Kontinuitetna enačba v diferencialni obliki

$$\frac{\partial \rho}{\partial t} + \nabla(\rho \vec{v}) = 0$$

Zapis v obliki snovskega odvoda:

$$\frac{D\rho}{Dt} = -\rho \nabla \vec{v} = 0$$

Poenostavitve:

Stacionaren tok:

$$\nabla(\rho \vec{v}) = 0$$

Nestisljiva tekočina ($\rho = konst$)

$$\nabla \vec{v} = 0$$

Gibalna enačba v diferencialni obliki

$$\rho \frac{D\vec{v}}{Dt} = -\nabla p - \nabla \underline{\tau} + \rho \vec{g}$$

Gibalna enačba v kartezičnem koordinatnem sistemu (x,y,z):

x: $$\rho \left[\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z}\right] = -\frac{\partial p}{\partial x} - \left(\frac{\partial}{\partial x}\tau_{xx} + \frac{\partial}{\partial y}\tau_{yx} + \frac{\partial}{\partial z}\tau_{zx}\right) + \rho g_x$$

y: $$\rho \left[\frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z}\right] = -\frac{\partial p}{\partial y} - \left(\frac{\partial}{\partial x}\tau_{xy} + \frac{\partial}{\partial y}\tau_{yy} + \frac{\partial}{\partial z}\tau_{zy}\right) + \rho g_y$$

z: $$\rho \left[\frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z}\right] = -\frac{\partial p}{\partial z} - \left(\frac{\partial}{\partial x}\tau_{xz} + \frac{\partial}{\partial y}\tau_{yz} + \frac{\partial}{\partial z}\tau_{zz}\right) + \rho g_z$$

Poenostavitve gibalne enačbe:

Eulerjev fluid (neviskozen)

$$\rho \frac{D\vec{v}}{Dt} = -\nabla p + \rho \vec{g}$$

Zapis strižnih tenzorjev za Newtonovski (viskozen) fluid:

$$\tau_{xx} = -2\mu \frac{\partial v_x}{\partial x} + \frac{2}{3}\mu(\nabla \cdot \vec{v})$$

$$\tau_{yy} = -2\mu \frac{\partial v_y}{\partial y} + \frac{2}{3}\mu(\nabla \cdot \vec{v})$$

$$\tau_{zz} = -2\mu \frac{\partial v_z}{\partial z} + \frac{2}{3}\mu(\nabla \cdot \vec{v})$$

$$\tau_{xy} = \tau_{yx} = -\mu\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right)$$

$$\tau_{xz} = \tau_{zx} = -\mu\left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right)$$

$$\tau_{yz} = \tau_{zy} = -\mu\left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right)$$

Newtonovski fluid (Navier-Stokesova enačba)

$$\rho\frac{D\vec{v}}{Dt} = -\nabla p + \mu\nabla^2\vec{v} + \rho\vec{g}$$

x: $$\rho\left[\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right] = -\frac{\partial p}{\partial y} - \mu\left(\frac{\partial^2}{\partial x^2}v_x + \frac{\partial^2}{\partial y^2}v_x + \frac{\partial^2}{\partial z^2}v_x\right) + \rho g_x$$

y: $$\rho\left[\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right] = -\frac{\partial p}{\partial y} - \mu\left(\frac{\partial^2}{\partial x^2}v_y + \frac{\partial^2}{\partial y^2}v_y + \frac{\partial^2}{\partial z^2}v_y\right) + \rho g_y$$

z: $$\rho\left[\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right] = -\frac{\partial p}{\partial y} - \mu\left(\frac{\partial^2}{\partial x^2}v_z + \frac{\partial^2}{\partial y^2}v_z + \frac{\partial^2}{\partial z^2}v_z\right) + \rho g_z$$

Naloga 1:

a.) Hitrostno polje toka tekočine je podano z enačbo:

$\vec{v} = (a_1x + b_1y + c_1z)i + (a_2x + b_2y + c_2z)j + (a_3x + b_3y + c_3z)k$

Izrazi pogoj pri katerem bo ta enačba predstavljala tok nestisljive tekočine:

$a_1 + b_2 + c_3 = 0$

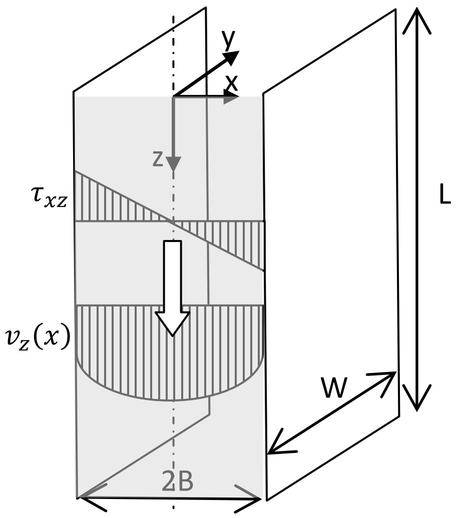
b.) Poznamo dve komponenti hitrostnega polja toka tekočine:

$v_x = a(x^2 + y^2)$ in $v_z = 0$

Izračunaj komponento hitrosti v smeri y:

$v_y = -2axy + C$

Naloga 2: Laminaren tok med vzporednima ploščama



Obravnavamo navpični, navzdol usmerjeni, laminarni, stacionarni, polnorazvit, dvodimenzionalen tok nestisljive tekočine med dvema vzporednima ploščama širine W, ki sta razmaknjeni za 2B. Tok je simetričen glede na ravnino x=0.

1. Zapišite gibalno enačbo za obravnavani primer:

$$0 = -\frac{\partial p}{\partial z} - \frac{\partial}{\partial x}\tau_{xz} + \rho g$$

2. Z integracijo gibalne enačbe določite odvisnost $\tau_{xz}(x)$. Pri tem upoštevajte, da je tok gibalne količine glede na smer toka na sredini med ploščama enak 0.

$$\tau_{xz}(x) = \left(\rho g - \frac{dp}{dz}\right)x$$

3. Upoštevajte, da je tekočina Newtonovska in zapišite odvisnost z-komponente hitrosti v smeri x:

$$v_z(x) = \frac{1}{2\mu}\left(\frac{dp}{dz} - \rho g\right)(x^2 - B^2)$$

4. Izpeljite masni pretok:

$$\dot{m} = \frac{2}{3}\frac{W\rho}{\mu}\left(\rho g - \frac{dp}{dz}\right)B^3$$

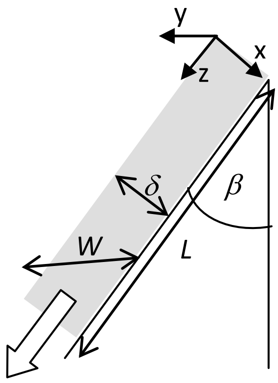
5. Izračunajte padec tlaka na razdalji L:

$$\Delta p = \left(\rho g - \frac{3}{2}\nu\frac{\dot{m}}{W^3}\right)L$$

6. Izračunajte celotno silo na plošči na razdalji L:

$$F_{tr} = \frac{3\nu\dot{m}}{B^2}$$

Naloga 3: Laminaren tok filma na poševni površini



Po ravni plošči širine W, nagnjeni pod kotom β (glej sliko) teče navzdol film kapljevine s konstantno debelino δ. Predpostavimo, da je tok tekočine nestisljiv, laminaren, stacionaren, polnorazvit in dvodimenzionalen.

1. Zapišite gibalno enačbo v z smeri:

$$0 = -\frac{\partial}{\partial x}\tau_{xz} + \rho g \cos\beta$$

2. Določite velikost strižne napetosti na steni. Predpostavite, da je strižna napetost na gladini filma enaka 0.

$$\tau_{xz}(x) = -\rho g \cos\beta\, x$$

3. Ob predpostavki Newtonovske tekočine določite profil hitrosti $v_z(x)$

$$v_z(x) = \frac{1}{2\mu}\rho g \cos\beta\,(\delta^2 - x^2)$$

4. Določite volumski pretok filma

$$\dot{V} = \frac{W\rho g \cos\beta}{3\mu}\delta^3$$

5. Določite povprečno in maksimalna hitrost filma ter razmerje med njima

$$\overline{v} = \frac{\rho g \cos\beta\delta^2}{3\mu}$$

$$v_{max} = \frac{1}{2\mu}\rho g \cos\beta\,\delta^2$$

$$\frac{v_{max}}{\overline{v}} = \frac{3}{2}$$

6. Določite silo trenja na ploščo

$$F_{tr} = -WL\rho g \cos\delta$$