Topics in Combinatorics

Fall 2011

Questions for the oral exam

normal font:	definitions, examples and algorithms you should know
italics:	theorems you should be able to state (without proof)
bold font:	theorems for which you should know at least the idea of the proof

1. Partitions
   size, length, conjugate partition, diagram, *pentagonal number theorem*, partial orderings, **generating function for** $p(n)$, $z(\lambda)$, *description of conjugacy classes of permutations*

2. Formal power series
   algebra $K[[x]]$, basic operations, units, valuation, metric, convergence, composition, *important expansions* (*e.g.* generalized binomial theorem), ordinary and exponential generating function, Catalan numbers

3. Exponential formula and Lagrange inversion
   combinatorial interpretation of operations on exponential generating functions, *exponential formula*, inverse of a formal power series, *Lagrange inversion*, **Cayley's formula**

4. Algebra of symmetric functions
   definition of Λ^n and Λ, monomial, elementary, complete homogeneous, power sum symmetric functions, *fundamental theorem of symmetric functions*

5. Scalar product and involution ω
   Definition of $\langle \cdot, \cdot \rangle$ and ω, **non-degeneracy** and **non-negativity** of $\langle \cdot, \cdot \rangle$, **involutivity** of ω, **isometricity**

6. Generating functions
   Formal power series $H(t)$, $E(t)$, $P(t)$ and **relations** between them, **expansions** of $\prod_{i,j}(1-x_iy_j)^{-1}$ and $\prod_{i,j}(1+x_iy_j)$, *connection to duality of bases*

7. Specializations
   Reduction of the number of variables, principal specializations, other examples of specializations

8. Combinatorial definition of Schur functions
   semistandard Young tableaux, Kostka numbers, $s_{\lambda/\mu}$ **is a symmetric function**

9. RSK algorithm
   bumping, insertion, reverse insertion, *bijectivity of RSK*, dual RSK algorithm, growth diagrams, *symmetry of RSK*, **main consequences of RSK**

10. Classical Schur function identities
    Classical definition of Schur functions, *Jacobi-Trudi identity*, *Murnaghan-Nakayama rule*

11. Other classical results
    Greene's theorem, Knuth transformations, *Knuth equivalence and insertion tableaux*, jeu-de-taquin slides, *jeu-de-taquin equivalence of tableaux and Knuth equivalence of reading words*, Schützenberger evacuation, *involutivity of* evac, *RSK on* w_0ww_0, *Littlewood-Richardson rule*, *hook-length formula*