Topics in Combinatorics 2011

Homework 6 (due November 18)

1. Use `sage` to find all 12 transition matrices between the bases $\{m_\lambda\colon \lambda \vdash 7\}$, $\{e_\lambda\colon \lambda \vdash 7\}$, $\{h_\lambda\colon \lambda \vdash 7\}$, $\{p_\lambda\colon \lambda \vdash 7\}$ of Λ^7. Which of the matrices have only non-negative integers as elements?

2. For $f \in \Lambda^n$, define $f_k \in \Lambda^{nk}$ by

$$f_k(x_1, x_2, \ldots) = f(x_1^k, x_2^k, \ldots).$$

   Show that

$$\omega f_k = (-1)^{n(k-1)}(\omega f)_k.$$

3. (a) Prove that

$$h_n = \sum_{\lambda \vdash n} \epsilon_\lambda \binom{\ell(\lambda)}{m_1(\lambda), m_2(\lambda), \ldots} e_\lambda.$$

   (b) Define the *forgotten symmetric function* $f_\lambda = \omega(m_\lambda)$. Prove that

$$f_\lambda = \epsilon_\lambda \sum_\mu a_{\lambda\mu} m_\mu,$$

   where $a_{\lambda\mu}$ is the number of all distinct permutations $(\alpha_1, \ldots, \alpha_\ell)$ of the sequence $(\lambda_1, \ldots, \lambda_\ell)$ such that for every j, $\mu_1 + \ldots + \mu_j = \alpha_1 + \ldots + \alpha_i$ for some $i = i(j)$.