Analiza 1
20. domača naloga

(1) Izračunaj naslednje integrale

(a) $\int (\sqrt{x}+1)(x-\sqrt{x}+1)\,dx$ (b) $\int \left(3^{4-5x}+\frac{e^{a/x}}{x^2}\right)dx$ (c) $\int \frac{1}{3x-\sqrt{x}}\,dx$

(d) $\int e^x\sqrt{a-be^x}\,dx$ (e) $\int \frac{dx}{\sqrt{(1+x^2)\ln(x+\sqrt{1+x^2})}}$ (f) $\int \operatorname{tg}^3 x\,dx$

(g) $\int \frac{1}{x^{5/4}-x^{3/4}}\,dx$ (h) $\int \frac{\sqrt{2+x^2}-\sqrt{2-x^2}}{\sqrt{4-x^4}}\,dx$ (i) $\int \frac{x}{\sqrt{a^2-x^2}}\,dx$

(j) $\int e^{3-x^2}x\,dx$ (k) $\int \frac{1-3x}{3+2x}\,dx$ (l) $\int x\ln\frac{2-x}{2+x}\,dx$

(m) $\int \frac{\sqrt{x}+\ln x}{x}\,dx$ (n) $\int \frac{e^{\operatorname{arctg} x}+x\ln(1+x^2)+1}{1+x^2}\,dx$ (o) $\int \frac{x\cos x}{\sin^2 x}\,dx$

(p) $\int \sqrt{\frac{\arcsin x}{1-x^2}}\,dx$ (q) $\int \arcsin x\,dx$ (r) $\int \frac{x-\operatorname{arctg}^3 2x}{1+4x^2}\,dx$

(a) $\frac{2}{5}x^{5/2}+x+C$	(b) $C-\frac{3^{4-5x}}{5\ln 3}-\frac{1}{a}e^{a/x}$	(c) $\frac{2}{3}\ln\lvert 3\sqrt{x}-1\rvert+C$
(d) $C-\frac{2}{3b}(a-be^x)^{3/2}$	(e) $2\sqrt{\ln(x+\sqrt{1+x^2})}+C$	(f) $\frac{1}{2\cos^2 x}+\ln\lvert\cos x\rvert+C$
(g) $2\ln\left\lvert\frac{\sqrt[4]{x}-1}{\sqrt[4]{x}+1}\right\rvert+C$	(h) $\arcsin\frac{x}{\sqrt{2}}-\ln(x+\sqrt{2+x^2})+C$	(i) $C-\sqrt{a^2-x^2}$
(j) $C-\frac{1}{2}e^{3-x^2}$	(k) $-\frac{3}{2}x+\frac{11}{4}\ln\lvert 3+2x\rvert+C$	(l) $\frac{x^2}{2}\ln\frac{2-x}{2+x}-2x+2\ln\left\lvert\frac{x+2}{x-2}\right\rvert+C$
(m) $2\sqrt{x}+\frac{1}{2}\ln^2 x+C$	(n) $e^{\operatorname{arctg} x}+\operatorname{arctg} x+\frac{1}{4}\ln^2(1+x^2)+C$	(o) $-\frac{x}{\sin x}+\frac{1}{2}\ln\frac{1-\cos x}{1+\cos x}+C$
(p) $\frac{2}{3}(\arcsin x)^{3/2}+C$	(q) $x\arcsin x+\sqrt{1-x^2}+C$	(r) $\frac{1}{8}\ln(1+4x^2)-\frac{1}{8}\operatorname{arctg}^4(2x)+C$

(2) (a) Naj bo f zvezno odvedljiva funkcija in C realno število. Dokaži enakost

$$\int \big(f(x)+f'(x)\big)\ \mathrm{e}^x\ dx = f(x)\ \mathrm{e}^x + C\,.$$

(b) S pomočjo točke (a) izračunaj integral

$$\int \frac{x}{(1+x)^2}\ \mathrm{e}^x\ dx\,.$$

Integral je $\frac{e^x}{1+x}+C$.