

Topics in Combinatorics 2011

Bonus homework (due January 27)

- (a) Note that the diagram of a partition can be uniquely decomposed as the union of a vertical strip and a diagram of an even partition. Conclude that

$$\prod_i \frac{1}{1-x_i^2} \prod_{i < j} \frac{1}{1-x_i x_j} = \sum_{\lambda} s_{\lambda},$$

where the sum on the right is over even partitions λ .

- (b) Compute $h_n[h_2]$.
- Suppose that $[\lambda/\mu]$ is a translation of a straight shape $[\nu]$. Prove that $Q_{\lambda/\mu}(t)$ is a polynomial multiple of $Q_{\nu}(t)$.
- For $\pi \in \mathfrak{S}_n$, denote by $\text{Fix}(\pi)$ the set of fixed points of π . It is clear that $\chi(\pi) = |\text{Fix}(\pi)| - 1$ is a class function. Prove that χ is actually an irreducible character of the symmetric group corresponding to some partition.