolikšno energijo izgubi delec α z energijo 5,5MeV, ko preleti list zlata debeline 1,5μm. Gostota zlata je 19,3 g/cm3, vrstno število 79 in masno število 197. Povprečna ionizacijska energija za težje elemente je sorazmerna z njihovim vrstnim številom: $I \approx E_I Z$, kjer je $E_I = 9{,}1\mathrm{eV}$. (=0,7 MeV)

Bethe-Blochova formula

$$-\frac{dE}{\rho dx} = K\frac{Z}{A}\frac{{z_1}^2}{\beta^2}\left[\ln\left(\frac{2\,m_e c^2}{I}\,\frac{\beta^2}{1-\beta^2}\right) - \beta^2\right]$$

$$K = 4\pi\,N_a r_e^2 m_e c^2 = 0{,}307\ \mathrm{MeV}\frac{\mathrm{cm}^2}{\mathrm{g}}$$

$$I \approx Z\ \cdot\ 13{,}5\ \mathrm{eV}$$

Velja za $\frac{v}{c} = \beta > 0.1$ in $z_1 < 13,\ \mathrm{M} \gg m_e$