Kontinuitetna enačba v cilindričnem koordinatnem sistemu $(\boldsymbol{r}, \boldsymbol{\theta}, \boldsymbol{z})$

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial \mathrm{r}}(\rho \mathrm{r} \mathrm{v_r}) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho \mathrm{v_\theta}) + \frac{\partial}{\partial \mathrm{z}}(\rho \mathrm{v_z}) = 0$$

Gibalna enačba v cilindričnem koordinatnem sistemu $(\boldsymbol{r}, \boldsymbol{\theta}, \boldsymbol{z})$

r: $$\rho\left[\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\vartheta}{r}\frac{\partial v_r}{\partial \theta} + \frac{{v_\theta}^2}{r} + v_z\frac{\partial v_r}{\partial z}\right] = -\frac{\partial p}{\partial r}$$

$$-\left[\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rr}) + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} - \frac{\tau_{\theta\theta}}{r} + \frac{\partial \tau_{rz}}{\partial z}\right] + \rho g_r$$

ϑ: $$\rho\left[\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}\right] = -\frac{1}{r}\frac{\partial p}{\partial \theta}$$

$$-\left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{r\theta}) + \frac{1}{r}\frac{\partial \tau_{\theta\theta}}{\partial \theta} - \frac{\tau_{\theta\theta}}{r} + \frac{\partial \tau_{\theta z}}{\partial z}\right] + \rho g_\theta$$

z: $$\rho\left[\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right] = -\frac{1}{r}\frac{\partial p}{\partial z}$$

$$-\left[\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) + \frac{1}{r}\frac{\partial \tau_{\theta z}}{\partial \theta} + \frac{\partial \tau_{zz}}{\partial z}\right] + \rho g_z$$

Komponente strižnega tenzorja za Newtonovski fluid:

$$\tau_{rr} = -\mu\left[2\frac{\partial v_r}{\partial r} - \frac{2}{3}(\nabla\cdot\vec{v})\right]$$

$$\tau_{\vartheta\vartheta} = -\mu\left[2\left(\frac{1}{r}\frac{\partial v_\vartheta}{\partial \vartheta} + \frac{v_r}{r}\right) - \frac{2}{3}(\nabla\cdot\vec{v})\right]$$

$$\tau_{zz} = -\mu\left[2\frac{\partial v_z}{\partial z} - \frac{2}{3}(\nabla\cdot\vec{v})\right]$$

$$\tau_{r\vartheta} = \tau_{\vartheta r} = -\mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\vartheta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \vartheta}\right]$$

$$\tau_{\vartheta z} = \tau_{z\vartheta} = -\mu\left[\frac{\partial v_\vartheta}{\partial z} + \frac{\partial v_z}{\partial \vartheta}\right]$$

$$\tau_{zr} = \tau_{rz} = -\mu\left[\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}\right],$$

kjer je

$$(\nabla\cdot\vec{v}) = \frac{1}{r}\frac{\partial}{\partial \mathrm{r}}(\mathrm{r v_r}) + \frac{1}{r}\frac{\partial \mathrm{v_\theta}}{\partial \theta} + \frac{\partial \mathrm{v_z}}{\partial \mathrm{z}}$$

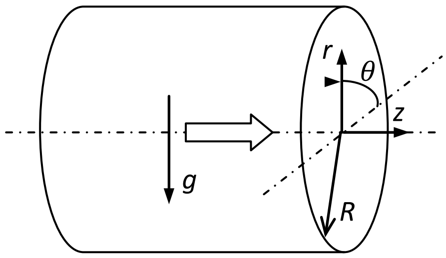
Navier-Stokesova enačba v cilindričnih koordinatah:

r: $$\rho\left[\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{{v_\theta}^2}{r} + v_z\frac{\partial v_r}{\partial z}\right] = -\frac{\partial p}{\partial r}$$
$$+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2}{\partial z^2}v_r\right] + \rho g_r$$

θ: $$\rho\left[\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}\right] = -\frac{1}{r}\frac{\partial p}{\partial \theta}$$
$$+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2}\right] + \rho g_\theta$$

z: $$\rho\left[\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right] = -\frac{1}{r}\frac{\partial p}{\partial z}$$
$$+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z$$

Naloga 1: Hagen- Poiseuillev tok



Obravnavamo laminaren, osnosimetričen, polnorazvit tok nestisljive tekočine v vodoravni cevi polmera *R*. Predpostavimo še, da se tlak prečno na smer toka ne spreminja.

1. Zapišite gibalno enačbo v smeri toka:

   $$0 = -\frac{\partial p}{\partial z} - \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz})$$

2. Določite strižno napetost $\tau_{rz}(r)$

   $$\tau_{rz} = -\frac{1}{2}r\frac{dp}{dz}$$

3. V primeru Newtonovske tekočine zapišite profil hitrosti:

   $$v_z = \frac{1}{4\mu}\left(\frac{dp}{dz}\right)[r^2 - R^2]$$

4. S pomočjo volumskega pretoka, izračunajte povprečno hitrost in izrazite razmerje med maksimalno in povprečno hitrostjo:

   $$\dot{V} = \frac{\pi}{8\mu}\left(\frac{dp}{dz}\right)R^4$$

   $$\overline{v} = -\frac{1}{8\mu}\left(\frac{dp}{dz}\right)R^2$$

   $$\frac{v_{max}}{\overline{v}} = 2$$

5. Zapišite tlačni padec na razdalji *L*.

   $$\Delta p = -\frac{8\mu\overline{v}}{R^2}L$$

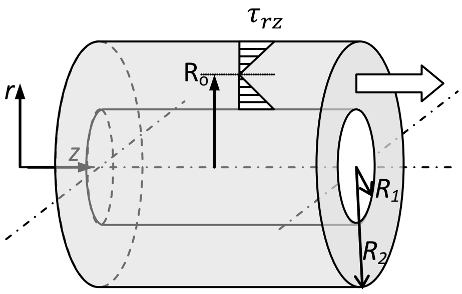
6. S pomočjo Darcy-Weisbachove enačbe in *Re* števila izračunajte koeficient linijski izgub λ za laminaren tok v cevi:

   Darcy-Wisbachova enačba: $p_1 - p_2 = \lambda\left(\frac{L}{D}\right)\frac{\rho\overline{v}^2}{2}$

   Reynoldsovo število za tok v cevi: $Re = -\frac{\overline{v}D}{\nu}L$

   Koeficient linijskih izgub: $\lambda = \frac{Re}{64}$

Naloga 2: Tok v obročni cevi



Rešujemo laminaren, osnosimetričen, polnorazvit tok nestisljive tekočine med dvema valjema polmerov R_1 in R_2 kot prikazuje slika. Pri tem upoštevajte, da je tok strižna napetost pri R_o enaka 0.

1. Gibalna enačba v smeri toka:

$$0 = -\frac{\partial p}{\partial z} - \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz})$$

2. Komponenta strižnega tenzorja:

$$\tau_{rz} = \frac{1}{2}\left(\frac{dp}{dz}\right)\left(\frac{{R_0}^2}{r} - r\right)$$

3. Profil hitrosti v smeri toka za Newtonovsko tekočino:

$$v_z = \frac{1}{2}\left(\frac{dp}{dz}\right)\left(\frac{1}{2}r^2 - {R_0}^2 \ln r\right) + C_2$$

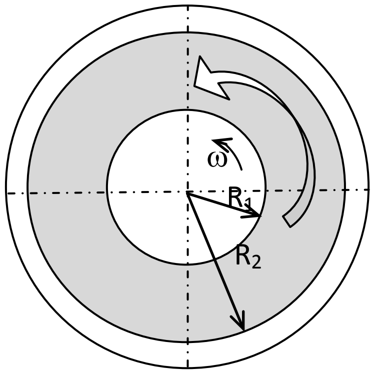
Ob upoštevanju robnih pogojev $v_z(R_1) = v_z(R_2) = 0$ sledi rešitev:

$$v_z = \frac{1}{4\mu}\frac{dp}{dz}\left[\left(r^2 - {R_1}^2\right) - \left(\frac{{R_2}^2 - {R_1}^2}{2\ln\frac{R_2}{R_1}}\right)\ln\frac{r}{R_1}\right]$$

4. Masni pretok

$$\dot{m} = \frac{\pi\rho}{8\mu}\frac{dp}{dz}\left[\left({R_1}^4 - {R_2}^4\right) + \frac{\left({R_2}^2 - {R_1}^2\right)^2}{\ln\frac{R_2}{R_1}}\right]$$

Naloga 3: Krožni Couettov tok



Določite profil hitrosti in strižno napetost med dvema koncentričnima valjema. Zunanji valj polmera R_2 miruje, notranji valj premera R_1 pa se vrti s konstantno kotno hitrostjo ω.

1. Profil hitrosti

   $$v_\theta(r) = \frac{1}{2}C_1 r + \frac{C_2}{r}$$

   Ob upoštevanju robnih pogojev $v_\theta(R_1) = \omega R_1$ in $v_\theta(R_2) = 0$ sledi rešitev:

   $$v_\theta(r) = \frac{\omega {R_1}^2}{\left({R_1}^2 - {R_2}^2\right)}\left(r - \frac{R_2^2}{r}\right)$$

2. Strižna napetost

   $$\tau_{r\theta} = -\mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)\right]$$

   $$\tau_{r\theta} = -\mu\frac{3\omega {R_1}^2 {R_2}^2}{\left({R_1}^2 - {R_2}^2\right)}\left(\frac{1}{r^2}\right)$$