(3) stacionarne točke $\equiv \nabla = 0$

$4x^3 + 4y = 0$ in $4y^3 + 4x = 0$

$(x,y) \in \{(0,0), (1,-1), (-1,1)\}$ (5)

Hessejeva matrika $\begin{bmatrix} 12x^2 & 4 \\ 4 & 12y^2 \end{bmatrix}$ (5)

$\nabla(0,0) : -16 < 0 \to$ ni lok ekstrem! (5)

$\nabla(1,-1)$ in $(-1,1)$: $144 - 16 > 0$ (5)

obakrat $f_{xx} > 0$

torej oba lokalna minimuma!

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(4) $\min\ 2\pi r^2 + 2\pi r h$

p.p. $\pi r^2 h = V_0$

Lagrange: $2\pi r^2 + 2\pi r h + \lambda(V_0 - \pi r^2 h)$ (5)

$4\pi r + 2\pi h = \lambda 2\pi r h$

$2\pi r = \lambda \pi r^2$ (10)

$\lambda = \frac{2}{r}$

$V_0 = \pi r^2 h$

$2r + \cancel{2h} = 2h$ (5)

$\boxed{h = 2r}$

$r = \sqrt[3]{\frac{V_0}{2\pi}}$

tudi $2\pi r^2 + \frac{2V_0}{r}$, odvajamo

$h = \sqrt[3]{8 V_0/\pi}$

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(5) $\vec{r}(t) = (x(t), y(t), z(t))$

(a) $\vec{t} = \frac{\dot{\vec{r}}}{\|\dot{\vec{r}}\|}$ $\qquad \vec{b} = \frac{\dot{\vec{r}} \times \ddot{\vec{r}}}{\|\dot{\vec{r}} \times \ddot{\vec{r}}\|}$

$\vec{n} = \vec{b} \times \vec{t}$ (5) each

$= \frac{(t^3/3,\ t^2/2,\ t)}{\sqrt{\dots}}$

$= \frac{(2t^3, 3t^2, 6t)}{\sqrt{4t^6 + 9t^4 + 36t^2}} = \frac{(2t^2, 3t, 6)}{\sqrt{4t^4 + 9t^2 + 36}}$

$\ddot{\vec{r}} = (t^2, t, 1)$

∇ $t = 0$ $(\vec{t}, \vec{n}, \vec{b})$ ni definiran!

$(2t^3, 3t^2, 6t) \times (t^2, t, 1)$

$= (3t^2 - 6t^2, 6t^3 - 2t^3, 2t^4 - 3t^4) = (-3t^2, 4t^3, -t^4)$

$\vec{b} = \frac{(-3t^2, 4t^3, -t^4)}{\sqrt{9t^4 + 16t^6 + t^8}} = \frac{(-3, 4t, -t^2)}{\sqrt{9 + 16t^2 + t^4}}$

$\vec{n} = \vec{b} \times \vec{t}$

$= \frac{(-3, 4t, -t^2) \times (2t^2, 3t, 6)}{\sqrt{\dots} \cdot \sqrt{\dots}}$

$= \frac{(24t + 3t^3, -2t^4 + 18, -9t - 8t^3)}{\sqrt{(4t^4 + 9t^2 + 36)(9 + 16t^2 + t^4)}}$

(b) $\vec{r}(2) = \left(\frac{8}{3}, \frac{4}{3}, 3\right)$ (1)

$\boxed{R(c) = \vec{r}(2) + c\,\vec{t}(2)}$ (3)

$\vec{t}(2) = \left(\frac{8}{2\sqrt{34}}, \frac{6}{2\sqrt{34}}, \frac{6}{2\sqrt{34}}\right)$

$\sqrt{64 + 36 + 36} = \sqrt{136} = 2\sqrt{34}$

$= \frac{1}{\sqrt{34}}(4, 3, 3)$ (1)

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MATEMATIKA 2 (FMT) – 2. KOLOKVIJ – REŠITVE

(1) (a) $(\nabla T)(x,y,z) = -T_0\,(4x,\ 2y,\ 6z)\, e^{-(2x^2+y^2+3z^2)}$. (10)

(b) $\hat{e} = \dfrac{(1,-1,\sqrt{2})}{\sqrt{1+1+2}} = \left(\frac{1}{2}, -\frac{1}{2}, \frac{1}{\sqrt{2}}\right)$ enotska smer.

$P = (1,0,2)$

$$\frac{d\,T(P+t\hat{e})}{dt} = \langle \nabla T(P), \hat{e} \rangle = -T_0\left(\frac{1}{2}, -\frac{1}{2}, \frac{1}{\sqrt{2}}\right)\cdot(4,0,12)\cdot e^{-(2+0+12)}$$

$$= -T_0\left(2 + \frac{12}{\sqrt{2}}\right)e^{-14}$$

$$= -T_0\,\frac{12+2\sqrt{2}}{\sqrt{2}}\,e^{-14} = -2T_0\,\frac{6+\sqrt{2}}{\sqrt{2}}\,e^{-14}$$ (5)

(c) raste: v smeri gradienta: (5)

$$-\frac{(4,0,12)}{\sqrt{16+144}}T_0 = -\frac{(4,0,12)}{\sqrt{160}}T_0 = -\frac{(1,0,3)}{\sqrt{10}}T_0$$ (3)

pade: v nasprotni smeri od gradienta $+(1,0,3)/\sqrt{10}\cdot T_0$ (2)

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(2) $p = \dfrac{nRT}{V} = p(V,T)$ (5)

$p(V,T) \overset{\text{TAYLOR DO 2. REDA}}{=}$

$$p(V_0,T_0) + \frac{\partial p}{\partial V}(V_0,T_0)(V-V_0) + \frac{\partial p}{\partial T}(V_0,T_0)(T-T_0)$$

$$+ \frac{1}{2}\frac{\partial^2 p}{\partial V^2}(V_0,T_0)(V-V_0)^2 + \frac{1}{2}\frac{\partial^2 p}{\partial T^2}(V_0,T_0)(T-T_0)^2$$

$$+ \frac{\partial^2 p}{\partial V \partial T}(V_0,T_0)(V-V_0)(T-T_0)$$ (10)

$$= \frac{nRT_0}{V_0} - \frac{nRT_0}{V_0^2}(V-V_0) + \frac{nR}{V_0}(T-T_0) + \frac{nRT_0}{V_0^3}(V-V_0)^2$$

$$- \frac{nR}{V_0^2}(T-T_0)(V-V_0).$$ (5)

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