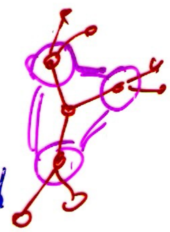
Connectivity of fullerenes

- **Fullerene** is a cubic + 3-connected planar graph with $\forall$ face of size 5 or 6.

- 3-connectivity $\Rightarrow$ 3-edge-connectivity
  $\kappa(G) \leq \lambda(G) \leq \rho(G)$

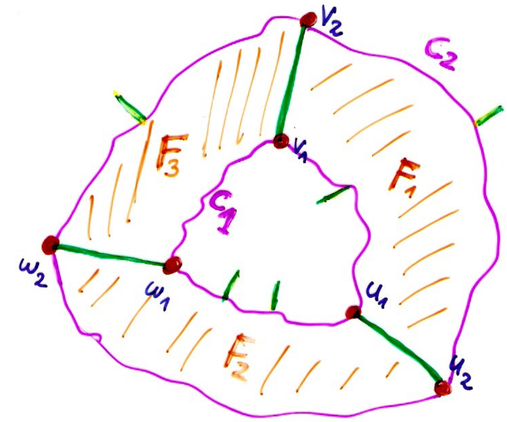


- No fullerene is 4-(edge-)-connected

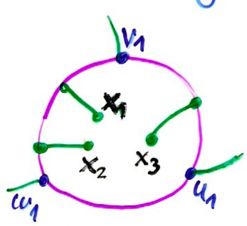
- A graph G is cyclically k-edge-connected if G cannot be separated into 2 components, each containing a cycle, by removing of $< k$ edges.
- A graph G is cyclically k-connected if it cannot be separated into ≥ 2 components, each containing a cycle, by removing of $< K$ vertices.

Thm (Došlić) Every fullerene is cyclically 4-edge-connected

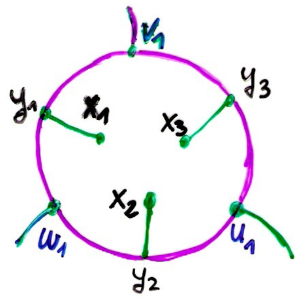
P: Suppose G contradicts the claim.



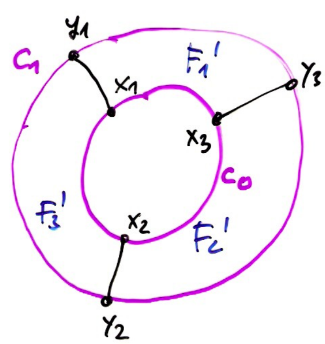
- $v_1 \neq w_1 \neq u_1$ and $v_2 \neq w_2 \neq u_2$
- F_1, F_2, F_3 are 5- or 6- faces
- C_1, C_2 are cycles
- C_1, C_2 are of length 6
- Considering C_1 we have 2 situations



and



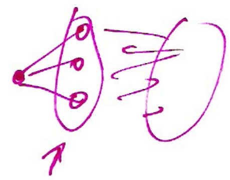
- $X_1 \neq X_2 \neq X_3$; otherwise 3- or 4- cycle is obtained
- Observe that the faces F_1', F_2', F_3', and the cycle C_0 exists.

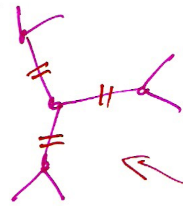


- We obtain similar situation as in begining

$\rightarrow\leftarrow$ with the fact that G is finite.

- Ali je vsak 3-prerez

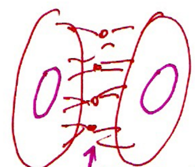


- 

Ali vsak po porezavah 3-prerez tipa

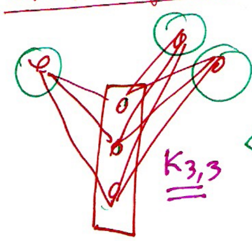
prerez

ciklična ~~povezanost~~

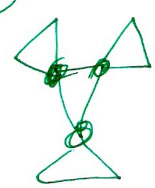


prerez ≥ 3

ciklična povezanost po točkah.

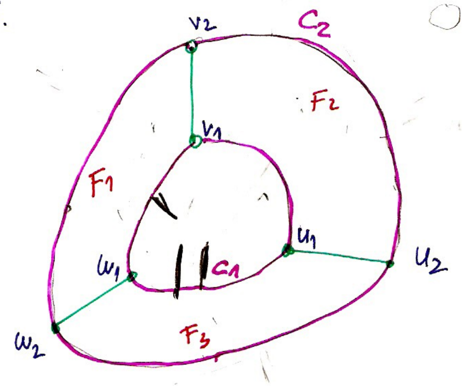


$K_{3,3}$



Izrek (Došlić) Vsak fuleren je ciklično po povezavah 4-povezan.

D: Recimo, da ne drži:



- velja
  $v_1 \neq w_1 \neq u_1$
  $v_2 \neq w_2 = u_2$

- F_1, F_2, F_3 so lica velikosti 5 ali 6.

- C_1 in C_2 cikla

k_2 št povezav ki peljejo ven

k_1 št. povez. ki pelj noter.

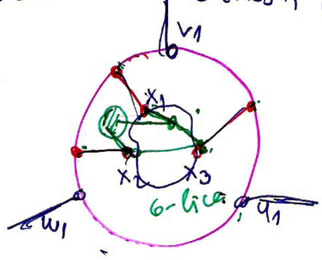
- $3 \leq k_1 + k_2 \leq 6$

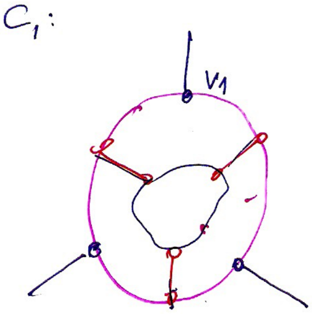
$3 \leq k_1$ $\quad$ $k_i = 0$ $\quad$ $k_i \geq 1$

$3 \leq k_2$ $\quad$ Ker je G po povezavah 3-povezan sledi $k_i \geq 3$.

$\Rightarrow k_1 = 3$ in $k_2 = 3$

- Ločimo dve možnosti pri C_1:

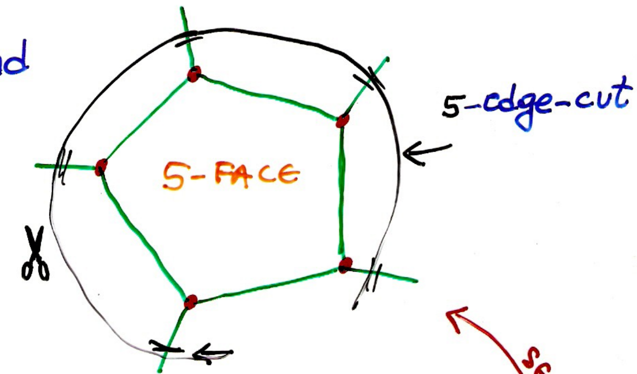




- $x_1 \neq x_2 \neq x_3 \neq x_1$

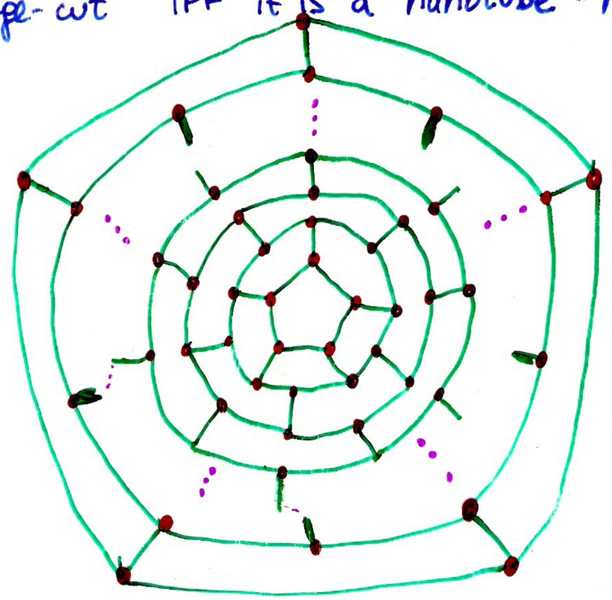
Thm (Došlić) Every fullerene is cyclically 5-edge connected.

- "5" is the best bound



- Question: How the cyclic 5-edge-cuts look?

- Def 5-edge-cut is trivial, if it bounds a pentagon.
- $\forall$ fullerene has precisely 12 trivial cyclic 5-edge-cuts.
- Thm A fullerene has a non-trivial cyclic 5-edge-cut IFF it is a nanotube N_K, $K>0$



NANOTUBE N_K