# Formule za Analizo 1

Na listu so zbrane nekatere bolj zapletene formule, za katere ne pričakujemo, da se jih boste naučili na pamet. Seveda pa morate znati vse ostale formule.

## Trigonometrija

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \qquad \sin\alpha\cos\beta = \frac{1}{2}\big(\sin(\alpha+\beta)+\sin(\alpha-\beta)\big)$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \qquad \cos\alpha\cos\beta = \frac{1}{2}\big(\cos(\alpha+\beta)+\cos(\alpha-\beta)\big)$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2} \qquad \sin\alpha\sin\beta = \frac{1}{2}\big(\cos(\alpha-\beta)-\cos(\alpha+\beta)\big)$$

## Integral

$$\int e^{ax}\sin(bx)\,dx = \frac{e^{ax}}{a^2+b^2}\big(a\sin(bx) - b\cos(bx)\big) + C$$

$$\int e^{ax}\cos(bx)\,dx = \frac{e^{ax}}{a^2+b^2}\big(a\cos(bx) + b\sin(bx)\big) + C$$

$$\int \frac{dx}{\sqrt{ax^2+bx+c}} = \begin{cases} \frac{1}{\sqrt{a}}\ln\left|2ax+b+2\sqrt{a}\sqrt{ax^2+bx+c}\right| + C & \text{za } a>0 \\ -\frac{1}{\sqrt{-a}}\arcsin\frac{2ax+b}{\sqrt{D}} + C & \text{za } a<0\,,\ D=b^2-4ac \end{cases}$$

Trigonometrični substituciji v integrale tipa $\int R\big(\sin x, \cos x\big)\,dx$:

$$u = \operatorname{tg}\frac{x}{2},\quad \sin x = \frac{2u}{1+u^2},\quad \cos x = \frac{1-u^2}{1+u^2},\quad dx = \frac{2du}{1+u^2}$$

$$t = \operatorname{tg} x,\quad \sin^2 x = \frac{t^2}{1+t^2},\quad \cos^2 x = \frac{1}{1+t^2},\quad dx = \frac{dt}{1+t^2}$$

Eulerjeva substitucija v integrale tipa $\int R\big(x, \sqrt{ax^2+bx+c}\big)\,dx$:

$$a>0:\ \sqrt{ax^2+bx+c} = \sqrt{a}(u-x)$$

$$a<0:\ \sqrt{ax^2+bx+c} = \sqrt{a(x-x_1)(x-x_2)} = \sqrt{-a}(x-x_1)u$$

Dolžina loka, ploščina izseka, prostornina in površina vrtenine (okrog $x$ osi):

– parametrično podane krivulje

$$s = \int \sqrt{\dot{x}^2(t)+\dot{y}^2(t)}\,dt \qquad p = \frac{1}{2}\int (x(t)\dot{y}(t) - y(t)\dot{x}(t))\,dt$$

$$V = \pi\int y^2(t)\dot{x}(t)\,dt \qquad P = 2\pi\int y(t)\sqrt{\dot{x}^2(t)+\dot{y}^2(t)}\,dt$$

– krivulje podane v polarnih koordinatah

$$s = \int \sqrt{r^2(t)+\dot{r}^2(t)}\,dt \qquad p = \frac{1}{2}\int r^2(t)\,dt$$

$$V = \pi\int r^2(t)\sin^2 t(\dot{r}(t)\cos t - r(t)\sin t)\,dt \qquad P = 2\pi\int r(t)\sin t\sqrt{r^2(t)+\dot{r}^2(t)}\,dt$$

## Taylorjeve vrste

$$e^x = \sum_{n=0}^{\infty}\frac{x^n}{n!}\ (R=\infty) \qquad \sin x = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}\ (R=\infty)$$

$$\cos x = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}\ (R=\infty) \qquad \ln(1+x) = \sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n}\ (R=1)$$

$$(1+x)^\alpha = \sum_{n=0}^{\infty}\binom{\alpha}{n}x^n\ (R=1)$$