10.1. Dana sta vektorska prostora

$$U = Lin\left\{\begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \\ 1 \end{bmatrix}\right\},$$

in

$$V = Lin\left\{\begin{bmatrix} 2 \\ -1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -2 \\ 2 \end{bmatrix}\right\}.$$

Ugotovi, ali velja $U \subset V$, $V \subset U$ oziroma $U = V$.

10.2. Dana sta vektorska prostora

$$U = Lin\{x^3 - x, x^3 - x^2\},$$

in

$$V = \{p \in \mathbb{R}_3[x] \colon p(1) = 0\}.$$

Ugotovi, ali velja $U \subset V$, $V \subset U$ oziroma $U = V$.

10.3. V prostoru $\mathbb{R}^4$ sta dana podprostora U in V. Prostor U ima bazo

$$\mathcal{B}_1 = \left\{\begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}\right\}$$

prostor V pa bazo

$$\mathcal{B}_2 = \left\{\begin{bmatrix} 1 \\ 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -1 \\ 1 \end{bmatrix}\right\}.$$

Poišči bazi podprostorov $U \cap V$ in $U + V$.

10.4. Dana sta vektorska podprostora

$$U = \{p \in \mathbb{R}_3[x] \colon p(-1) = p(1) = 0\} \quad \text{in} \quad V = \{p \in \mathbb{R}_3[x] \colon p'''(0) = p'(0) = 0\}$$

v prostoru $\mathbb{R}_3[x]$ polinomov stopnje največ 3. Poišči baze prostorov $U + V$ in $U \cap V$.

10.5. Dana sta vektorska podprostora

$$U = \{p \in \mathbb{R}_3[x], p(0) = p'(0) = 0\}$$

in

$$V = Lin\{x^3 - x + 1, x^3 - x^2, x^2 - x + 1\}$$

v prostoru $\mathbb{R}_3[x]$ polinomov stopnje največ 3. Poišči baze prostorov U, V, $U+V$ in $U \cap V$.

10.6. V prostoru $\mathbb{R}_3[x]$ polinomov stopnje največ 3 sta dana podprostora

$$U = Lin\{x^3 - x - 1, x^2 + x + 1, x^3 - x^2 - 2x - 2\}$$

in

$$V = \{p(x) = ax^3 + bx^2 + bx - a; a, b \in \mathbb{R}\}.$$

Poišči baze podprostorov $U, V, U + V$ in $U \cap V$.

10.7. V prostoru $\mathbb{R}_3[x]$ polinomov stonje največ 3 sta dana podprostora

$$U = \{p \in \mathbb{R}_3[x], p(1) = p(-1), p''(0) = 2p(1)\}$$

in

$$V = Lin\{x^2, 1\}.$$

Poišči bazi prostorov $U + V$ in $U \cap V$.

Rešitve:

10.1. $U = V$

10.2. $U \subset V$

10.3. baza $U + V = \left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ 2 \end{bmatrix} \right\}$, baza $U \cap V = \left\{ \begin{bmatrix} 2 \\ 0 \\ -1 \\ 1 \end{bmatrix} \right\}$

10.4. baza $U + V = \{x^3 - x, x^2 - 1, x^2\}$, baza $U \cap V = \{x^2 - 1\}$

10.5. baza $U = \{x^3, x^2\}$, baza $V = \{x^3 - x + 1, x^3 - x^2\}$,
baza $U + V = \{x^3, x^2, x^3 - x + 1\}$, baza $U \cap V = \{x^3 - x^2\}$

10.6. baza $U = \{x^3 - x - 1, x^2 + x + 1\}$, baza $V = \{x^3 - 1, x^2 + x\}$,
baza $U + V = \{x^3 - x - 1, x^2 + x + 1, x^3 - 1, x^2 + x\}$, baza $U \cap V = \{\}$

10.7. baza $U + V = \{x^3 - x, x^2, 1\}$, baza $U \cap V = \{x^2\}$