Topics in Combinatorics 2011

Homework 11 (due December 23)

1. Suppose $\lambda \vdash n$, $\mu \vdash k \leq n$. Add $n-k$ copies of 1 to μ to obtain $\tau \vdash n$. Prove that
$$\chi^\lambda(\tau) = \sum_{\nu \vdash k} f^{\lambda/\nu} \chi^\nu(\mu).$$

2. Show that s_δ can be expressed as a polynomial in $\{p_i \colon i \text{ odd}\}$.

3. The entries $K^{-1}_{\lambda\mu}$ of the matrix K^{-1} (inverse of the Kostka matrix K) are called *inverse Kostka numbers*. We have
$$m_\lambda = \sum_\mu K^{-1}_{\lambda\mu} s_\mu,$$
$$s_\mu = \sum_\lambda K^{-1}_{\lambda\mu} h_\lambda.$$

   No simple formula for $K^{-1}_{\lambda\mu}$ is known, but there are formulas for special choices of λ and/or μ, such as the one that follows.
   Prove that if $\mu = (a, 1^b)$ is a hook, then
$$K^{-1}_{\lambda\mu} = (-1)^{\ell(\lambda)+\ell(\mu)} \frac{(\ell(\lambda)-1)!}{\prod_{i \geq 1} m_i(\lambda)!} \lambda'_a.$$

   Hint. Prove that $s_\mu = \sum_{i=0}^b (-1)^i h_{a+i} e_{b-i}$ and then use Problem 3a from Homework 6.