Analiza 1
22. domača naloga

(1) Izračunaj naslednje integrale

(a) $\int \frac{1-\sqrt{x+1}}{1+\sqrt[3]{x+1}}\,dx$ (b) $\int \frac{dx}{\sqrt{1-4x-x^2}}$ (c) $\int \sqrt{x^3+x^4}\,dx$

(d) $\int \frac{\cos x}{1+\cos x}\,dx$ (e) $\int \frac{dx}{\cos^4 x\sin^4 x}$ (f) $\int \sin(3x)\sin(\pi x)\,dx$

(g) $\int \frac{\sin^4 x}{\cos^3 x}dx$ (h) $\int \frac{\cos 3x}{\cos^2 x}\,dx$ (i) $\int \frac{1}{3-3\sin^2 x+5\cos x}\,dx$

(j) $\int \cos\sqrt{5x}\,dx$ (k) $\int e^{\sin x}\sin 2x\,dx$ (l) $\int (x-\sin x)^3\,dx$

(m) $\int \frac{e^{2x}}{\operatorname{sh}^4 x}\,dx$ (n) $\int \frac{e^x(1+e^x)}{\sqrt{1-e^{2x}}}\,dx$ (o) $\int \operatorname{sh} x\sqrt[3]{\frac{1+\operatorname{ch} x}{2+\operatorname{ch} x}}\,dx$

(p) $\int \frac{\ln x}{(1+x)^2}\,dx$ (q) $\int \frac{\ln\cos x}{\cos^2 x}\,dx$ (r) $\int \frac{\arcsin x}{x^2}\,dx$

(s) $\int \mathrm{e}^{2x}\sin(\mathrm{e}^x)\,dx$ (t) $\int \operatorname{ch} x\cos x\,dx$ (u) $\int \frac{x^3\arccos x}{\sqrt{1-x^2}}\,dx$

(a) $6\left(-\frac{(x+1)^{7/6}}{7}+\frac{(x+1)^{5/6}}{5}+\frac{(x+1)^{2/3}}{4}-\frac{\sqrt{x+1}}{3}-\frac{\sqrt[3]{x+1}}{2}+\sqrt[6]{x+1}+\frac{\ln(1+\sqrt[3]{x+1})}{2}-\operatorname{arctg}\sqrt[6]{x+1}\right)$

(b) $\arcsin\frac{x+2}{\sqrt{5}}$

(c) $\left(\frac{x^2}{3}+\frac{x}{12}-\frac{1}{8}\right)\sqrt{x^2+x}+\frac{\ln|x+\frac{1}{2}+\sqrt{x^2+x}|}{16}$

(d) $x+\operatorname{ctg} x-\frac{1}{\sin x}$

(e) $\frac{1}{3}(\operatorname{tg}^3 x-\operatorname{ctg}^3 x)+3(\operatorname{tg} x-\operatorname{ctg} x)$

(f) $\frac{\sin(\pi-3)x}{2\pi-6}-\frac{\sin(\pi+3)x}{2\pi+6}$

(g) $\sin x+\frac{\sin x}{2\cos^2 x}+\frac{3}{4}\ln\frac{1-\sin x}{1+\sin x}$

(h) $4\sin x+\frac{3}{2}\ln\frac{1-\sin x}{1+\sin x}$

(i) $\frac{1}{5}\ln\left|\frac{\operatorname{tg}\frac{x}{2}+1}{\operatorname{tg}\frac{x}{2}-1}\right|-\frac{3}{10}\operatorname{arctg}\frac{\operatorname{tg}\frac{x}{2}}{2}$

(j) $\frac{2}{5}(\sqrt{5x}\sin\sqrt{5x}+\cos\sqrt{5x})$

(k) $2e^{\sin x}(\sin x-1)$

(l) $\frac{x^4+3x^2}{4}+3x^2\cos x-6x\sin x-\frac{3}{4}x\sin 2x-5\cos x-\frac{3}{8}\cos 2x-\frac{\cos^3 x}{3}$

(m) $-\frac{8}{3}\frac{3e^{4x}-3e^{2x}+1}{(e^{2x}-1)^3}$

(n) $\arcsin e^x-\sqrt{1-e^{2x}}$

(o) $-\frac{2}{3}\frac{1}{t-1}+\frac{2t+1}{t^2+t+1}+\frac{1}{6}\ln\frac{(t-1)^2}{t^2+t+1}-\frac{7}{3\sqrt{3}}\operatorname{arctg}\frac{2t+1}{\sqrt{3}}$, kjer je $t=\sqrt[3]{\frac{1+\operatorname{ch} x}{2+\operatorname{ch} x}}$

(p) $\ln\left|\frac{x}{x+1}\right|-\frac{\ln x}{1+x}$

(q) $\operatorname{tg} x(1+\ln\cos x)-x$

(r) $-\frac{\arcsin x}{x}+\frac{1}{2}\ln\frac{1-\sqrt{1-x^2}}{1+\sqrt{1-x^2}}$

(s) $\sin e^x-e^x\cos e^x$

(t) $\frac{1}{2}(\operatorname{sh} x\cos x+\operatorname{ch} x\sin x)$

(u) $-\frac{(2+x^2)\sqrt{1-x^2}}{3}\arccos x-\frac{x^3+6x}{9}$

(2) Izračunaj limito

$$\lim_{n\to\infty}\frac{1+\sqrt[n]{e}+\sqrt[n]{e^2}+\cdots+\sqrt[n]{e^{n-1}}}{n}.$$

$e-1$

(3) Izračunaj limito

$$\lim_{n\to\infty}\frac{\ln\frac{n+(e-1)}{n}+2\ln\frac{n+2(e-1)}{n}+3\ln\frac{n+3(e-1)}{n}+\cdots+n\ln\frac{n+n(e-1)}{n}}{n^2}.$$

$\frac{e^2-3}{4(e-1)^2}$