**Prerequisites for the course "Permutation groups"**

Student should be familiar with the following notions. Details can be found in any textbook on group theory, for example, in "Rotman, An Introduction to the Theory of Groups, Graduate Texts in Mathematics, Springer Verlag".

- group, subgroup, normal subgroups, cosets by a subgroup;
- order of an element, Lagrange's theorem;
- normaliser, centraliser, center of a group;
- homomorphism and isomorphism of groups;
- quotients of groups and isomorphism theorems;
- direct product of groups;
- automorphisms of groups;
- Sylow's theorems;
- generating sets of group;
- finitely presented groups.

**Exercise.** All the groups below are assumed to be finite.

- Let $G$ be a fgroup. Suppose that for every $x \in G$, $x^2 = 1$ holds. Show that $G$ is abelian.
- Give an example of a group $G$ and a normal subgroup $H \lhd G$ such that $G$ contains no subgroup isomorphic to $G/H$.
- Suppose $H$ and $K$ are subgroups of $G$ and that their orders are coprime. Show that $H \cap K = 1$.
- Let $H \lhd G$ and let $f\colon G \to G/H$ be the corresponding quotient projection. Suppose that, for some $g \in G$, the order $m$ of $f(g)$ is coprime to $|H|$. Show that then $f^{-1}(f(g))$ contains an element of order $m$. Give an example which shows that the assumption on coprimeness of the orders of $f(g)$ and $H$ is needed.

- Let $p$ be a prime divisor of $G$ and let $H \lhd G$ satisfying $|H| < p$. Suppose that the Sylow $p$-subgroup of $G/H$ is normal in $G/H$. Show that the Sylow $p$-subgroup of $G$ is also normal in $G$.
- List all groups (up to isomorphism) of order at most 9.
- Let $D_n$ be the dihedral group of order $2n$ (the group of rotations and reflections of a regular $n$-gon in its action on the vertices of the $n$-gon).
  - Show that $D_3 \cong S_3$ (the symmetric group on 3 elements);
  - Determine all the subgroups of $D_n$;
  - Show that any finite group that can be generated by two involutions (elements of order 2) is a dihedral group;
  - Show that $D_n \cong \langle r, t \mid r^n, t^2, (rt)^2 \rangle$;
  - Determine $\mathrm{Aut}(D_n)$.