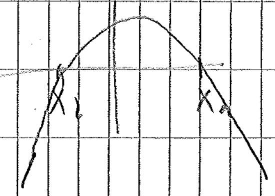
2.1 (3) $x \in (0,\infty)$;; $\log : (0,\infty) \to \mathbb{R}$

(6) $-x^2+5x+2 \geq 0$;; $\sqrt{\cdot} : [0,\infty) \to [0,\infty)$, $1/\cdot : \mathbb{R}\setminus\{0\} \to \mathbb{R}$

$$x_{1,2} = \frac{-5 \pm \sqrt{25+8}}{-2} = \begin{cases} (5+\sqrt{33})/2 = x_1 \\ (5-\sqrt{33})/2 = x_2 \end{cases}$$

$$\Leftrightarrow \quad x \in \left(\frac{5-\sqrt{33}}{2}, \frac{5+\sqrt{33}}{2} \right) \quad (3)$$



Tada

$$D_f = (0,\infty) \cap \ldots = \left(0, \frac{5+\sqrt{33}}{2}\right) \quad (3)$$

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4. f je integrabilna, i u točkama dolazi $\tilde{f}$ pored konvergentne! (3)

a)
$$\tilde{f}(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{2\pi n x}{b-a} + \sum_{n=1}^{\infty} b_n \sin\frac{2\pi n x}{b-a}$$

$a=-1$, $b=2$

$$a_0 = \frac{2}{b-a}\int_a^b f(x)\,dx = \frac{1}{3}(4-1) = 1 \quad (3)$$

Za $n \in \mathbb{N}$:

$$a_n = \frac{2}{b-a}\int_a^b f(x)\cos\left(\frac{2\pi n}{b-a}x\right)dx$$

$$= \frac{2}{3}\int_{-1}^{2} x\cos\left(\frac{2\pi n}{3}x\right)dx = \frac{1}{\pi n}\left[2\sin\left(\frac{4\pi n}{3}\right) - \sin\left(\frac{2\pi n}{3}\right) + \frac{3}{2\pi n}\left(\cos\left(\frac{4\pi n}{3}\right) - \cos\left(\frac{2\pi n}{3}\right)\right)\right]$$

$$\left(x\sin\left(\frac{2\pi n}{3}x\right)\right)' = \sin\left(\frac{2\pi n}{3}x\right) + x\cos\left(\frac{2\pi n}{3}x\right)\frac{2\pi n}{3} \quad (6)$$

$$b_n = \frac{2}{b-a}\int_a^b f(x)\sin\frac{2\pi n x}{b-a}\,dx$$

$$= \frac{2}{3}\int_{-1}^{2} x \sin\left(\frac{2\pi n}{3}x\right)dx$$

$$\left(x\cos\left(\frac{2\pi n}{3}x\right)\right)' = \cos\left(\frac{2\pi n}{3}x\right) - x\frac{2\pi n}{3}\sin\left(\frac{2\pi n}{3}x\right)$$

$$= \frac{1}{\pi n}\left[\frac{3}{2\pi n}\left(\sin\left(\frac{4\pi n}{3}\right) + \sin\left(\frac{2\pi n}{3}\right)\right) - \left(2\cos\left(\frac{4\pi n}{3}\right) + \cos\left(\frac{2\pi n}{3}\right)\right)\right]$$

$$a_n = -\frac{3}{\pi n}\sin\left(\frac{2\pi n}{3}\right) = -\frac{3}{\pi n}\begin{cases} \sqrt{3}/2 & n \equiv 1\ (3) \\ 0 & n \equiv 0\ (3) \\ -\sqrt{3}/2 & n \equiv -1\ (3) \end{cases} \quad (1)$$

$$b_n = -\frac{3}{\pi n}\cos\left(\frac{2\pi n}{3}\right) = -\frac{3}{\pi n}\begin{cases} -\frac{1}{2} & n \equiv 1\ (3) \\ 1 & n \equiv 0\ (3) \\ -\frac{1}{2} & n \equiv -1\ (3) \end{cases} \quad (1)$$

(6)

5 (b) $\tilde{f}(-1) = \dfrac{f(2)+f(-1)}{2} = \dfrac{2+(-1)}{2} = \dfrac{1}{2}$ (3)

$\tilde{f}(0) = f(0) = 0$ (4)

$\tilde{f}(2) = \tilde{f}(-1) = \frac{1}{2}$ (3)

$\boxed{30}$

5.
$$\frac{\partial f}{\partial r} = \left(4r^3\cos^4\theta + 2r\cos^2\theta + 4r^3\cos^2\theta\sin^2\theta\right) + \left(4r^3\sin^4\theta + 2r\sin^2\theta + 4r^3\sin^2\theta\cos^2\theta\right) \quad (5)$$

$$= 4r^3\left(\cos^4\theta + 2\cos^2\theta\sin^2\theta + \sin^4\theta\right) + 2r\left(\cos^2\theta+\sin^2\theta\right) = 4r^3 + 2r$$

$$\frac{\partial f}{\partial \theta} = \left(4r^4\cos^3\theta + 2r^2\cos\theta + 4r^4\cos\theta\sin^2\theta\right)(-\sin\theta) + \left(4r^4\sin^3\theta + 2r^2\sin\theta + 4r^4\sin\theta\cos^2\theta\right)(\cos\theta) = 0 \quad (5)$$

#16 NO CALCULATORS

(b) $\frac{\partial F}{\partial \theta} = 0 \Rightarrow F$ je konstantna v 2. koordinati.
(10)

(c) Če vzamemo $\theta = 0$, $x(r,\theta) = r$, $y(r,\theta) = 0$,
torej
$g(r) = F(r,0) = \underline{r^4 + r^2}$.
(5)

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MAT 2 (FMT) – 1. kolokvij (4.12.2014) REŠITVE

1. DA. (5)
(0) $d: \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}$, za $\{x,y\} \subset \mathbb{R}^3$

(1) $d \geq 0$, $d(x,y) = 0 \iff x = y$ (ker $|x_1 - y_1| + |x_2^3 - y_2^3| + |x_3^5 - y_3^5| = 0$
$\Rightarrow |x_1 - y_1| = 0$ in $|x_2^3 - y_2^3| = 0$ in $|x_3^5 - y_3^5| = 0 \Rightarrow$
$x_1 = y_1$, $x_2^3 = y_2^3$ & $x_3^5 = y_3^5 \Rightarrow x_1 = y_1$ & $x_2 = y_2$ & $x_3 = y_3$
(uporabimo: $a + b = 0$, $a \geq 0, b \geq 0 \Rightarrow a = b = 0$ in injektivnost $^3, ^5$)). (5)

(2) $d(x,y) = d(y,x)$ za $\{x,y\} \subset \mathbb{R}^3$. (5)

(3) trikotniška neenakost: $\{x,y,z\} \subset \mathbb{R}^3$: (5)

$d(x,z) = |x_1 - z_1| + |x_2^3 - z_2^3| + |x_3^5 - z_3^5|$

$\leq |x_1 - y_1| + |y_1 - z_1| + |x_2^3 - y_2^3| + |y_2^3 - z_2^3| +$
$|x_3^5 - y_3^5| + |y_3^5 - z_3^5|$

$= d(x,y) + d(y,z)$

$(\mathbb{R}, |\cdot|)$ je metrični prostor

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3. Ima, limita je $(1, 0, 0)$ (5)
(5)

limita $\mathcal{E}$, $n^2 \to \infty$ $\therefore 1/n^2 \to 0$

sin omejena, $e^{-\sqrt{n}} \to 0$, ker $\sqrt{n} \to \infty$. (5)

$\log(n+1) \to \infty$, torej $1/\log(n+1) \to 0$.

(5)

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