Integralske metode analize – povprečenje po kontrolnem volumnu

Splošna enačba spremembe lastnosti ψ v kontrolnem volumnu:

$$\int_V \frac{\partial \psi}{\partial t} dV + \oint_A \psi(\vec{v}\cdot d\vec{A}) = -\oint_A \vec{\psi}\cdot d\vec{A} - \oint_A \vec{\psi}_0 \cdot d\vec{A} + \int_V \dot{\psi}_g dV$$

Masna bilanca

$\psi = \rho,\quad \vec{\psi} = 0,\quad \vec{\psi}_0 = 0,\quad \dot{\psi}_g = 0$

$$\int_V \frac{\partial \rho}{\partial t} dV + \oint_A \rho(\vec{v}\cdot d\vec{A}) = 0$$

Povprečenje po površini

$$\langle f\rangle = \frac{1}{A}\int_A f\, dA$$

$$\underbrace{\int_V \frac{\partial \rho}{\partial t} dV}_{\dot{m}} - \underbrace{A_1\rho_1\langle v\rangle_1}_{\dot{m}_1} + \underbrace{A_2\rho_2\langle v\rangle_2}_{\dot{m}_2} = 0$$

Stacionaren primer

$$\dot{m}_1 = \dot{m}_2$$

Gibalna enačba – ohranitev gibalne količine v volumnu

$$\int_V \frac{\partial \vec{\psi}}{\partial t} dV + \oint_A (\vec{v}\otimes\vec{\psi})dA = \oint_A \underline{\psi} d\vec{A} - \oint_A \psi_0 d\vec{A} + \int_V \dot{\vec{\psi}}_g dV$$

$\vec{\psi} = \rho\vec{v},\qquad \vec{\psi}_0 = 0,\qquad \underline{\psi} = \underline{P} = p\,\underline{I} + \underline{\tau}$

Splošen zapis

$$\int_V \frac{\partial(\rho\vec{v})}{\partial t} dV + \oint_A \rho\vec{v}(\vec{v}\cdot d\vec{A}) = -\oint_A \underline{P} d\vec{A} + \int_V \dot{\vec{\psi}}_g\, dV$$

Stacionaren primer – bilanca po kontrolnem volumnu

$$-\int_{A_1} \rho\vec{v}(\vec{v}\cdot d\vec{A}) + \int_{A_2} \rho\vec{v}(\vec{v}\cdot d\vec{A}) = -\int_{A_1} p\vec{n}dA + \int_{A_2} p\vec{n}dA - \vec{F}_w + \vec{F}_e$$

Povprečenje po ploskvi

$$A_1\rho_1\langle\vec{v}v\rangle_1 + A_2\rho_2\langle\vec{v}v\rangle_2 + A_1\langle p\rangle_1\vec{n}_1 + A_2\langle p\rangle_2\vec{n}_2 + \vec{F}_w - \vec{F}_e = 0$$

Upoštevamo masno bilanco

$$\rho_1 A_1 = \frac{\dot m}{\langle v\rangle_1} \quad \rho_2 A_2 = \frac{\dot m}{\langle v\rangle_2}$$

$$-\dot m \frac{\langle v^2\rangle_1}{\langle v\rangle_1}\cos\alpha_1 + \dot m \frac{\langle v^2\rangle_2}{\langle v\rangle_2}\cos\alpha_2 - A_1 p_1 \cos\alpha_1 + A_2 p_2 \cos\alpha_2 + F_{w_x} - F_{e_x} = 0$$

$C_v = \frac{\langle v\rangle^2}{\langle v^2\rangle}$ …. porazdelitveni koeficient hitrosti

Energijska bilanca

$$\psi = \rho\left(u + \frac{1}{2}v^2 + gz\right) = \rho e$$

$$\int_V \frac{\partial \rho e}{\partial t} dV = \dot E$$

$$\oint_A \rho e(\vec v \cdot d\vec A) = \int_{A_1} \rho e\left(\vec v \cdot d\vec A\right) + \int_{A_2} \rho e(\vec v \cdot d\vec A)$$

Splošen zapis

$$\dot E + \int_{A_1} \rho e\left(\vec v \cdot d\vec A\right) + \int_{A_2} \rho e\left(\vec v \cdot d\vec A\right) = \dot Q - \dot W_{tr} - \int_{A_1} p_1 \vec v \cdot d\vec A - \int_{A_2} p_2 \vec v \cdot d\vec A$$

Stacionaren primer ($\dot E = 0$)

$$\langle ev\rangle = \langle\left(u + \frac{1}{2}v^2 + gz\right)v\rangle = \langle uv\rangle + \frac{1}{2}\langle v^3\rangle + g\langle zv\rangle$$

$$A_1\rho_1\left[\langle uv\rangle_1 + \frac{1}{2}\langle v^3\rangle_1 + gz_1\langle v\rangle_1\right] - A_2\rho_2\left[\langle uv\rangle_2 + \frac{1}{2}\langle v^3\rangle_2 + gz_2\langle v\rangle_2\right] - \dot Q + \dot W_{tr} - A_1 p_1\langle v\rangle_1$$

Upoštevamo masno bilanco in entalpijo

$$\dot m = \rho_1 A_1 \langle v\rangle_1 \quad h = u + \frac{p}{\rho}$$

Bilančna Bernoullijeva enačba

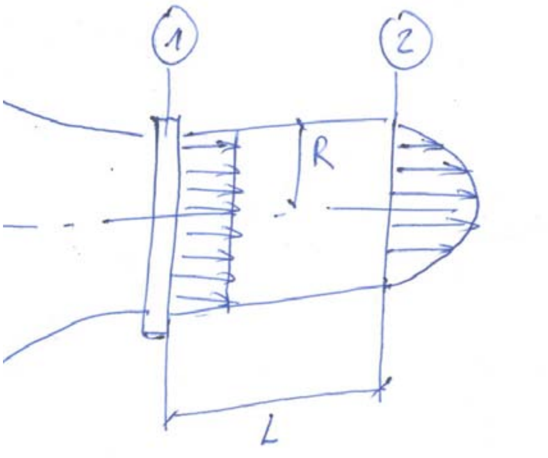
$$h_1 - h_2 + \frac{1}{2}\left(\frac{\langle v\rangle_1^2}{c_e} - \frac{\langle v\rangle_2^2}{c_e}\right) + g(z_1 - z_2) + \frac{\dot Q}{\dot m} - \frac{\dot W_{tr}}{\dot m} + \frac{p_1}{\rho_1} - \frac{p_2}{\rho_2} = 0$$

$C_e = \frac{\langle v\rangle^3}{\langle v^3\rangle}$ …. porazdelitveni koeficient energije

Ohranitev mehanske energije brez trenja

$$\frac{1}{2}\rho_1\langle v_1\rangle^2 - \frac{1}{2}\rho_2\langle v_2\rangle^2 + \rho g(z_1 - z_2) = 0$$

Naloga 1: Izstop iz cevi



Cev polmera R in dolžine L je pritrjena na izstop iz rezervoarja. Na prerezu (1) je začetna hitrost v_o konstantna, na prerezu (2) pa dobimo paraboličen profil hitrosti:

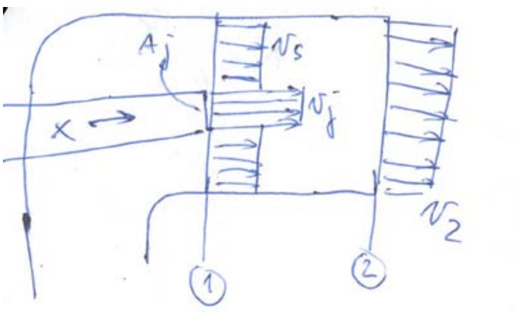
$$v(r) = Br^2 + C$$

Tok je stacionaren. Določi silo trenja na cev v odvisnosti od premera cevi, začetne hitrosti, tlaka in gostote.

Rešitev:

$$F_{w,x} = \pi R^2 \left[\rho\left(-\frac{1}{3}v_0^2\right) + p_1 - p_2\right]$$

Naloga 2: Vodni injektor



V primeru injektorja na sliki, voda, ki izstopa iz šobe preseka $A_j = 4{,}645 \cdot 10^{-3} m^2$ s hitrostjo $v_j = 27{,}4\ m/s$ vleče sekundarni tok, ki ima na prerezu (1) hitrost $v_s = 3\ m/s$. Tlak na prerezu (1) je $p_1 = 68950\ Pa$. Celotna površina toka na prerezu (2) je $A_2 = 55{,}74 \cdot 10^{-3} m^2$. Trenje zanemarimo in predpostavimo konstanten izstopni profil hitrosti v_2. Določi tlak in hitrost na prerezu (2).

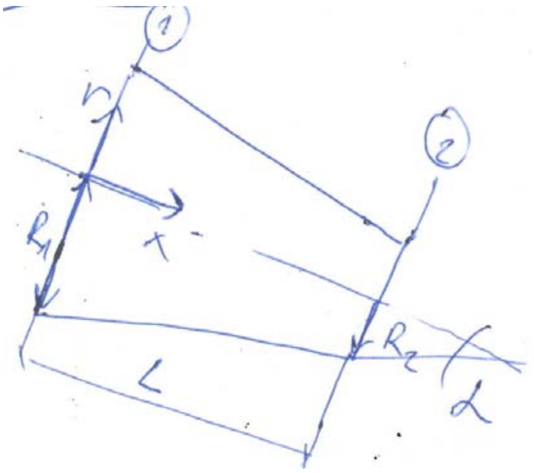
Rešitev:

$$p_2 = p_1 + \rho[\langle v^2\rangle_1 - \langle v^2\rangle_2]$$

$$\langle v^2\rangle_1 = \frac{1}{A}\left[{v_j}^2 A_j + {v_s}^2 (A - A_j)\right]$$

$$\langle v^2\rangle_2 = \frac{1}{A}\left[v_j A_j + v_s (A - A_j)\right]$$

3. Naloga: Cev spremenljivega prereza



Nestisljiva kapljevina se pretaka po osnosimetrični cevi spremenljivega prereza v smeri x. Poznamo masni pretok v cevi $\dot{m}$, gostoto ρ, tlaka na vstopu in izstopu $p_1$in p_2 ter geometrijo cevi (L, R_1, R_2, α). Na vsaki lokaciji se hitrost po preseku cevi spreminja po sledeči zakonitosti : $v(x,r) = v(x,0)\left(1-\frac{r}{R(x)}\right)^{\frac{1}{8}}$.

Oblika cevi je takšna, da se povprečna hitrost po prerezu spreminja na naslednji način: $\langle v\rangle_x = \langle v\rangle_1\left(1+k\frac{x}{L}\right)$. Določite silo trenja s katero deluje stena cevi na kapljevino.

Rešitev:

Sila trenja, s katero deluje stena cevi na kapljevino

$$F_{w,x} = \pi R_1^2 \rho \langle v^2\rangle_1 \cos\alpha - \pi R_2^2 \rho \langle v^2\rangle_2 \cos\alpha + A_1 p_1 \cos\alpha - A_2 p_2 \cos\alpha + g\frac{\rho\pi R_1^2 L}{\left(\frac{R_1^2}{R_2^2}-1\right)} \ln(1+\frac{R_1^2}{R_2^2}-1)\cdot sin\alpha$$

$$\langle v^2\rangle_1 = \frac{32}{45}\left(\frac{153}{128}\frac{\dot{m}}{\rho\pi R_1^2}\right)^2$$

$$\langle v^2\rangle_2 = \frac{32}{45}\left(\frac{153}{128}\frac{\dot{m}}{\rho\pi R_2^2}\right)^2$$