Group Theory 2013/14 – homework 4

In the last two problems try to avoid the use of Mackey's decomposition formula (Webb's book, p. 68).

(1) Using induction, find the character table of the group

$$G = \langle (1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6\ \ 7)\,, (2\ \ 3\ \ 5)\,(4\ \ 7\ \ 6)\rangle.$$

You can apply `GAP` to facilitate your computations, but all the hand proofs need to be provided.

(2) Let $b(G)$ be the maximum of the degrees of all irreducible characters of G. Prove that if H is a subgroup of G, then $b(H) \leq b(G) \leq |G : H| b(H)$.

(3) Let H and K be subgroups of G and $G = HK$. Let φ be a class function of H. Prove that $\varphi \uparrow_H^G \downarrow_K^G = \varphi \downarrow_{H\cap K}^G \uparrow_{H\cap K}^K$.

(4) Let H be a subgroup of G and let ψ be a character of H. Let K be a subgroup of G and suppose that $\psi \uparrow_H^G \downarrow_K^G$ is an irreducible character of K.
  (a) Show that $\langle \psi \uparrow_H^G \downarrow_K^G, \psi \downarrow_{H\cap K}^H \uparrow_{H\cap K}^K \rangle \neq 0$.
  (b) Conclude that $|G : H| \leq |K : H \cap K|$.
  (c) Prove that $HK = G$.

This homework is due 13 January 2014.