Permutation groups — graduate course Spring 2014

GRADUATE COURSE IN MATHEMATICS

ASSIGNMENT NO. 1

1. Let G be an abelian group acting faithfully on a set Ω. Show that the action is semiregular. **(10 points)**

2. Find two actions of a group G that are isomorphism but not equivalent. **(15 points)**

3. Let G act transitively on a set Ω, $|\Omega| \geq 2$, and let $\emptyset \neq \Delta \subseteq \Omega$. Show that Δ is not a block of this action if and only if for every pair $(\alpha, \beta) \in \Omega \times \Omega$ there exists $g \in G$ such that $\alpha \in \Delta^g$ but $\beta \notin \Delta^g$. **(25 points)**

4. Let $\mathbb{F}$ be a finite field, $|\mathbb{F}| \geq 4$, and let $G = \mathrm{PGL}(2, \mathbb{F})$ act on $\mathbb{F} \cup \{\infty\}$ in the usual way. Analyse the structure of the group G_∞. Show that G_ω acts transitively on $\Omega \setminus \{\omega\}$ for every $\omega \in \Omega$. Moreover, prove that $G_{\omega\delta}$ acts transitively on $\Omega \setminus \{\omega, \delta\}$ for every pair $\omega, \delta \in \Omega$. **(25 points)**

5. Let G be a finite group and let $\rho\colon G \times G \to \mathrm{Sym}(G)$ be the group action of $G \times G$ on G defined by:

$$\omega^{\rho(g,h)} = g^{-1}\omega h, \text{ for all } \omega, g, h \in G.$$

Under what condition on G will this action be faithful? When is it going to be primitive? **(25 points)**