Group Theory 2013/14 – homework 2

(1) Consider the following permutations in S_7:

$$a = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{pmatrix} \text{ and } b = \begin{pmatrix} 2 & 5 & 3 \end{pmatrix}\begin{pmatrix} 4 & 6 & 7 \end{pmatrix}.$$

Find the character table of $G = \langle a, b \rangle$.

(2) Let G be a group of order n and denote $\epsilon = e^{2\pi i/n}$. Let σ be an automorphism of the field $\mathbb{Q}(\epsilon)$. Let χ be a character of G. Define $\chi^\sigma : G \to \mathbb{C}$ by the rule $\chi^\sigma(g) = \sigma(\chi(g))$ (why is this definition OK?). Show that χ^σ is a character of G, and that it is irreducible if and only if χ is irreducible.

(3) Let G be a finite abelian group. Show that $\mathrm{Irr}(G)$ is a group under pointwise multiplication. Prove that the obtained group is isomorphic to G.

(4) Let χ be a character of an abelian group A. Prove that

$$\sum_{x \in A} |\chi(x)|^2 \geq |A|\chi(1).$$

This homework is due 2 December 2013.