(3) $xy' + 3y = x^3y^2 \quad y(1) = 1$

$z := \frac{1}{y}$ (3) $\quad z' = -\frac{1}{y^2}y'$

$-xz' + 3z = x^3$

$z' - \frac{3}{x}z = -x^2$ (2) lin dif 1 redu, nehomogen

homogeni dio $\qquad$ variacija konstante

$z' = \frac{3}{x}z \qquad c'x^3 + 3cx^2 - 3cx^2 = -x^2$

$\frac{dz}{z} = 3\frac{dx}{x} \qquad c' = -\frac{1}{x}$, npr.

$c = -\ln x$ (4)

$z = cx^3$ (4)

(2)

$\Rightarrow z = Cx^3 - x^3\ln x$

$\Rightarrow y = 1/((C - \ln x)x^3)$ (2)

početni uvjet $\Rightarrow C = 1$. (2)

$\Rightarrow y = \frac{1}{x^3(1-\ln x)}$. (1)

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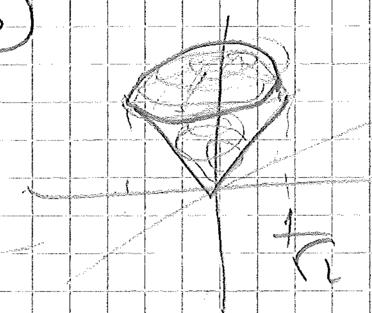
(4)

$x^2 + y^2 + (\sqrt{x^2+y^2})^2 = 1$

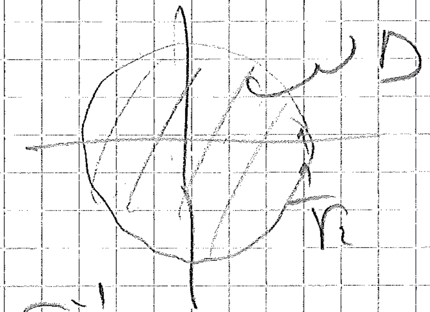
$x^2 + y^2 = \frac{1}{2} = \left(\frac{1}{\sqrt{2}}\right)^2$

(3)



$\iint_D (\sqrt{1-x^2-y^2} - \sqrt{x^2+y^2})\,dP$

(5)



$= \int_0^{\sqrt{2}^{-1}} dr\, r \int_0^{2\pi} d\varphi \left(\sqrt{1-r^2} - r\right)$ (4)

φ pdone

$= 2\pi \left[-\frac{1}{3}(1-r^2)^{3/2} - \frac{r^3}{3} \right]_0^{\sqrt{2}^{-1}}$ (3) (3)

$= 2\pi \left[\frac{1}{3}\left(1 - \frac{2}{2\sqrt{2}}\right) \right]$

(2)

$= \frac{2\pi}{3}\left(1 - \frac{1}{\sqrt{2}}\right)$

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(5)

a) $\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \times \left(3x^2 e^{-y} z, -x^3 e^{-y} z + 2y, x^3 e^{-y}\right)$

$\left(-x^3 e^{-y} + x^3 e^{-y}, 3x^2 e^{-y} - 3x^2 e^{-y}, -3x^2 e^{-y} z + 3x^2 e^{-y} z\right) = (0, 0, 0)$ (5) (5)

b) DA, rot $= 0$ na enostavno povezanem območju, (2)

(x_0, y_0, z_0)

$\vec{r}(t) = t(x_0, y_0, z_0), \quad t \in [0, 1]$

$\varphi(x_0, y_0, z_0) = \int_0^1 \vec{R}(\vec{r}(t)) \cdot \dot{\vec{r}}(t)\, dt$ (4)

$= \int_0^1 \left[3x_0^3 z_0 e^{-t y_0} t^3 - x_0^3 z_0 y_0 t^4 e^{-t y_0} + 2t y_0^2 + x_0^3 z_0 t^3 e^{-t y_0}\right] dt$ (2)

$= \frac{4 x_0^3 z_0}{y_0^4} \int_0^{y_0} e^{-u} u^3\, du - \frac{x_0^3 z_0}{y_0^4} \int_0^{y_0} e^{-u} u^4\, du + y_0^2$ (1)

$= \frac{x_0^3 z_0}{y_0^4} \left[\int_0^{y_0} (4u^3 - u^4) e^{-u}\, du\right] + y_0^2$

$= \frac{x_0^3 z_0}{y_0^4} \left(u^4 e^{-u} \Big|_0^{y_0}\right) + y_0^2$

$= x_0^3 z_0 e^{-y_0} + y_0^2$ (1)

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MATEMATIKA 2 (FMT) – 2. PISNI IZPIT

(1) $\frac{\partial f}{\partial x} = \left(4x - 2x(2x^2+y^2)\right) e^{-(x^2+y^2)} = 0$ (4)

$\frac{\partial f}{\partial y} = \left(2y - 2y(2x^2+y^2)\right) e^{-(x^2+y^2)} = 0$ (4)

$\Rightarrow \quad 2x = x(2x^2+y^2)$ &

$y = y(2x^2+y^2)$

$\Leftrightarrow \left(x = 0 \text{ ali } 2 = 2x^2+y^2\right)$ in

$\left(y = 0 \text{ ali } 1 = 2x^2+y^2\right)$. (6)

$\Rightarrow (x, y) \in \{(0,0), (0, \pm 1), (\pm 1, 0)\}$, (2) (2) (2)

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(2) $\vec{r}(t) = (t^2, t - t^3/3)$

$\dot{\vec{r}}(t) = (2t, 1 - t^2)$ (8)

$\vec{t}(t) = \frac{(2t, 1-t^2)}{\sqrt{4t^2 + (1-t^2)^2}} = \frac{(2t, 1-t^2)}{1+t^2}$

enačba tangente v točki $t = 2$

$\vec{r}(2) + u\, \vec{t}(2) = (4, -\frac{2}{3}) + w(4, -3)$ (4)

(4) (4) $w = \dots u$

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