MATEMATIKA I.

3. domača naloga;

1.) a) $f(x) = \sqrt{x - x^2} + \arcsin\sqrt{x}$

$D_f = ?$ $\quad \underline{D_f = [0,1]}$

- $\arcsin\sqrt{x} \rightarrow D_f = (-1,1)$
- $\sqrt{x-x^2} \rightarrow D_f:\ x - x^2 \geq 0$
  $x(1-x) \geq 0$
  $x_1 = 0$
  $x_2 = 1$

$\Big\} D_f = [0,1]$

b) $f(x) = \sqrt{|4-x| + |2x-1| - 5}$

$D_f = ?$ $\quad \underline{D_f = (-\infty, 0] \cup [2, \infty)}$

- $|4-x| + |2x-1| - 5 \geq 0$

$$|4-x| = \begin{cases} 4-x; & x \leq 4 \\ -4+x; & x > 4 \end{cases} \quad ; \quad |2x-1| = \begin{cases} 2x-1; & x \geq \frac{1}{2} \\ -2x+1; & x < \frac{1}{2} \end{cases}$$

	1.		2.		3.
$\lvert 4-x\rvert$	$\oplus$		$\oplus$		$\ominus$
$\lvert 2x-1\rvert$	$\ominus$	$\frac{1}{2}$	$\oplus$	4	$\oplus$

1) $\boxed{x < \frac{1}{2}}$ $\quad$ 2) $\boxed{1 < x \leq 4}$ $\quad$ 3) $\boxed{x > 4}$

b) $f(x) = \sqrt{|4-x| + |2x-1| - 5}$

$D_f = ?$ $\quad D_f = (-\infty, 0] \cup [2; \infty)$

- $|4-x| + |2x-1| - 5 \geq 0$

$$|4-x| = \begin{cases} 4-x; & x \leq 4 \\ -4+x; & x > 4 \end{cases} \quad ; \quad |2x-1| = \begin{cases} 2x-1; & x \geq \frac{1}{2} \\ -2x+1; & x < \frac{1}{2} \end{cases}$$

	⊕ 1.	⊕ 2.	⊖ 3.
$\lvert 4-x\rvert$	⊖	⊕	⊕
$\lvert 2x-1\rvert$	⊖	⊕	⊕

$\frac{1}{2}$ $\qquad$ 4

1.) $\boxed{x < \frac{1}{2}}$

$4 - x - 2x + 1 - 5 \geq 0$

$-3x \geq 0$

$x \leq 0$

$x \in (-\infty, 0]$

2.) $\boxed{\frac{1}{2} \leq x \leq 4}$

$4 - x + 2x - 1 - 5 \geq 0$

$-2 + x \geq 0$

$x \geq 2$

$x \in [2, 4]$

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3) $\boxed{x > 4}$

$-4 + x + 2x - 1 - 5 \geq 0$

$-10 + 3x \geq 0$

$x \geq \frac{10}{3}$

$x \in (4, \infty)$

c) $f(x) = \ln\left(\left|\frac{x+3}{x-2}\right| - 1\right)$

$D_f = ?$ $\quad D_f = (-\frac{1}{2}, 2) \cup (2, \infty)$

$$|x+3| = \begin{cases} x+3; & x \geq -3 \\ -x-3; & x < -3 \end{cases}$$

$$|x-2| = \begin{cases} x-2; & x \geq 2 \\ -x+2; & x < 2 \end{cases}$$

- $\left|\frac{x+3}{x-2}\right| - 1 > 0$; $x \neq 2$

	⊖ 1.	⊕ 2.	⊕ 3.
$\lvert x+3\rvert$	⊖	⊕	⊕
$\lvert x-2\rvert$	⊖	⊖	⊕

-3 $\qquad$ 2

1.) $\boxed{x < -3}$

$\frac{-x-3}{-x+2} > 1$

$\frac{-(x+3)}{-(x-2)} > 1 \;/\cdot(x-2)$

$-x-3 < x-2$

$-3 < -2$

Ni rešitve!

2.) $\boxed{-3 \leq x < 2}$

$\frac{x+3}{-x+2} > 1$

$\frac{-(-x-3)}{-(x-2)} > 1 \;/\cdot(x-2)$

$-x-3 < (x-2)$

$-2x < 1 \;/\cdot(-2)$

$x > -\frac{1}{2}$

$x \in (-\frac{1}{2}, 2)$

3.) $\boxed{x \geq 2}$

$\frac{x+3}{x-2} > 1 \;/\cdot(x-2)$

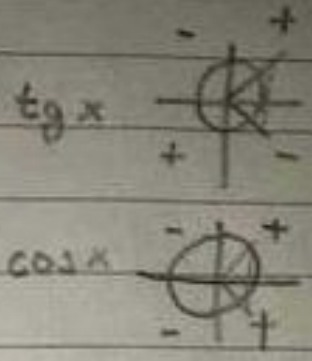
$x+3 > x-2$

$3 > -2$

$x \in [2, \infty)$

2) a) $f(x) = x\,tg\,x - \cos x$

SODA ali LIHA F.?



• SODA $\mapsto f(-x) = f(x)$; LIHA $\mapsto f(-x) = -f(x)$

$f(-x) = -x\,tg(-x) - \cos(-x) =$

$= (-x)\cdot(-tg(x)) - \cos x =$

$= x\cdot tg(x) - \cos(x) = f(x)$

Funkcija je soda.

b) $f(x) = x^2(2^x - 2^{-x})$

SODA ali LIHA?

• SODA $\mapsto f(-x) = f(x)$; LIHA $\mapsto f(-x) = -f(x)$:

$f(-x) = (-x)^2(2^{-x} - 2^{-(-x)}) =$

$= x^2(2^{-x} - 2^x) = -x^2(2^x - 2^{-x}) = -f(x)$

Funkcija je liha.

c) $f(x) = x^3 + x^2$

SODA ali LIHA F.?

SODA $\mapsto f(-x) = f(x)$; LIHA $\mapsto f(-x) = -f(x)$

$f(-x) = (-x)^3 + (-x)^2 = -x^3 + x^2 \neq f(x)$

$f(-x) = -x^3 + x^2 = -(x^3 - x^2) \neq -f(x)$

Funkcija ni niti liha, niti soda.

3) $f(x) = 2x+1$
$g(x) = (x-1)^2$

- $f \circ f = 2(2x+1)+1 = 4x+3$
- $f \circ g = 2[(x-1)^2]+1 = 2x^2-4x+3$
- $g \circ f = ((2x+1)-1)^2 = 4x^2$
- $g \circ g = ((x-1)^2-1)^2 = x^4-4x^3+4x^2$

4) a) $2^{|2x-3|} = 7$

$x = 2$

$2^{|2x-3|} = 7$

$|2x-3| = \begin{cases} 2x-3; & x \geq \frac{3}{2} \\ -2x+3; & x < \frac{3}{2} \end{cases}$

$|2x+3|$ — 1. | + 2.

$\frac{3}{2}$

$\log_a b = c \Leftrightarrow a^c = b$

1.) $\boxed{x < \frac{3}{2}}$

$2^{(-2x+3)} = 7$

$\log_2 7 = -2x+3$

$\frac{\log 7}{\log 2} = -2x+3$

$\frac{\log 7}{\log 2} - 3 = -2x \;/:(-2)$

$\frac{\log 7}{-2\log 2} + \frac{3}{2} = x_1$

$x_1 = \frac{3}{2} - \frac{\log 7}{2\log 2} = \frac{1}{2}\left(3 - \frac{\log 7}{\log 2}\right)$

2.) $\boxed{x \geq \frac{3}{2}}$

$2^{2x-3} = 7$

$\log_2 7 = 2x-3$

$\frac{\log 7}{\log 2} + 3 = 2x \;/:2$

$x_2 = \frac{3}{2} + \frac{\log 7}{2\log 2} = \frac{1}{2}\left(3 + \frac{\log 7}{\log 2}\right)$

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b) $2\sin\left(3x-\frac{\pi}{2}\right)=\sqrt{3} \quad /:2$

$\sin\left(3x-\frac{\pi}{2}\right)=\frac{\sqrt{3}}{2}$

$3x_1-\frac{\pi}{2}=\frac{\pi}{3}+k2\pi$

$3x_1=\frac{5\pi}{6}+k2\pi \quad /:3$

$x_1=\frac{5\pi}{18}+\frac{k2\pi}{3}$

$3x_2-\frac{\pi}{2}=\frac{2\pi}{3}+k2\pi$

$3x_2=\frac{7\pi}{6}+k2\pi \quad /:3$

$x_2=\frac{7\pi}{18}+\frac{k2\pi}{3}$

$k\in\mathbb{Z}$

c) $2\cos\left(2x+\frac{\pi}{3}\right)=\sqrt{2} \quad /:2$

$\cos\left(2x+\frac{\pi}{3}\right)=\frac{\sqrt{2}}{2}$

$2x_1+\frac{\pi}{3}=\frac{\pi}{4}+k2\pi$

$2x_1=-\frac{\pi}{12}+k2\pi \quad /:2$

$x_1=-\frac{\pi}{24}+k\pi$

$2x_2+\frac{\pi}{3}=\frac{7\pi}{4}+k2\pi$

$2x_2=\frac{17\pi}{12}+k2\pi$

$x_2=\frac{17\pi}{24}+k\pi$

$k\in\mathbb{Z}$

d) $\sin x+\sin 2x=0$

$\sin x+2\sin x\cdot\cos x=0$

$\sin x(1+2\cos x)=0$

$\sin x_1=0$

$x_1=k\pi$

$2\cos x=-1$

$\cos x=-\frac{1}{2}$

$x_2=\frac{2\pi}{3}+2k\pi$

$x_3=\frac{4\pi}{3}+2k\pi$

e) $\sin^2 x-2\cos^2 x+\sin x=-\frac{3}{4}$

$\sin^2 x-2+2\sin^2 x+\sin x=-\frac{3}{4}$

$3\sin^2 x+\sin x-\frac{5}{4}=0$

$t=\sin x$

$3t^2+t-\frac{5}{4}=0 \quad /\cdot 4$

$12t^2+4t-5=0$

$t_{1,2}=\frac{-4\pm\sqrt{16+4\cdot 5\cdot 12}}{24} \Rightarrow t_1=-\frac{5}{6}, \; t_2=\frac{1}{2}$

$t_1=\sin x=-\frac{5}{6}$

$x_1=\arcsin\left(-\frac{5}{6}\right)$

$x_2=\pi-\arcsin\left(-\frac{5}{6}\right)$

$t_2=\sin x=\frac{1}{2}$

$x_3=\frac{\pi}{6}+k2\pi$

$x_4=\frac{5\pi}{6}+k2\pi$