(5)

(4)

$$\text{(3)} \Rightarrow \lambda^{\alpha+\beta} \, y^{\alpha+\beta} \left(\frac{\alpha}{w}\right)^{\alpha} \left(\frac{\beta}{r}\right)^{\beta} = y$$

$$\lambda = \left(\frac{w}{\alpha}\right)^{\frac{\alpha}{\alpha+\beta}} \left(\frac{r}{\beta}\right)^{\frac{\beta}{\alpha+\beta}} y^{\frac{1}{\alpha+\beta}-1}$$

$$\Rightarrow L^* = \left(\frac{\alpha}{w}\right)^{\frac{\beta}{\alpha+\beta}} \left(\frac{r}{\beta}\right)^{\frac{\beta}{\alpha+\beta}} y^{\frac{1}{\alpha+\beta}}$$

$$= \left(\frac{\alpha r}{w\beta}\right)^{\frac{\beta}{\alpha+\beta}} y^{\frac{1}{\alpha+\beta}}$$

$$K^* = \left(\frac{w\beta}{\alpha r}\right)^{\frac{\alpha}{\alpha+\beta}} y^{\frac{1}{\alpha+\beta}} \quad (5)$$

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(3) homogene $\lambda^2 - \lambda - 2 = 0$

$(\lambda - 2)(\lambda + 1) = 0$

$\lambda_1 = 2 \quad \lambda_2 = -1$ (4)

$y_H(x) = C_1 e^{2x} + C_2 e^{-x}$ (4)

Nastavek:

$y_P(x) = Ax^2 + Bx + C$ (4)

$2A - 2Ax - B - 2Ax^2 - 2Bx - 2C = x^2$ (4)

$A = -\frac{1}{2} \rightarrow B = \frac{1}{2} \rightarrow C = -\frac{3}{4}$ (4)

$$\rightsquigarrow y(x) = C_1 e^{2x} + C_2 e^{-x} - \frac{x^2}{2} + \frac{x}{2} - \frac{3}{4}.$$

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$p\,dx + q\,dy = 0$

$\mu p\,dx + \mu q\,dy = 0$

$$\underbrace{(3xy + y^2)}_{p}dx + \underbrace{(x^2 + xy)}_{q}dy = 0$$

$$\frac{\partial}{\partial y}(\mu p) = \frac{\partial}{\partial x}(\mu q)$$

$\mu p_y = \mu' q + \mu q_x$

$\mu(p_y - q_x) = \mu' q$

$$\frac{p_y - q_x}{q} = \frac{\mu'}{\mu} \quad (4)$$

$$\frac{d\mu}{\mu} = \frac{3x + 2y - 2x - y}{x^2 + xy}dx \quad (2)$$

$$= \frac{x+y}{x(x+y)}dx = \frac{dx}{x}$$

$\Rightarrow \mu = x$ (2) samo enega rabimo!

$$\underbrace{(3x^2y + y^2x)}_{\frac{\partial u}{\partial x}}dx + \underbrace{(x^3 + x^2y)}_{\frac{\partial u}{\partial y}}dy = 0 \text{ eksaktne.} \quad (2)$$

$\Rightarrow u = x^3y + \frac{1}{2}x^2y^2 + c(x)$ (4)

$$\frac{\partial u}{\partial x} = 3x^2y + xy^2 + c'(x) = 3x^2y + y^2x$$

$\Rightarrow c(x) = D = \text{konst.}$ (4)

$\Rightarrow u = x^3y + \frac{1}{2}x^2y^2 + D$ (1)

rešitev ($u = \text{const.}$)

$\Rightarrow \boxed{x^3y + \frac{1}{2}x^2y^2 = \text{const.}}$ (1)

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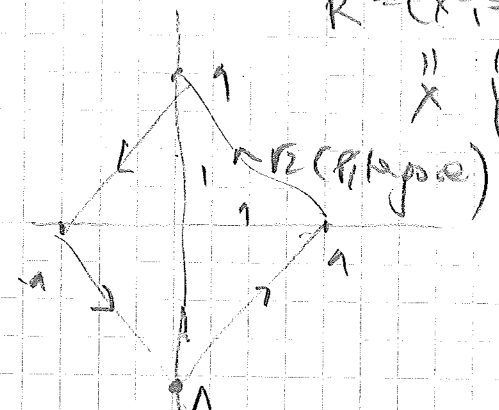
(5) $\vec{R}=(x^2,5x,0)$

$\quad\;\; X \;\; Y \;\; Z$



$\oint_{\partial K} \vec{R}\cdot d\vec{r} = \iint_K \langle \mathrm{rot}\,\vec{R}, \vec{n}\rangle\, dS$, $\vec{n} = (0,0,1)$

$= \iint_K \left(\frac{\partial Y}{\partial x} - \frac{\partial X}{\partial y}\right) dx\,dy$ (5)

$= \iint_K (5-0)\,dx\,dy = 5 \cdot$ ploščina kvadrata (5) (5)

$= 5\cdot(\sqrt{2})^2 = 10.$ (5)

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MATEMATIKA 2 (FMT) – 1. IZPIT

(1) $\kappa = \frac{|y''|}{(1+y'^2)^{3/2}} = \frac{12x^2}{(1+16x^6)^{3/2}}$ (4) (4)

$\lim_{\pm\infty} \kappa = 0 \qquad \kappa|_{\mathbb{R}\setminus\{0\}} > 0$

⇒ max, bo dosežen v stacionarni točki / ali $x \neq 0$

(5) ⇔ $24x(1+16x^6)^{3/2} - 12x^2\frac{3}{2}(1+16x^6)^{1/2}16\cdot 6\cdot x^5 = 0 \;/\cdot\frac{1}{24x(1+16x^6)^{1/2}}$

⇔ $(1+16x^6) - 12\cdot 6x^6 = 0$ (2)

⇔ $1 = 56x^6$

$x = \pm\sqrt[6]{1/56}$. (5)

⇒ $\kappa_{max} = \frac{12\sqrt[3]{1/56}}{(1+16\cdot\frac{1}{56})^{3/2}} = \frac{6}{\sqrt[3]{7}\,(1+\frac{2}{7})^{3/2}}$ (1)

$= \frac{2\cdot 7^{3/2}}{9\cdot 7^{1/3}} = \frac{2}{9}\,7^{7/6}.$

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(2) Lagrange

$wL + rK - \lambda(L^\alpha K^\beta - q)$ (5)

$w - \lambda\alpha L^{\alpha-1}K^\beta = 0$ (1) $\qquad L^\alpha K^\beta = q$ (3)

$r - \lambda\beta L^\alpha K^{\beta-1} = 0$ (2) (5)

(3)&(1) ⇒ $w - \lambda\alpha q/L = 0$ $\qquad$ (3)&(2) ⇒ $r - \lambda\beta q/K = 0$

$L = \frac{\alpha\lambda q}{w}$ $\qquad K = \frac{\lambda\beta q}{r}$