(b.2) ogljišča. Torej kandidati: (2)

x,y	$(1,1)$	$(1,-1)$	$(-1,1)$	$(-1,-1)$
f	$e\sin(1)$	$e^{-1}\sin(1)$	$e^{-1}\sin(1)$	$-e\sin(1)$

$(1,\frac{\pi}{4})$	$(-1,\pi/4)$
$-e^{-\pi/4}\frac{\sqrt{2}}{2}$	$e^{-\pi/4}\frac{\sqrt{2}}{2}$

Ker $e > e^{-1}, e^{-\pi/4}$, $\sin(1) > \sin(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$ $\Rightarrow$ gl. max v $(1,1)$ (1)

Podobno gl. min v $(-1,-1)$ (1).

↑ Lahko tudi ugenemo!

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(2)

$$\kappa(0) = \frac{y''(0)}{(1+(y'(0))^2)^{3/2}} \; (3) = \frac{e^0}{(1+(e^0)^2)^{3/2}} = \frac{1}{2\sqrt{2}} \; (4)$$

$y'(2) = e^2$ (2)

$\Rightarrow$ tangenta: $y - y(2) = y'(2)(x-2)$ (3)

$y = e^2 + e^2(x-2)$

$= e^2 x + (e^2 - 2)$ (2)

normala: $y - y(2) = (-1/y'(2))(x-2)$ (4)

$y = e^2 + e^{-2}(x-2)$ (2)

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(3) $V_0 = 100\,l$

$c_0 = 0.1\,kg/l$

(a) $\phi_1^* = 5\,l/min$

$c_1^* = 0.2\,kg/l$

$m_1^* = 15\,kg$

(b) $\phi_2^* = 5\,l/min$

$m_2^* = 5\,kg$

$c_2^* = 0$

masa soli v vodi

$dm_i = \phi_i^* c_i^* dt - \phi_i^* c_i dt \quad / V_0$

$dc_i = \frac{\phi_i^*}{V_0}(c_i^* - c_i)$ (5)

(a) $i=1$

$$\int_{c_0}^{m_1^*/V_0} \frac{dc_1}{c_1^* - c_1} \; (1) = \int_0^T \frac{\phi_1^*}{V_0}\, dt \; (2)$$

$$-\ln\frac{c_1^* - \frac{m_1^*}{V_0}}{c_1^* - c_0} = \frac{\phi_1^*}{V_0} T \; (2)$$

$$T = \frac{V_0}{\phi_1^*} \ln\left(\frac{1 - \frac{m_1^*}{V_0 c_1^*}}{1 - \frac{c_0}{c_1^*}}\right)^{-1} \; (2)$$

$= 20\,min \ln\left(\frac{1/4}{1/2}\right)^{-1} = 20 \ln 2\; min$

$\doteq 13.86\,min$

(b) $i=2$

$$\int_{c_0}^{m_2^*/V_0} \frac{dc_2}{c_2} \; (1) = -\int_0^T \frac{\phi_2^*}{V_0}\, dt \; (2)$$

$$-\ln\frac{m_2^*/V_0}{c_0} = \frac{\phi_2^*}{V_0} T \; (2)$$

$$T = \frac{V_0}{\phi_2^*} \ln\frac{c_0}{m_2^*/V_0} = 20\ln 2\; min \; (2)$$

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MATEMATIKA 2 (FMT) – 3. PISNI IZPIT

$e^{xy}\sin(y)$ $[-1,1]^2$.

KANDIDATI

(1) (a) v notranjosti, stacionarne točke (2)

(2) $\begin{cases} \partial_x f = y e^{xy} \sin(y) = 0 \\ \partial_y f = e^{xy}(x\sin(y) + \cos(y)) = 0 \end{cases}$

$e^{xy} > 0$

$\Leftrightarrow$ $y\sin(y) = 0$ & $x\sin(y) = -\cos(y)$ (1)

$\Rightarrow y \neq 0$

$\Rightarrow \sin(y) = 0$

ni stacionarnih točk (1)

(b) na robu: (1)

(b.1) v notranjosti daljic, stacionarne točke (1)

• $\{1\}\times(-1,1)$ $\frac{d}{dy}\left(e^{y}\sin(y)\right) = 0$ (1)

$\Leftrightarrow$ $e^{y}(\sin(y) + \cos(y)) = 0$ (1)

$\Leftrightarrow$ $\sin(y) + \cos(y) = 0$

$\Leftrightarrow$ $y = -\pi/4$ (1)

$\hookrightarrow y \in (-1,1)$

• $\{-1\}\times(-1,1)$ $\frac{d}{dy}\left(e^{-y}\sin(y)\right) = 0$ (1)

$\Leftrightarrow$ $e^{-y}(-\sin(y) + \cos(y)) = 0$ (1)

$\Leftrightarrow$ $\sin(y) = \cos(y)$

$\Leftrightarrow$ $y = \pi/4$ (1)

$y \in (-1,1)$

• $(-1,1)\times\{1\}$ $\frac{d}{dx}\left(e^{x}\sin(1)\right) = 0$ / ne bo rešitev (1)

• $(-1,1)\times\{-1\}$ $\frac{d}{dx}\left(e^{x}\sin(-1)\right) = 0$ / — II — (1)

(4) $\int_0^\infty dx\, e^{-x} \int_0^x dy \left(\frac{y}{x}\right)^{a-1}\left(1-\frac{y}{x}\right)^{b-1} x^{b-1}\, x^{a-1}\, \frac{x}{x}$ (5)

$= \int_0^\infty dx\, e^{-x} \int_0^1 u^{a-1}(1-u)^{b-1}\, du\; x^{a+b-2}\, x$ (5)

$= \int_0^\infty dx\, e^{-x}\, B(a,b)\, x^{a+b-1}$ (5)

$= B(a,b)\,\Gamma(a+b) = \Gamma(a)\Gamma(b)$. (5)

20.

(5.) $\iiint_C \nabla\cdot\vec{R}\, dV$

$\vec{R} = (x\sqrt{x^2+y^2},\, y\sqrt{x^2+y^2},\, 0)$

$C =$; $h = 5$; $r = R$.

$= \iint_{\partial C} \vec{R}\cdot\vec{n}\, dS$ (6)

$\vec{n} = \frac{(x,y)}{\sqrt{x^2+y^2}}$ na plašču $\parallel \vec{R}$ (6)

$= R^2\, pl$ (4)

$\vec{n} = (0,0,\pm 1)$ na o.p. $\perp \vec{R}$

$= R^2\, h\, 2\pi R$ (4)

$= 2\pi h R^3$

$= 2\pi\, 5\cdot 4 = \underline{40\pi}$.

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