Analiza 1
21. domača naloga

(1) Izračunaj naslednje integrale.

(a) $\displaystyle\int e^{\sin^2 x}\sin 2x\,dx$ (b) $\displaystyle\int (x^2-2x+5)e^{-x}\,dx$ (c) $\displaystyle\int \ln(x+\sqrt{1+x^2})\,dx$

(d) $\displaystyle\int x\sin x\cos x\,dx$ (e) $\displaystyle\int \sin(\ln x)\,dx$ (f) $\displaystyle\int \sin^3 x\ln(\cos x)\,dx$

(g) $\displaystyle\int \frac{\sin^3(\ln x)\cos(\ln x)}{x}\,dx$ (h) $\displaystyle\int \frac{1+\cos^2 x}{1+\cos 2x}\,dx$ (i) $\displaystyle\int x^2\operatorname{arctg} x\,dx$

(j) $\displaystyle\int \frac{\operatorname{arctg}\sqrt{x}}{\sqrt{x}(1+x)}\,dx$ (k) $\displaystyle\int e^{\sqrt{x}}\,dx$ (l) $\displaystyle\int (\arcsin x)^2\,dx$

(m) $\displaystyle\int \frac{1}{2\operatorname{ch} x+\operatorname{sh} x+2}\,dx$ (n) $\displaystyle\int x^5e^{-x^2}\,dx$ (o) $\displaystyle\int (3x^2-2x+1)\ln x\,dx$

(a) $e^{\sin^2 x}$	(b) $-(x^2+5)e^{-x}$	(c) $x\ln(x+\sqrt{1+x^2})-\sqrt{1+x^2}$
(d) $\frac{1}{8}\sin 2x-\frac{x}{4}\cos 2x$	(e) $\frac{x}{2}\big(\sin(\ln x)-\cos(\ln x)\big)$	(f) $\frac{\cos^3 x}{9}(3\ln(\cos x)-1)+\cos x(1-\ln(\cos x))$
(g) $\frac{1}{4}\sin^4(\ln x)$	(h) $\frac{1}{2}(\operatorname{tg} x+x)$	(i) $\frac{x^3}{3}\operatorname{arctg} x-\frac{1}{6}x^2+\frac{1}{6}\ln(1+x^2)$
(j) $\operatorname{arctg}^2\sqrt{x}$	(k) $2(\sqrt{x}-1)e^{\sqrt{x}}$	(l) $x(\arcsin^2 x-2)+2\sqrt{1-x^2}\arcsin x$
(m) $\ln\frac{3e^x+1}{e^x+1}$	(n) $-(\frac{x^4}{2}+x^2+1)e^{-x^2}$	(o) $(x^3-x^2+x)\ln x-\frac{x^3}{3}+\frac{x^2}{2}-x$

(2) Izračunaj integrale racionalnih funkcij.

(a) $\displaystyle\int \frac{x^2+2x+4}{x^2+x+1}\,dx$ (b) $\displaystyle\int \frac{3x+2}{x^3+x}\,dx$ (c) $\displaystyle\int \frac{x^2+x+1}{x^4+4x^2}\,dx$

(d) $\displaystyle\int \frac{3x+5}{(x^2+2x+2)^2}\,dx$ (e) $\displaystyle\int \frac{4-x^4}{4+x^4}\,dx$ (f) $\displaystyle\int \frac{1}{x^3(1+x^2)^2}\,dx$

(g) $\displaystyle\int \frac{1}{x^4+x^2+1}\,dx$ (h) $\displaystyle\int \frac{x^3+4x^2-2x+1}{x^4+x}\,dx$ (i) $\displaystyle\int \frac{x^2-x+14}{(x-4)^3(x-2)}\,dx$

(j) $\displaystyle\int \frac{1}{(x^4-1)^2}\,dx$ (k) $\displaystyle\int \frac{x+1}{x^3-4x^2+5x-2}\,dx$ (l) $\displaystyle\int \frac{x^6-2x^4+3x^3-9x^2+4}{x^5-5x^3+4x}\,dx$

(m) $\displaystyle\int \frac{x^4}{(1-x^5)^7}\,dx$ (n) $\displaystyle\int \frac{x^3+1}{(x+1)(x^2+1)^2}\,dx$ (o) $\displaystyle\int \frac{x^4+1}{x^3-x^2+x-1}\,dx$

(p) $\displaystyle\int \frac{x^2}{1+x^6}\,dx$ (q) $\displaystyle\int \frac{4x^3+6x^2-15x-5}{(x^2-4x+4)(x^2+x+1)}\,dx$

(a) $x+\frac{1}{2}\ln(x^2+x+1)+\frac{5}{\sqrt{3}}\operatorname{arctg}\frac{2x+1}{\sqrt{3}}$	(b) $\ln\frac{x^2}{x^2+1}+3\operatorname{arctg} x$	(c) $\frac{3}{8}\operatorname{arctg}\frac{x}{2}+\frac{1}{8}\ln\frac{x^2}{x^2+4}-\frac{1}{4x}$
(d) $\frac{x-\frac{1}{2}}{x^2+2x+2}+\operatorname{arctg}(x+1)$	(e) $\frac{1}{2}\ln\frac{x^2+2x+2}{x^2-2x+2}+\operatorname{arctg}\frac{2x}{2-x^2}-x$	(f) $-\frac{1}{2}(\frac{1}{x^2}+\frac{1}{x^2+1})+\ln\frac{1+x^2}{x^2}$
(g) $\frac{1}{4}\ln\frac{x^2+x+1}{x^2-x+1}+\frac{1}{2\sqrt{3}}\operatorname{arctg}\frac{x\sqrt{3}}{1-x^2}$	(h) $\log\frac{x^3-x^2+x}{(1+x)^2}+\frac{2}{\sqrt{3}}\operatorname{arctg}\frac{2x-1}{\sqrt{3}}$	(i) $2\ln\left\lvert\frac{x-4}{x-2}\right\rvert+\frac{3}{x-4}-\frac{13}{2(x-4)^2}$
(j) $\frac{3}{16}\ln\left\lvert\frac{x+1}{x-1}\right\rvert+\frac{3}{8}\operatorname{arctg} x-\frac{x}{4(x^4-1)}$	(k) $\frac{2}{x-1}+3\ln\left\lvert\frac{x-2}{x-1}\right\rvert$	(l) $\frac{x^2}{2}+\frac{1}{2}\ln\left\lvert\frac{x^2(x+1)^3(x-1)}{(x+2)^2}\right\rvert$
(m) $\frac{1}{30}(x^5-1)^{-6}$	(n) $\operatorname{arctg} x+\frac{1}{2(x^2+1)}$	(o) $\frac{(x+1)^2}{2}+\frac{1}{2}\ln\frac{(x-1)^2}{x^2+1}-\operatorname{arctg} x$
(p) $\frac{1}{3}\operatorname{arctg} x^3$	(q) $-\frac{3}{x-2}+\frac{4}{\sqrt{3}}\operatorname{arctg}\frac{2x+1}{\sqrt{3}}+\ln\frac{(x-2)^6}{x^2+x+1}$	

Opomba: V zgornjih nedoločenih integralih je povsod treba dodati splošno konstanto. V nekaterih integralih je uporabljen naslednji "adicijski izrek" za funkcijo arctg:

$$\operatorname{arctg} x\pm\operatorname{arctg} y=\operatorname{arctg}\frac{x\pm y}{1\mp xy}\,.$$

Namen tega je le, da so formule krajše; vam rezultatov ni treba preoblikovati na ta način.