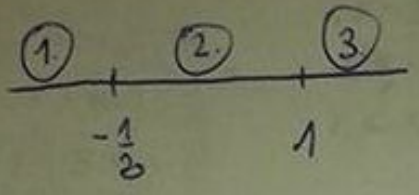
# MATEMATIKA I.; 2. DOMAČA NALOGA;

1) $|2x+1| < |x-1| + x$

- $|2x+1| = \begin{cases} 2x+1; & x \geq -\frac{1}{2} \\ -2x-1; & x < -\frac{1}{2} \end{cases}$
- $|x-1| = \begin{cases} x-1; & x \geq 1 \\ -x+1; & x < 1 \end{cases}$
- 


① $\boxed{x < -\frac{1}{2}}$:

$-2x-1 < -x+1+x$

$-2x < 2 \;/:(-2)$

$x > -1$

$x \in \left(-1, -\frac{1}{2}\right)$

② $\boxed{-\frac{1}{2} \leq x < 1}$:

$2x+1 < -x+1+x$

$2x < 0 \;/:2$

$x < 0$

$x \in \left[-\frac{1}{2}, 0\right)$

③ $\boxed{x \geq 1}$:

$2x+1 < x-1+x$

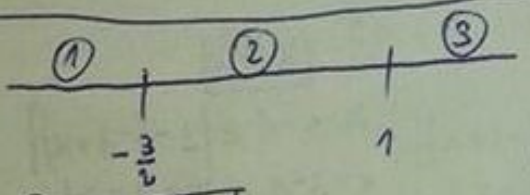
$2x+1 < 2x-1$

$0 < -2$

Ni rešitve.

SKUPNI INTERVAL: $\boxed{x \in (-1, 0)}$

2) $|x-1| + x \leq |2x+3|$

- $|x-1| = \begin{cases} x-1; & x \geq 1 \\ -x+1; & x < 1 \end{cases}$
- $|2x+3| = \begin{cases} 2x+3; & x \geq -\frac{3}{2} \\ -2x-3; & x < -\frac{3}{2} \end{cases}$
- 


① $\boxed{x < -\frac{3}{2}}$:

$-x+1+x \leq -2x-3$

$2x \leq -4 \;/:2$

$x \leq -2$

$x \in \left[-2, -\frac{3}{2}\right)$

② $\boxed{-\frac{3}{2} \leq x < 1}$:

$-x+1+x \leq 2x+3$

$-2x \leq 2 \;/:(-2)$

$x \geq -1$

$x \in [-1, 1)$

③ $\boxed{x \geq 1}$

$x-1+x \leq 2x+3$

$2x-1 \leq 2x+3$

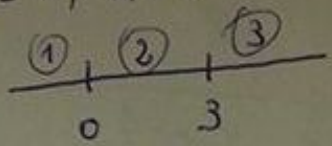
$-1 \leq 3$

Ni rešitve!

SKUPNI INTERVAL: $\boxed{x \in \left[-2, -\frac{3}{2}\right) \cup [-1, 1)}$

3) $|x-3|-1 < |3-|x||$

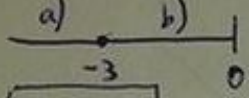
• $|x| = \begin{cases} x; & x \geq 0 \\ -x; & x < 0 \end{cases}$ • $|x-3| = \begin{cases} x-3; \\ -x+3; & x \end{cases}$




① $\boxed{x < 0}$

$-x+3-1 < |3+x|$

$|3+x| = \begin{cases} 3+x; & x \geq -3 \\ -3-x; & x < -3 \end{cases}$




a) $\boxed{x < -3}$

$-x+3-1 < -3-x$

$0 < -7$

Ni rešitve!

b) $\boxed{-3 \leq x < 0}$

$-x+3-1 < 3+x$

$-2x < 1 \;/:(-2)$

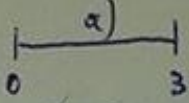
$x > -\frac{1}{2}$

$x \in \left(-\frac{1}{2}, 0\right)$

② $\boxed{0 \leq x < 3}$

$-x+3-1 < |3-x|$

$3-x = \begin{cases} 3-x; & x \geq 3 \\ -3+x; & x < 3 \end{cases}$




a) $-x+3-1 < 3-x$

$-1 < 0$

$x \in [0, 3)$

③ $\boxed{x \geq 3}$

$x-3-1 < |3-x|$

$x-3-1 < -3+x$

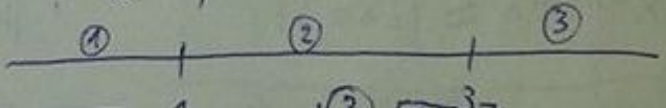
$-1 < 0$

$x \in [3, \infty)$

SKUPNI INTERVAL: $\boxed{x \in \left(-\frac{1}{2}, \infty\right)}$

4) $|x-3|-1 \leq |2-|1-x||$

• $|1-x| = \begin{cases} 1-x; & x \leq 1 \\ -1+x; & x > 1 \end{cases}$ • $|x-3| = \begin{cases} x-3; & x \geq 3 \\ -x+3; & x < 3 \end{cases}$



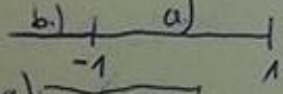

① $\boxed{x \leq 1}$:

$-x+3-1 \leq |2-(1-x)|$

$-x+3-1 \leq |2-1+x|$

$-x+3-1 \leq |1+x|$

$|1+x| = \begin{cases} 1+x; & x \geq -1 \\ -1-x; & x < -1 \end{cases}$




a) $\boxed{-1 \leq x \leq 1}$

$-x+3-1 \leq 1+x$

$-2x \leq -1 \;/:(-2)$

$x \geq \frac{1}{2}$

$x \in \left[\frac{1}{2}, 1\right]$

b) $\boxed{x < -1}$

$-x+3-1 \leq 1-x$

$3 \leq 0$

Ni rešitve!

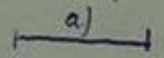
② $\boxed{1 < x < 3}$

$-x+3-1 \leq |2-(-1+x)|$

$-x+3-1 \leq |2+1-x|$

$-x+3-1 \leq |3-x|$

$|3-x| = \begin{cases} 3-x; & x \leq 3 \\ -3+x; & x > 3 \end{cases}$




a) $\boxed{x \leq 3}$

$-x+3-1 \leq 3-x$

$-1 \leq 0$

Ni rešitve!

③ $\boxed{x \geq 3}$

$x-3-1 \leq |2-(-1+x)|$

$x-3-1 \leq |2+1-x|$

$x-4 \leq |3-x|$


a) $\boxed{x \geq 3}$

$x-4 \leq -3+x$

$-4 \leq -3$

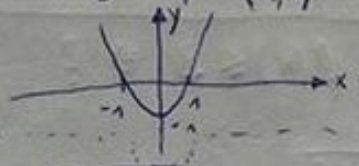
Ni rešitve.

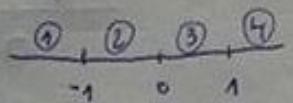
SKUPNI INTERVAL: $\boxed{x \in \left[\frac{1}{2}, 1\right]}$

$|x^2-1| \leq |x|$

$\bullet\ |x| = \begin{cases} x; & x \geq 0 \\ -x; & x < 0 \end{cases}$

$\bullet\ |x^2-1| = \begin{cases} x^2-1; & x \in (-\infty,-1] \cup [1,\infty) \\ -x^2+1; & x \in (-1,1) \end{cases}$




① $\boxed{x \leq -1}$

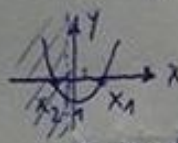
$x^2-1 \leq -x$

$x^2+x-1 \leq 0$

$x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\Rightarrow x_1 = -\frac{1}{2} + \frac{\sqrt{5}}{2}$

$x_2 = -\frac{1}{2} - \frac{\sqrt{5}}{2}$




$x \in \left[\frac{-1-\sqrt{5}}{2}; -1\right]$

② $\boxed{-1 < x < 0}$

$-x^2+1 \leq -x$

$-x^2+x+1 \leq 0 \ / \cdot(-1)$

$x^2-x-1 \geq 0$

$x_{1,2} = \frac{1 \pm \sqrt{(-1)^2-4\cdot1\cdot(-1)}}{2\cdot1} \Rightarrow$

$\Rightarrow x_1 = \frac{1}{2} + \frac{\sqrt{5}}{2}$

$x_2 = \frac{1}{2} - \frac{\sqrt{5}}{2}$

$x \in \left(-1; \frac{1}{2} - \frac{\sqrt{5}}{2}\right]$

③ $\boxed{0 \leq x < 1}$

$-x^2+1 \leq x$

$-x^2-x+1 \leq 0 \ / \cdot(-1)$

$x^2+x-1 \geq 0$

$x_{1,2} = \frac{-1 \pm \sqrt{1^2-4\cdot1\cdot(-1)}}{2\cdot1} \Rightarrow$

$\Rightarrow x_1 = -\frac{1}{2} + \frac{\sqrt{5}}{2}$

$x_2 = -\frac{1}{2} - \frac{\sqrt{5}}{2}$

$x \in \left[-\frac{1}{2} + \frac{\sqrt{5}}{2}; 1\right)$

④ $\boxed{x \geq 1}$

$x^2-1 \leq x$

$x^2-x-1 \leq 0$

$x \in \left[\frac{1}{2} + \frac{\sqrt{5}}{2}; \infty\right)$

⑥ $||ax-b|-2| = 3$

$\bullet\ |ax-b| = \begin{cases} ax-b; & x \geq \frac{b}{a} \\ -ax+b; & x < \frac{b}{a} \end{cases}$

$x_1 = -1$

$x_2 = 0$

$x_3 = 1$

$a, b = ?$

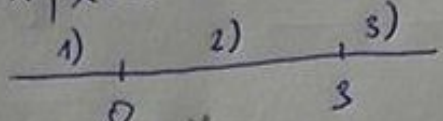
SKUPNI INTERVAL:

$x \in \left[\frac{-1-\sqrt{5}}{2}; \frac{1-\sqrt{5}}{2}\right] \cup \left[-\frac{1}{2} + \frac{\sqrt{5}}{2}; 1\right] \cup \left[\frac{1}{2} + \frac{\sqrt{5}}{2}; \infty\right)$

⑦ $f(x) = |x| + |x-3| - 2$

$f(x) \leq 3 \Rightarrow ?$

• $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$ • $|x-3| = \begin{cases} x-3, & x \geq 3 \\ -x+3, & x < 3 \end{cases}$




1) $\boxed{x < 0}$

$f(x) = -x - x + 3 - 2 =$

$= -2x + 1$

2) $\boxed{x \in [0,3)}$

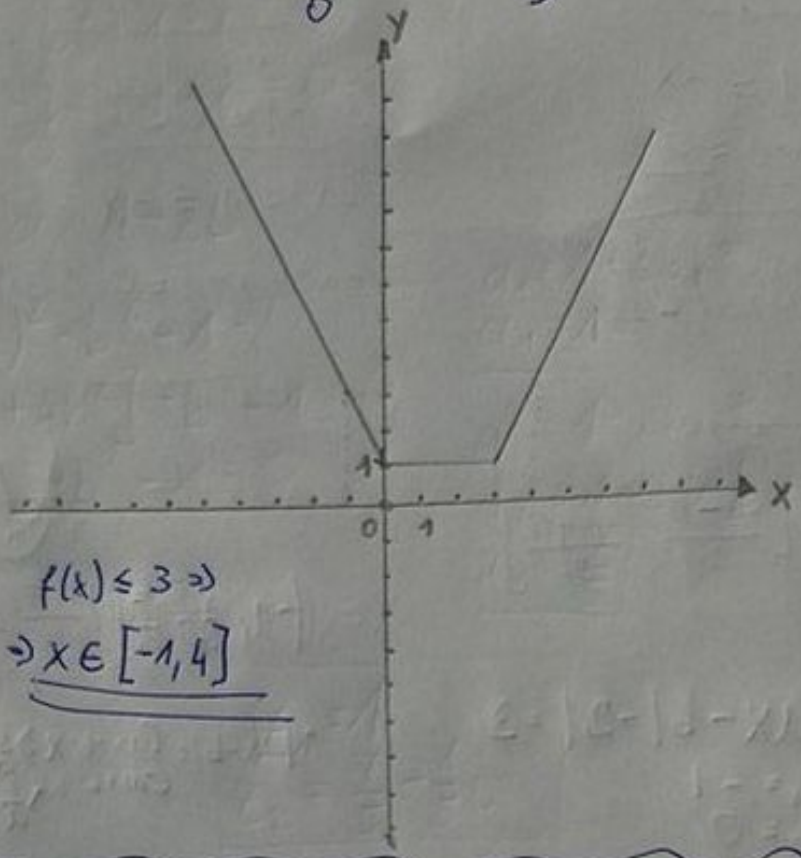
$f(x) = x - x + 3 - 2 =$

$= 1$

3) $\boxed{x \geq 3}$

$f(x) = x + x - 3 - 2 =$

$= 2x - 5$




$f(x) \leq 3 \Rightarrow$

$\Rightarrow x \in [-1, 4]$

⑧ $\frac{2-3i}{2+3i} + \frac{3-i}{3-4i} = \frac{(2-3i)(2-3i)}{(2+3i)(2-3i)} + \frac{(3-i)(3+4i)}{(3-4i)(3+4i)} =$

$= \frac{4-6i-6i-9}{13} + \frac{9+12i-3i+4}{25} = \frac{-5-12i}{13} + \frac{13+9i}{25} =$

$= \frac{25(-5-12i) + 13(13+9i)}{13 \cdot 25} = \frac{-125-300i+169+117i}{325} = \frac{44-183i}{325}$

⑨ $\frac{(1+i)^{15} - 3i}{2+i+\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)^{27}} =$

$= \frac{-128+125i}{2+i+i\frac{\sqrt{3}}{2} - \frac{1}{2}} =$

$= \frac{-128+125i}{\frac{3}{2} + \frac{2\sqrt{3}}{2}i} = \frac{\left(-128+125i\right)\left(\frac{3}{2} - \frac{2\sqrt{3}}{2}i\right)}{\frac{9}{4} + \frac{4 \cdot 3}{4}} =$

$= \frac{-192 + 128\sqrt{3}i + \frac{375}{2}i + 125\sqrt{3}}{\frac{21}{4}} =$

$= \frac{(125\sqrt{3} - 192) + (128\sqrt{3} + 187)i}{5\frac{1}{4}} =$

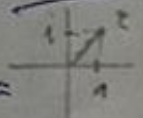
• $(1+i)^{15} =$

$= |\sqrt{2}|^{15}\left(\cos 15 \cdot \frac{\pi}{4} + i \sin 15 \cdot \frac{\pi}{4}\right) =$

$= 2^7\sqrt{2}\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right) =$

$= 128\sqrt{2}\left(-\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right) =$

$= -128 + 128i$




$|z| = \sqrt{1^2+1^2} = \sqrt{2}$

$\alpha = 45° = \frac{\pi}{4}$

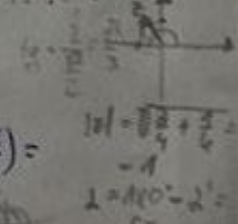
• $\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)^{27} =$

$= 1\left(\text{cis } 27\frac{5\pi}{6}\right) = 1\left(\text{cis } 3\frac{5\pi}{6}\right) =$

$= 1\left(\text{cis } \frac{15\pi}{6}\right) = 1\left(\text{cis } \frac{5\pi}{3}\right) = 1\left(\text{cis } \frac{2\pi}{3}\right) =$

$= \cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} =$

$= -\frac{1}{2} + i\frac{\sqrt{3}}{2}$




$|z| = \sqrt{\frac{3}{4} + \frac{1}{4}} = 1$

$\alpha = 180° - 2 \cdot \frac{5\pi}{6}$

$(\sqrt{3}-i)^{16} = 2^{16}\left(\text{cis}\,\frac{8}{16}\cdot\frac{11\pi}{6}\right) = 2^{16}\left(\text{cis}\,\frac{\pi}{3}\right) =$

$= 2^{16}\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right) = 2^{15}+i2^{15}\sqrt{3}$

$|z| = \sqrt{3+1} = 2$

$\operatorname{tg}\alpha = \frac{-1}{\sqrt{3}} \Rightarrow \frac{\pi}{6}$

$\Rightarrow \alpha = \frac{11\pi}{6}$

(11) $z^3 = (-1+i)^6$

$z_k = \sqrt[3]{\sqrt{2}}\left(\text{cis}\,\frac{\frac{3\pi}{4}+k2\pi}{n}\right)$

$k=0: z_0 = \sqrt[6]{2}\left(\text{cis}\,\frac{\frac{3\pi}{4}}{3}\right) = \sqrt[6]{2}\left(\text{cis}\,\frac{3\pi}{4\cdot 3}\right) =$

$= \sqrt[6]{2}\left(\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\right) = \sqrt[3]{\frac{1}{2}}+i\sqrt[3]{\frac{1}{2}}$

$k=1: z_1 = \sqrt[6]{2}\left(\text{cis}\,\frac{\frac{3\pi}{4}+2\pi}{3}\right) = \sqrt[6]{2}\left(\text{cis}\,\frac{\frac{11\pi}{4}}{3}\right) =$

$= \sqrt[6]{2}\left(\text{cis}\,\frac{11\pi}{12}\right)$

$k=2: z_2 = \sqrt[6]{2}\left(\text{cis}\,\frac{19\pi}{12}\right) = \sqrt[6]{2}\left(\text{cis}\,\frac{7\pi}{12}\right)$



$|w| = \sqrt{1^2+1^2} = \sqrt{2}$

$\operatorname{tg}\alpha = \frac{1}{-1} = -1 \Rightarrow \alpha = \frac{3\pi}{4}$

$w = (\sqrt{2})^6\left(\text{cis}\,\frac{3\pi}{4}\right) =$

$= 8\left(-\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\right) =$

$= -4\sqrt{2}+i4\sqrt{2}$

(12) $|z|-1 = 2\bar{z}-z-15i$

Fie $z = a+bi$, $\bar{z} = a-bi$

$|z| = \sqrt{a^2+b^2}$

$\sqrt{a^2+b^2}-1 = 2a-2bi-a-bi-15i$

$\sqrt{a^2+b^2} = (a+1)-i(3b+15)\ /^2$

$a^2+b^2 = (a+1)^2-2(a+1)(3b+15)i+(3b+15)^2$

$a^2+b^2 = a^2+2a+1-6ab\,i+30a\,i+6b\,i+30\,i+9b^2+90b+225$

$0 = -b^2+2a+1+9b^2+90b+225$

$0 = -6ab+30a+6b+30$

$8b^2+2a+90b+226 = 0 \Rightarrow a = -4b^2-45b-113$

$-6ab+30a+6b+30 = 0$

$24b^3+270b+678b-120b^2-1350b-3390+6b+30 = 0$

$24b^3-120b^2-402b-3294 = 0$

$12b^3-60b^2-201b-1647 = 0$

(13) $(2+i)z = |z|^2 + 1 + i$ ; $z = a+bi$ ; $|z| = \sqrt{a^2+b^2}$

$(2+i)(a+bi) = a^2+b^2+1+i$

$2a + 2bi + ai - b = a^2+b^2+1+i$

$2a + 2bi + ai - b - a^2 - b^2 - 1 - i = 0$

$2a - b - a^2 - b^2 - 1 = 0$

$2b + a - 1 = 0 \Rightarrow a = 1 - 2b$

$2(1-2b) - b - (1-2b)^2 - b^2 - 1 = 0$

$2 - 4b - b - [1 - 4b + 4b^2] - b^2 - 1 = 0$

$2 - 5b - 1 + 4b - 4b^2 - b^2 - 1 = 0$

$-5b^2 - b = 0$

$b(-5b - 1) = 0 \Rightarrow \boxed{b_1 = 0}$

$-5b_2 = 1 \;/:(-5)$

$\boxed{b_2 = -\frac{1}{5}}$

$a_1 = 1 - 2b_1$

$\boxed{a_1 = 1}$

$a_2 = 1 - 2b_2$

$a_2 = 1 - 2\left(-\frac{1}{5}\right)$

$\boxed{a_2 = \frac{7}{5}}$

$\boxed{z_1 = 1}$

$\boxed{z_2 = \frac{7}{5} - \frac{1}{5}i}$

(14) $|z - 3 - i| = |z + 2i|$ ; $z = a+bi$ ; $|z| = \sqrt{a^2+b^2}$

$|a + bi - 3 - i| = |a + bi + 2i|$

$|\underbrace{(a-3)}_{A} + i\underbrace{(b-1)}_{B}| = |\underbrace{a}_{C} + i\underbrace{(b+2)}_{D}|$

$\sqrt{(a-3)^2 + (b-1)^2} = \sqrt{a^2 + (b+2)^2} \;/^2$

$(a-3)^2 + (b-1)^2 = a^2 + (b+2)^2$

$a^2 - 6a + 9 + b^2 - 2b + 1 = a^2 + b^2 + 4b + 4$

$\text{Re}(z): -6a + 9 = 0 \Rightarrow a = \frac{3}{2}$

$\text{Im}(z): -2b + 1 = 4b + 4 \Rightarrow b = -\frac{1}{2}$

$\boxed{z = \frac{3}{2} - \frac{1}{2}i}$

# MATEMATIKA I. (2. DOMAĆA NALOGA)

(15) $z^5 = -32$

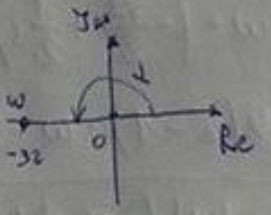
$z^n = w \Rightarrow w = -32$

$z_k = \sqrt[n]{|w|} \cdot \left(\cos\frac{\varphi + 2k\pi}{n} + i\sin\frac{\varphi+2k\pi}{n}\right)$

$w = |w|(\cos\varphi + i\sin\varphi)$

$w = 32(\cos\pi + i\sin\pi)$

$w = 32(-1) = -32$




$\varphi = 180° = \pi$

$|w| = 32$

$z_0 = \sqrt[5]{32} \cdot \left(\cos\frac{\pi+0}{5} + i\sin\frac{\pi+0}{5}\right) =$

$= 2 \cdot \left(\cos\frac{\pi}{5} + i\sin\frac{\pi}{5}\right)$

$z_1 = 2\left(\cos\frac{3\pi}{5} + i\sin\frac{3\pi}{5}\right)$

$z_2 = 2\left(\cos\frac{5\pi}{5} + i\sin\frac{5\pi}{5}\right) =$

$= 2(\cos\pi + i\sin\pi) =$

$= 2 \cdot (-1) = -2$

$z_3 = 2\left(\cos\frac{7\pi}{5} + i\sin\frac{7\pi}{5}\right)$

$z_4 = 2\left(\cos\frac{9\pi}{5} + i\sin\frac{9\pi}{5}\right)$

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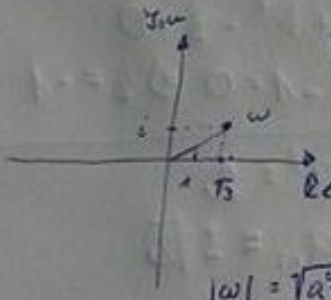
(16) $w = \sqrt{3} + i$

$\mathrm{Re}(w^4 z) = 0 \quad \wedge \quad |z| = 2$

$z = ?$




$\operatorname{tg}\varphi = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \Rightarrow$

$\Rightarrow \varphi = \frac{\pi}{6}$

$|w| = \sqrt{a^2+b^2} =$

$= \sqrt{3+1} = 2$

$w^4 = \mu = |w|^4(\cos 4\varphi + i\sin 4\varphi) =$

$= 16\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) =$

$= 16\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) =$

$= -8 + i8\sqrt{3}$

$\mu \cdot z = (-8 + i8\sqrt{3})(a+bi) =$

$= -8a - 8bi + i8\sqrt{3}a - 8\sqrt{3}b =$

$= \underbrace{(-8a - 8\sqrt{3}b)} + i(-8b + 8\sqrt{3}a)$

$\mathrm{Re}(\mu \cdot z) = 0 \wedge |z| = 2$

$-8a - 8\sqrt{3}b = 0 \wedge a^2 + b^2 = 4$

$a = -\sqrt{3}b \Rightarrow 3b^2 + b^2 = 4 \Rightarrow b_{1,2} = \pm 1$

$a_{1,2} = \mp\sqrt{3}$

$z_1 = \sqrt{3} - i$

$z_2 = -\sqrt{3} + i$

(13) $(1+i)_{\square} = |z|^2 + 1 + i$

(17) • $\omega = \left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)^{21}$

• $z(\bar{z} - 1) = \omega + 3$

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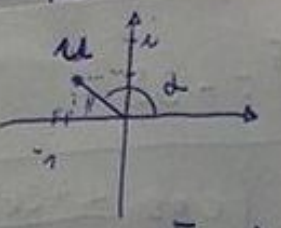
$\omega = |\omega|(\cos\alpha + i\sin\alpha)$

$\omega = u^n = |u|^n(\cos n\cdot\alpha + i\sin n\alpha)$

$\omega = 1\left(\cos 21\frac{5\pi}{6} + i\sin 21\frac{5\pi}{6}\right)$

$\omega = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = \underline{i}$

$|u| = \sqrt{\frac{3}{4} + \frac{1}{4}} = \sqrt{\frac{4}{4}} = 1$




$\operatorname{tg}\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \frac{1}{-\sqrt{3}} = -\frac{\sqrt{3}}{3} \Rightarrow$

$\Rightarrow \alpha = \frac{5\pi}{6}$

$z(\bar{z} - 1) = \omega + 3$

$z\bar{z} - z = \omega + 3$

$a^2 + b^2 - a - bi = i + 3$

$a^2 + b^2 - a - bi - i - 3 = 0$

Re: $a^2 + b^2 - 3 = 0$

Im: $-b - 1 = 0 \Rightarrow b = -1$

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$a^2 = 2$

$a_{1,2} = \pm\sqrt{2}$

$z_1 = \sqrt{2} - i$

$z_2 = -\sqrt{2} - i$