Determinante

79. Dana je matrika

$$A = \begin{bmatrix} 1 & 1 & 3 & 2 \\ 0 & 0 & 2 & 1 \\ 0 & 2 & a & 0 \\ 1 & a & 2 & 1 \end{bmatrix}.$$

Določi a tako, da bo matrika obrnljiva.

80. Reši enačbo, v kateri nastopa determinanta velikosti $n \times n$

$$\begin{vmatrix} 1 & 1 & 1 & \cdots & 1 & 1 \\ 1 & 1-x & 1 & \cdots & 1 & 1 \\ 1 & 1 & 2-x & \ddots & 1 & 1 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 1 & 1 & 1 & \ddots & n-2-x & 1 \\ 1 & 1 & 1 & \cdots & 1 & n-1-x \end{vmatrix} = 0$$

81. Izračunaj determinanto velikosti $n \times n$

$$\begin{vmatrix} -1 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & -1 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & -1 & 1 & \cdots & 0 & 0 \\ 0 & 0 & 0 & -1 & \ddots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \ddots & -1 & 1 \\ 1 & 1 & 1 & 1 & \cdots & 1 & 1 \end{vmatrix}$$

82. Izračunaj determinanto velikosti $n \times n$

$$\begin{vmatrix} 1 & 1 & 1 & \cdots & 1 & 1 \\ 1 & 0 & 1 & \cdots & 1 & 1 \\ 1 & 1 & 0 & \ddots & 1 & 1 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 1 & 1 & 1 & \ddots & 0 & 1 \\ 1 & 1 & 1 & \cdots & 1 & 0 \end{vmatrix}$$

83. Izračunaj determinanto velikosti $n \times n$

$$\begin{vmatrix} 1 & 2 & 3 & \cdots & n-2 & n-1 & n \\ 2 & 3 & 4 & \cdots & n-1 & n & 1 \\ 3 & 4 & 5 & \cdots & n & 1 & 2 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ n-2 & n-1 & n & \cdots & n-5 & n-4 & n-3 \\ n-1 & n & 1 & \cdots & n-4 & n-3 & n-2 \\ n & 1 & 2 & \cdots & n-3 & n-2 & n-1 \end{vmatrix}.$$

84. Naj bo n naravno število, večje od 2. Izračunaj naslednjo determinanto velikosti $n \times n$, ki ima na neoznačenih mestih ničle:

$$\begin{vmatrix} 1 & 1 & & & & \\ -2 & 1 & 1 & & & \\ & -2 & 1 & 1 & & \\ & & \ddots & \ddots & \ddots & \\ & & & -2 & 1 & 1 \\ 1 & & & & -2 & 1 \end{vmatrix}.$$

85. Izračunaj $2n \times 2n$ determinanto

$$\begin{vmatrix} 1 & 0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 2 & \cdots & 0 & 2 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & n-1 & n-1 & 0 & 0 & \cdots & 0 \\ 1 & 2 & \cdots & n-1 & n & n+1 & n+2 & \cdots & 2n \\ 0 & 0 & \cdots & 0 & n+1 & n+1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & n+2 & 0 & n+2 & \cdots & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 2n & 0 & 0 & \cdots & 2n \end{vmatrix}.$$

86. Dana je matrika $A = [a_{ij}]_{i,j=1}^{n} \in \mathbb{R}^{n,n}$. Naj bo

$$p(x) = \begin{vmatrix} a_{11}+x & a_{12}+x & \cdots & a_{1n}+x \\ a_{21}+x & a_{22}+x & \cdots & a_{2n}+x \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1}+x & a_{n2}+x & \cdots & a_{nn}+x \end{vmatrix}.$$

(a) Dokaži, da je p linearen polinom v spremenljivki x.
(b) Dokaži: $p(x) = (1-x)\det A + xp(1)$.